# Weakly-$C^1$ solutions for Kirchhoff Plates.

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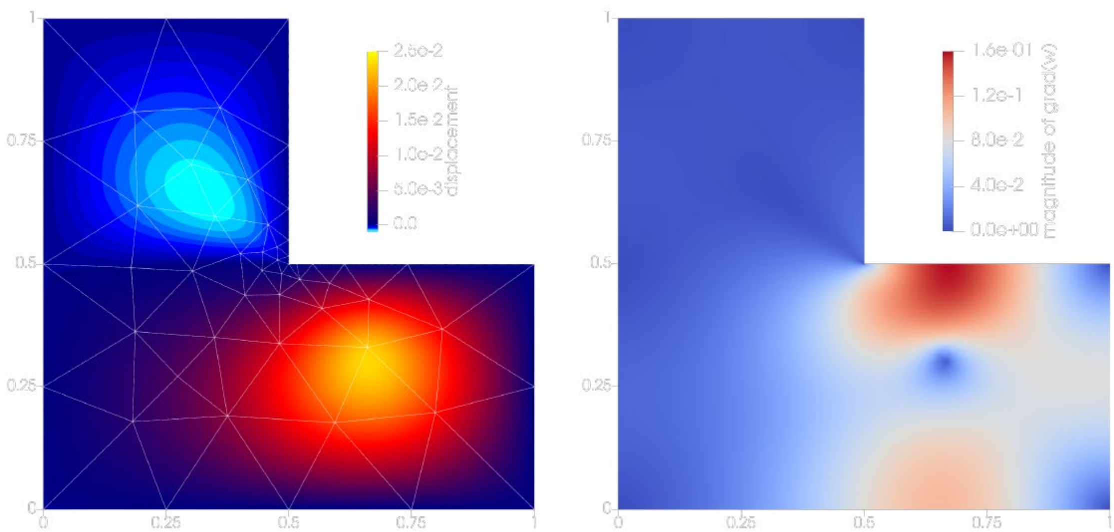
Feel free to contact the supervisors if you have any questions about the project.

During the design of real-world structures, it is important to understand the behaviour of a structure under given loads. There are various models to this end, one of which is the Kirchhoff plate model. From a mathematical point of view, the Kirchhoff plate model is a higher-order partial differential equation (PDE). The picture below shows an example solution of the Kirchhoff plate model computed in [1].




(a) (b)

The higher-order nature of the model means that direct finite element discretisations require $C^1$ smooth basis functions. These basis functions can easily be constructed using b-splines on simple geometries. However, when dealing with more complicated geometries, this becomes very difficult.

The supervisors recently developed a method to handle complex geometries in the context of the biharmonic problem (a simple higher-order PDE) [2]. This work is based on the method introduced by Ainsworth & Parker [1], which computes $C^1$ solutions while only using $C^0$ smooth basis functions. The method of Ainsworth & Parker [1] is limited to relatively simple geometries. The work of the supervisors, however, can handle complex geometries, though the solutions are no longer $C^1$ smooth.

The main goal of this project is to extend the supervisor's method to Kirchhoff plates. If this goes smoothly (pun intended), extensions to other models/materials/surfaces can be considered. While there will always be an implementational aspect to the project, there are also options to focus more on the theoretical understanding of the method.

## Milestones

- Literature and background knowledge:
  - Understanding (mixed) finite elements.
  - Working with b-spline finite elements (IGA).
  - Literature study on (mixed) finite elements for Kirchhoff plates.
  - Familiarising with the works by Ainsworth & Parker [1] and the supervisors [2].
- Method development:
  - Set up a computational strategy (based on [1, 2]) to tackle Kirchhoff plate problems.
  - Extend the method to surfaces if possible.
  - Explore options to deal with non-linear materials or models.
  - Develop theoretical tools to show well-posedness and/or error estimates in one of the above cases.
- Implementation and testing:
  - Implement the method in the open-source Julia package Mantis.jl.
  - Test the method on test cases of varying complexity.
- Documentation and thesis:
  - Write a thesis about the work.

## Prerequisites

- Basic knowledge of numerical methods and partial differential equations.
- Proficiency in at least one programming language, such as Julia, Python, or MATLAB.
- Familiarity with finite elements.

## References


[1] Ainsworth, M., & Parker, C. (2024). Computing $H^2$-Conforming Finite Element Approximations Without Having to Implement $C^1$-Elements. *SIAM Journal on Scientific Computing*, *46*(4), A2398–A2420. https://doi.org/10.1137/23M1615486

[2] Dekker, J, Palha, A., & Toshniwal, D. (2026). Weakly-$C^1$ Solutions to the Biharmonic Problem on Multi-Patch Domains. Manuscript available on request.