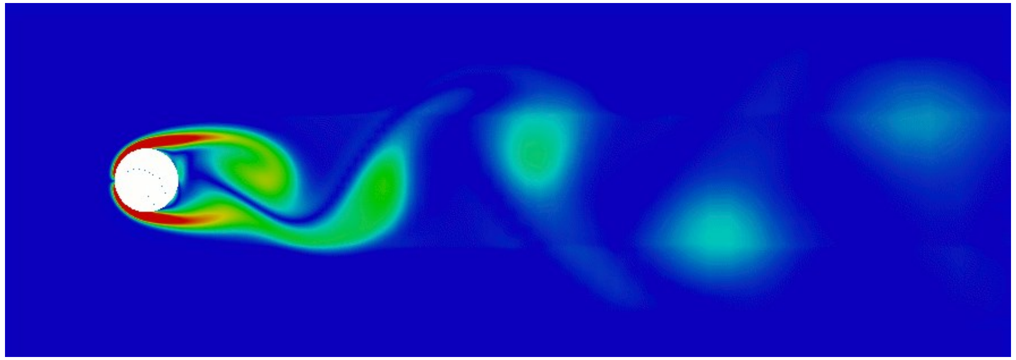
Fast simulation of fluid flows via nonlinear model reduction

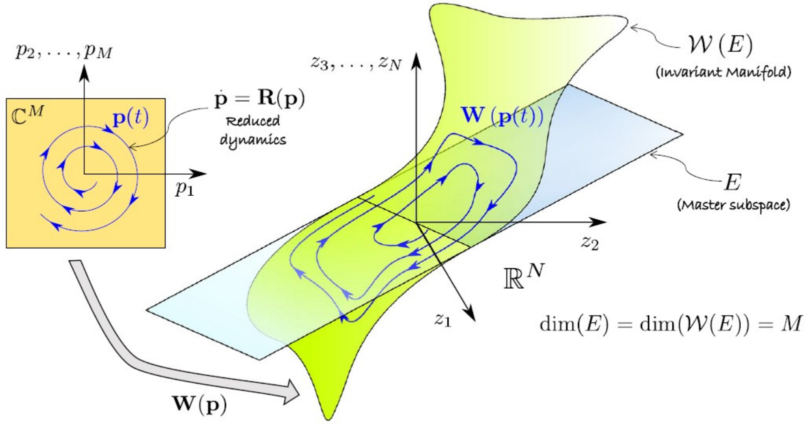
Background: Fluid flows are among the most complex physical phenomena and accurately modeling them often results in high-dimensional governing equations that are computationally challenging to simulate. Techniques based on dynamical systems theory offer rigorous, data-free reduced-order models (ROMs), enabling fast simulations of such complex nonlinear behavior. These techniques involve computing low-dimensional invariant manifolds (surfaces) from the governing equations. Importantly, these manifolds contain full system trajectories, and the resulting ROMs are thus guaranteed to preserve the underlying physical structure—such as mass conservation imposed by the continuity equation.

In this project, you will develop a structure-preserving numerical model for incompressible, nonlinear fluid flow and apply one or more nonlinear model reduction techniques to simplify the model you create.

Learning objectives: At the end of this project, you will be able to

- numerically model the incompressible Navier Stokes equation to accurately predict nonlinear fluid phenomena such as vortex shedding behind a cylinder [1],
- explain the importance of reduced-order models/invariant manifolds in analyzing nonlinear systems,
- compute invariant manifolds in spatially discretized PDEs using open-source packages [2, 3],
- use reduced dynamics to predict nonlinear phenomena in the full system,
- critically evaluate your results in comparison to other modelling and reduction techniques in the literature,
- identify key structures in the equations for the construction of an accurate ROM.





Relevant literature

1. Carini, Auteri, Giannetti (2015) Centre-manifold reduction of bifurcating flows. Journal of Fluid Mechanics. https://doi.org/10.1017/jfm.2015.3
2. Li, Thurnher, Xu, Jain (2025) Data-free non-intrusive model reduction for nonlinear finite element models via spectral submanifolds, Computer Methods in Applied Mechanics and Engineering, https://doi.org/10.1016/j.cma.2024.117590
3. Jain, Thurnher, Li, Haller (2025), SSMTool, Zenodo https://doi.org/10.5281/zenodo.4614201

Supervisors: Shobhit Jain, Artur Palha, Deepesh Toshniwal

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Learning activities:

- Literature study.
- Structure-preserving spatial discretization of the incompressible Navier-Stokes equation via available software packages
- Predicting nonlinear fluid phenomena such vortex shedding behind a cylinder and comparing results for several discretization choices.
- Computing center-manifolds in the discretized system to obtain ROM depending on the Reynold's number as a parameter using the MATLAB package, SSMTool.
- Comparing the low-dimensional dynamics against full system simulations or other linear reduction methods such as Proper Orthogonal Decomposition in terms of speed and accuracy.

Prerequisites:

- Affinity to programming (e.g., in MATLAB, Julia)
- Enthusiasm about the application of numerical methods to model and analyze real-world problems

Desirable:

- Prior experience with numerical modelling using finite elements
- Basic knowledge of dynamical systems/ODEs

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