Delft University of Technology

Project: Block Preconditioners for Monolithic Solvers of Very Large Floating Structures

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Motivation

There is a growing interest in **understanding the behavior of floating structures in offshore environments**. Some examples of applications where one can find a large floating structure are, for instance: floating solar farms, floating modular cities, or floating large infrastructures such as airports.

For engineering purposes, these examples can be modeled by means of a floating elastic plate in waves governed by potential flow theory. The elastic plate follows a transient bi-harmonic equation on the plate domain, while the flow is modeled by the Laplacian equation on the fluid domain; see [1]. This leads to a set of **coupled partial differential equations (PDEs) with variables defined in different topological dimensions**, that is, the flow potential is defined in the three-dimensional fluid domain and is coupled with the plate vertical motion, defined in the two-dimensional plate domain.

Figure 1: Floating photovoltaic farm (left), modular floating artificial islands (center), floating airport (right).

For simple geometries and simplified conditions, one can use semi-analytical methods to determine the motion of the structures under the effect of waves. However, for **complex geometries, several interacting modules, or complex wave configurations**, there is a high demand for **robust and efficient numerical solvers** to estimate the complex structural motion.

Problem definition

The problem can be simplified as follows. The flow potential, ϕ, is described by the **Laplacian boundary value problem**,

$$\begin{aligned} \Delta\phi &= 0 && \text{in } \Omega, \\ n\cdot\nabla\phi &= 0 && \text{on } \Gamma_N, \\ n\cdot\nabla\phi &= \partial_t\eta && \text{on } \Gamma_b. \end{aligned} \tag{1}$$

The structural vertical motion, η, satisfies the **beam/plate equation and the dynamic boundary condition**,

$$\begin{aligned} \rho_b h_b \partial_{tt}\eta + D\Delta^2\eta = p \qquad && \text{on } \Gamma_b, \\ p = -\rho_w\partial_t\phi - \rho_w g\eta && \text{on } \Gamma_b, \end{aligned} \tag{2}$$

where p is the pressure at the fluid-structure interface, ρ_w is the water density, g is the gravity acceleration, and ρ_b, h_b, and D are the structural density, thickness and rigidity, respectively.

Typical approaches to solve this type of problems use **segregated algorithms**, where the two separate problems for the fluid (1) and the structure (2) are solved independently and are coupled via an iterative process [4]. Depending on the material properties, the segregated approach leads to highly nonlinear systems. Alternatively, one can solve the problem using a **monolithic approach**, where all the coupled PDEs are solved at once, preserving the linear nature of the problem. However, this approach results in larger linear systems, which are generally more challenging to solve using iterative methods; see [2, 3] for comparisons of segregated and monolithic fluid-structure interaction algorithms.

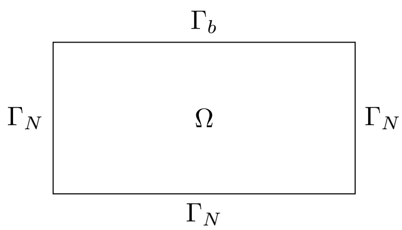

Figure 2: Simplified computational domain, Ω, with Neumann-type boundaries, Γ_N, and floating beam interface, Γ_b.

The goal of this project is to derive **efficient block-preconditioners for iterative solvers for the mixed-dimensional monolithic PDE system arising from modeling the interaction of floating structures in potential flow**. The developments will be done using **Gridap.jl** [5], a finite element software framework for the solution of PDEs in the **Julia programming language**. During the project, a **collaboration with Mocean**[1], marine engineering consultancy firm specialized in hydrodynamic and structural analysis of structures in offshore environments, is possible and is also aspired.

[1] https://www.mocean-offshore.com

Tasks

- Familiarize with the modeling of floating structures in potential flow and the resulting system of PDEs; see [1] for a broad overview of this problem.
- Install and familiarize with the Julia programming language[2] and the FEM package Gridap.jl[3].
- Familiarize with the implementation of the coupled PDEs (1) and (2) in Gridap.jl and perform a parametric study.
- Literature research on block preconditioners for coupled PDE problems.
- Implementation with Julia and investigation of the convergence properties and performance of different block preconditioners for the fully coupled monolithic formulation of the problem.

Contact

If you are interested in this project and/or have further questions, please contact Alexander Heinlein, a.heinlein@tudelft.nl, and Oriol Colomés, j.o.colomesgene@tudelft.nl.

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[2] https://julialang.org
[3] https://github.com/gridap/Gridap.jl