

Parametric Model Reduction for Imperfect Structures

Note: This thesis is offered in collaboration with mechanical engineering, Politecnico di Milano

Background: Complex mechanical structures are typically analysed using finite element (FE) models with hundreds of thousands to millions of degrees of freedom (DOFs). Computing their nonlinear dynamic response can be prohibitively expensive, as internal forces and tangent stiffness matrices must be repeatedly evaluated during time integration. Reduced-order models (ROM) address this challenge by describing the dynamics through a small number of dominant coordinates, extending the idea of modal reduction to nonlinear systems. In particular, spectral submanifolds (SSM) based methods provide a rigorous framework for constructing nonlinear ROMs and efficiently computing the frequency response of large FE models.

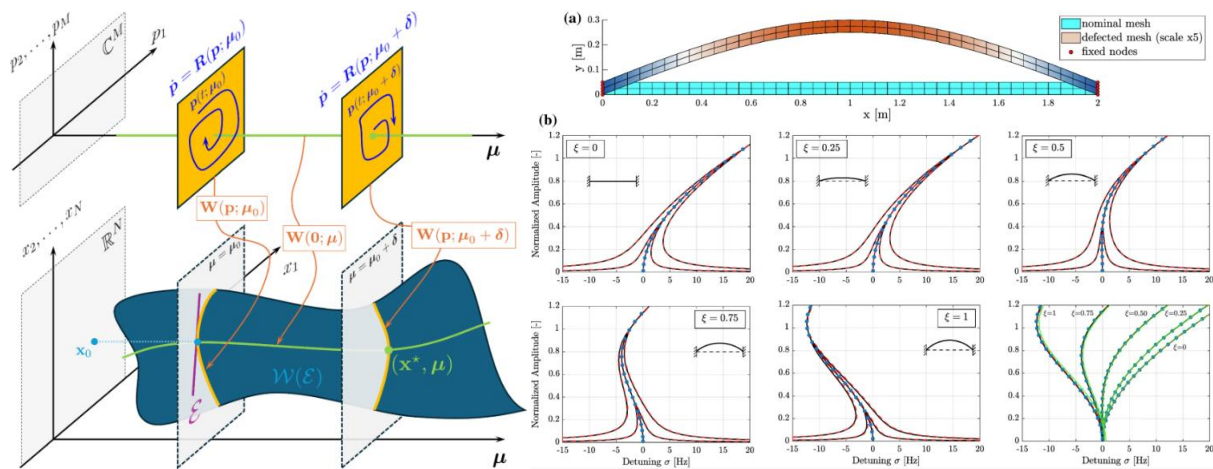


Figure: Geometric illustration of a parametric SSM [1] (left) Primary resonance of a clamped-clamped beam with an arc-defect, from [2] (right)

Project goal: The system response varies as system parameters change due to various factors. Accounting for these variations is essential to assess whether a nominal design remains effective in the presence of manufacturing imperfections. Even small geometric defects can substantially alter the nonlinear frequency response, as demonstrated in [2]. Predicting their influence is particularly relevant to MEMS devices, whose performance can be highly sensitive to geometric imperfections.

While SSM-based ROMs enable efficient simulations, their construction can still be computationally demanding. A recently developed parametric formulation addresses this limitation by incorporating parameter dependence into the reduced model [1]. A single ROM can thus be used to explore multiple configurations without repeating the reduction for each parameter value.

This master's thesis aims to combine a geometric parametrization inspired by [2] with the parametric SSM formulation developed in [1]. The student will develop, implement and validate the resulting approach on representative FE models, assessing its accuracy and computational efficiency. The goal is to enable rapid exploration of geometric variations and their effects on nonlinear dynamics.

References

- [1] Jain, S., Li, M. Computing parameter-dependent invariant manifolds for data-free nonlinear model reduction. *Nonlinear Dyn* **114**, 1012 (2026). <https://doi.org/10.1007/s11071-026-12814-z>
- [2] Marconi, J., Tiso, P., Quadrelli, D.E. *et al.* A higher-order parametric nonlinear reduced-order model for imperfect structures using Neumann expansion. *Nonlinear Dyn* **104**, 3039–3063 (2021). <https://doi.org/10.1007/s11071-021-06496-y>