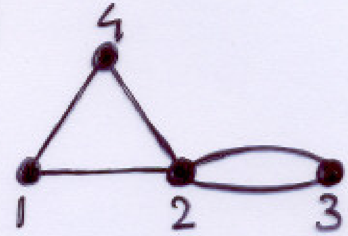
Good Will Hunting

Prijsvraag

G is de graaf



Vind:

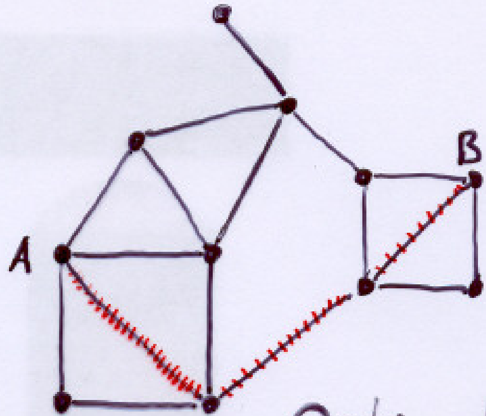
1) de structuurmatrix A

2) de matrix van aantallen 3-staps wandelingen

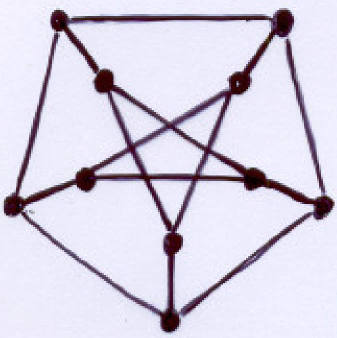
3) de genererende functie voor wandelingen van punt i naar punt j

4) de genererende functie voor wandelingen van 1 naar 3

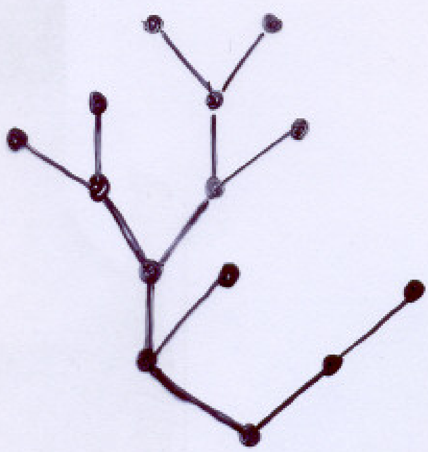
Grafen



Optimalisatie
Kortste weg

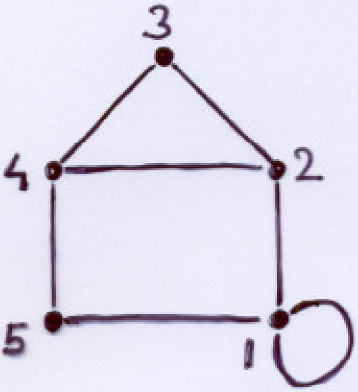


Symmetrische grafen
Algebra
Coderings theorie



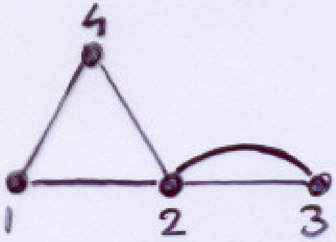
Datastructuren
Zoekbomen
Datacompressie

Structuurmatrix



$\longleftrightarrow$

	1	2	3	4	5
1	1	1	0	0	1
2	1	0	1	1	0
3	0	1	0	1	0
4	0	1	1	0	1
5	1	0	0	1	0



$\longleftrightarrow$

	1	2	3	4
1	0	1	0	1
2	1	0	2	1
3	0	2	0	0
4	1	1	0	0

$= A$

Opgave 1 ✓

Matrix algebra

Optellen

$$[A+B]_{i,j} = A_{i,j} + B_{i,j}$$

$$\begin{bmatrix} 8 & 1 & 6 \\ 3 & 5 & 7 \\ 4 & 9 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = \begin{bmatrix} 9 & 3 & 9 \\ 7 & 10 & 13 \\ 11 & 17 & 11 \end{bmatrix}$$

Vermenigvuldigen

$$[AB]_{i,j} = \sum_k A_{i,k} B_{k,j}$$

$$\begin{bmatrix} 2 & 1 & 3 \\ 3 & 2 & 1 \\ 1 & 3 & 2 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = \begin{bmatrix} 27 & 33 & 39 \\ 18 & 24 & \boxed{30} \\ 27 & 33 & 39 \end{bmatrix}$$

$$3 \cdot 3 + 2 \cdot 6 + 1 \cdot 9 = 30$$

$$A \cdot (B+C) = AB + AC$$

$$(AB)C = A(BC)$$

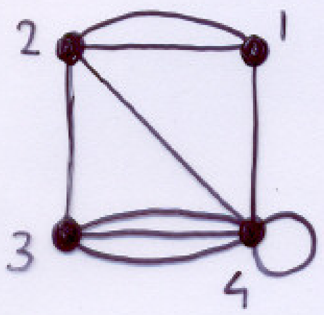
$AB \neq BA$ i.h.a.

$$O = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Wandelingen

G



Vb. $1 \to 2 \to 4 \to 4 \to 3 \to 4$

wandeling met 5 stappen.

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 2 & 0 & 1 \\ 2 & 2 & 0 & 1 & 1 \\ 3 & 0 & 1 & 0 & 3 \\ 4 & 1 & 1 & 3 & 1 \end{array} = A$$

A geeft aantal wandelingen met 1 stap.

A^2: aantal wandelingen met 2 stappen

A^3: aantal wandelingen met 3 stappen

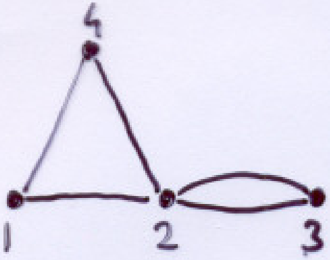
...

Vb. Wandeling 2 stappen van 1 naar 4

$$\begin{array}{lll} 1 \xrightarrow{0} 1 \xrightarrow{1} 4 & A_{1,1} \cdot A_{1,4} & = 0 \\ 1 \xrightarrow{2} 2 \xrightarrow{1} 4 & A_{1,2} \cdot A_{2,4} & = 2 \\ 1 \xrightarrow{0} 3 \xrightarrow{3} 4 & A_{1,3} \cdot A_{3,4} & = 0 \\ 1 \xrightarrow{1} 4 \xrightarrow{1} 4 & A_{1,4} \cdot A_{4,4} & = \underline{1} + \\ & & 3 \end{array}$$

$$\underbrace{\begin{bmatrix} 0 & 2 & 0 & 1 \\ 2 & 0 & 1 & 1 \\ 0 & 1 & 0 & 3 \\ 1 & 1 & 3 & 1 \end{bmatrix}}_{A} \cdot \underbrace{\begin{bmatrix} 0 & 2 & 0 & 1 \\ 2 & 0 & 1 & 1 \\ 0 & 1 & 0 & 3 \\ 1 & 1 & 3 & 1 \end{bmatrix}}_{A} = \underbrace{\begin{bmatrix} 5 & 1 & 5 & \boxed{3} \\ 1 & 6 & 3 & 6 \\ 5 & 3 & 10 & 4 \\ 3 & 6 & 4 & 12 \end{bmatrix}}_{A^2}$$

Wandelingen (2)



G

$$\begin{array}{c|cccc|} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 0 & 1 \\ 2 & 1 & 0 & 2 & 1 \\ 3 & 0 & 2 & 0 & 0 \\ 4 & 1 & 1 & 0 & 0 \\ \hline \end{array}$$

A

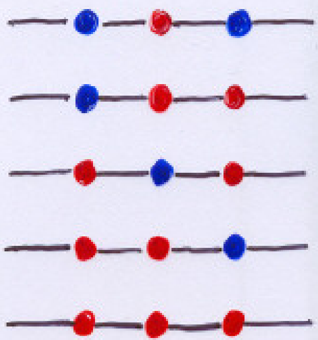
$$A^3 = \begin{array}{c|cccc|} & 1 & 2 & 3 & 4 \\ \hline 1 & 4 & 1 & 6 & 3 \\ 2 & 1 & 2 & 12 & 7 \\ 3 & 6 & 12 & 0 & 2 \\ 4 & 3 & 7 & 2 & 2 \\ \hline \end{array}$$

Opgave 2 ✓

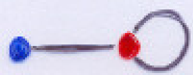
Telproblemen

Puzzel: Hoeveel kettingen van 10 rode of blauwe kralen zijn er, waarbij geen twee blauwe kralen naast elkaar zitten?

Vb.



Hint:



Telproblemen

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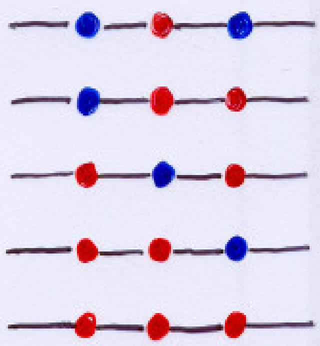
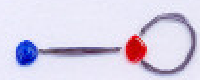


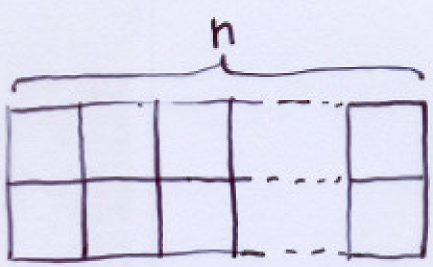
Hint:

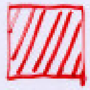
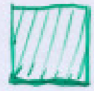


Oplossing:

$$\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^9$$

Variaties: Betegel



met  en  en ,

geén twee aangrenzende tegels van gelijke kleur!

etc.

Machtsverheffen

$$\underset{\substack{A^0\\(=I)}}{\begin{bmatrix}1&0\\0&1\end{bmatrix}}\quad \underset{A}{\begin{bmatrix}1&1\\1&0\end{bmatrix}}\quad \underset{A^2}{\begin{bmatrix}2&1\\1&1\end{bmatrix}}\quad \underset{A^3}{\begin{bmatrix}3&2\\2&1\end{bmatrix}}\quad \underset{A^4}{\begin{bmatrix}5&3\\3&2\end{bmatrix}}\quad \underset{A^5}{\begin{bmatrix}8&5\\5&3\end{bmatrix}}\quad \underset{A^6}{\begin{bmatrix}13&8\\8&5\end{bmatrix}}\quad \cdots$$

$$\boxed{A^2 - A - I = 0}$$

$x^2 - x - 1$ "karakteristiek polynoom van A"

Gulden snede

Genererende functies

$$f(z) := I + zA + z^2A^2 + z^3A^3 + z^4A^4 + \cdots$$

"Formele som" of "waslijn"

Truc:

1) $$(1-x)(1+x+x^2+x^3+\cdots) =$$
$$\begin{aligned} 1 + x + x^2 + x^3 + \cdots \\ - x - x^2 - x^3 - \cdots \end{aligned}$$
$$= 1$$

Dus $1 + x + x^2 + x^3 + \cdots = \dfrac{1}{1-x}$.

2) Neem $x := zA$

$$\underline{\underline{f(z) = (I - zA)^{-1}}}$$

3) $$f(z) = \begin{bmatrix} 1-z & -z \\ -z & 1 \end{bmatrix}^{-1} = \cancel{\begin{bmatrix} \frac{1}{1-x-x^2} & \frac{x}{1-x-x^2} \\ \frac{x}{1-x-x^2} & \frac{1-x}{1-x-x^2} \end{bmatrix}}$$

$$= \begin{bmatrix} \frac{1}{1-z-z^2} & \frac{z}{1-z-z^2} \\ \frac{z}{1-z-z^2} & \frac{1-z}{1-z-z^2} \end{bmatrix}$$

Staartdeling

Het aantal kettingen is gecodeerd in $\frac{1}{1-x-x^2}$

Hoe? Staartdeling!

$$1-x-x^2 \,/\, 1+0x+0x^2+\dots \,\backslash\; 1+1x+2x^2+3x^3+5x^4+\dots$$

$$\begin{array}{r}
\underline{1-x-x^2} \\
x+x^2 \\
\underline{x-x^2-x^3} \\
2x^2+x^3 \\
\underline{2x^2-2x^3-2x^4} \\
3x^3+2x^4 \\
\underline{3x^3-3x^4-3x^5} \\
5x^4+3x^5
\end{array}$$

```
In[14]:= A := {{1, -x, 0, -x}, {-x, 1, -2 x, -x}, {0, -2 x, 1, 0}, {-x, -x, 0, 1}};

In[15]:= MatrixForm[A]
```

$$\begin{pmatrix} 1 & -x & 0 & -x \\ -x & 1 & -2x & -x \\ 0 & -2x & 1 & 0 \\ -x & -x & 0 & 1 \end{pmatrix}$$

```
In[16]:= MatrixForm[Inverse[A]]
```

$$\begin{pmatrix} \frac{1-5x^2}{1-7x^2-2x^3+4x^4} & \frac{x+x^2}{1-7x^2-2x^3+4x^4} & \frac{2x^2+2x^3}{1-7x^2-2x^3+4x^4} & \frac{x+x^2-4x^3}{1-7x^2-2x^3+4x^4} \\ \frac{x+x^2}{1-7x^2-2x^3+4x^4} & \frac{1-x^2}{1-7x^2-2x^3+4x^4} & \frac{2x-2x^3}{1-7x^2-2x^3+4x^4} & \frac{x+x^2}{1-7x^2-2x^3+4x^4} \\ \frac{2x^2+2x^3}{1-7x^2-2x^3+4x^4} & \frac{2x-2x^3}{1-7x^2-2x^3+4x^4} & \frac{1-3x^2-2x^3}{1-7x^2-2x^3+4x^4} & \frac{2x^2+2x^3}{1-7x^2-2x^3+4x^4} \\ \frac{x+x^2-4x^3}{1-7x^2-2x^3+4x^4} & \frac{x+x^2}{1-7x^2-2x^3+4x^4} & \frac{2x^2+2x^3}{1-7x^2-2x^3+4x^4} & \frac{1-5x^2}{1-7x^2-2x^3+4x^4} \end{pmatrix}$$

```
In[17]:= Series[(2 x^2 + 2 x^3) / (1 - 7 x^2 - 2 x^3 + 4 x^4), {x, 0, 10}]
```

Out[17]= $2x^2 + 2x^3 + 14x^4 + 18x^5 + 94x^6 + 146x^7 + 638x^8 + 1138x^9 + 4382x^{10} + O[x]^{11}$

$\boxed{\text{Opgave 3,4 ✓}}$