

Course WI4221 Control of Discrete-Time Stochastic Systems in the Spring Semester of 2025

Homeworkset 1

13 February 2025 (Date Homework Set issued)

20 February 2025 (Date solution due;

however, contact the lecturer on 20 February at the latest, if you have trouble meeting this due date.)

See also Section 2.11 Exercises, of the lecture notes.

1. Exercise 2.11.3. From a conditional Gaussian measure to a random variable representation
2. Exercise 2.11.6. The concept of a sufficient statistic and conditional independence.
3. Consider a tuple of finite-valued random variables,

$$X = \{1, 3\} = \{a_1, a_2\} \subset \mathbb{R}, n_x = 1, n_{i_x} = 2, x : \Omega \rightarrow X,$$

$$Y = \{2, 4, 6\} = \{b_1, b_2, b_3\} \subset \mathbb{R}, n_y = 3, n_{i_y} = 3, y : \Omega \rightarrow Y,$$

$$1/12 = P(\{\omega \in \Omega \mid x(\omega) = a_1, y(\omega) = b_1\}),$$

$$2/12 = P(\{\omega \in \Omega \mid x(\omega) = a_1, y(\omega) = b_2\}),$$

$$1/12 = P(\{\omega \in \Omega \mid x(\omega) = a_1, y(\omega) = b_3\}),$$

$$3/12 = P(\{\omega \in \Omega \mid x(\omega) = a_2, y(\omega) = b_1\}),$$

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- (a) State the simple variable representations of x and of y and compute their parameters.
- (b) Calculate m_{i_x} , m_{i_y} , $E[x]$, and $E[y]$.
- (c) Are x and y independent random variables?
- (d) Calculate and compute the conditional expectation $E[x \mid F^y]$ in terms of a simple variable representation.

See other side for Reading Advice.

Reading Advice

The lecture notes in pdf format will be emailed to you if you have provided your email address to the lecturer by email or participated in the first lecture.

Lecture 1 Presented on 13th February 2025.

For this lecture you are advised to read of the lecture notes:

- Chapter 1, Section 1.1 or 1.3; and
- Chapter 2, primarily:
Section 2.7 Gaussian Random Variables and
Section 2.8 Conditional Expectation.

A reader who is interested in more probability theory is advised to read other sections of Chapter 2. Chapter 19 (Appendix C) contains additional theory on probability but the topics presented in that appendix are of an advanced level.

Lecture 2 To be presented on 20 February 2025. If you like to read about the topic to be discussed during Lecture 2 before the lecture is to be presented, then you could read parts of the Sections 3.1–3.3 of Chapter 3 of the lecture notes.

It should be clear that this reading is not required.