Master project

Fractional wave equations: analysis and approximations

Supervisors:

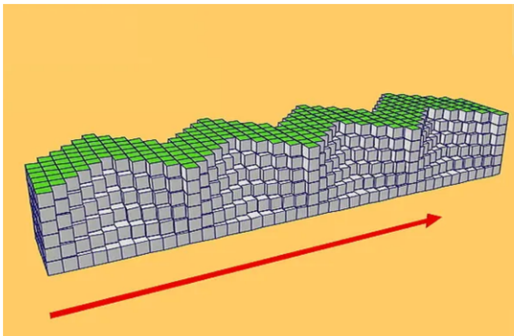
Kateryna Marynets (*Mathematical Physics group, DIAM*; K.Marynets@tudelft.nl)
Deepesh Toshniwal (*Numerical Analysis group, DIAM*; D.Toshniwal@tudelft.nl)

Background.

Fractional calculus has emerged as a powerful tool in mathematical modeling, garnering increasing attention from applied mathematicians and natural scientists in recent decades. Dynamical systems that incorporate fractional-order derivatives effectively capture *memory effects* and *anomalous diffusion*, offering a more nuanced representation of physical phenomena. Thanks to the nonlocal nature of fractional differential operators, they are particularly useful for modeling processes such as fluid flow through porous media (e.g., groundwater flow or filter), wave propagation on thin films and on an elastic medium. Besides the areas of applicability, one should also mention two additional advantages of fractional derivatives: the wide variety of their types, and the ability to adjust their order. This provides greater flexibility compared to traditional integer-order models. However, the nonlocal characteristics of fractional-order operators, along with the inherent nonlinearity of the models, pose significant challenges in both computational and analytical aspects. These complexities drive the need for advanced analytical and numerical methods to solve fractional initial and boundary value problems (IVPs and BVPs), enabling more accurate approximation and analysis of solutions.

Project description and methodology.

In this project you will derive new mathematical models for wave propagation, using fractional PDEs with Hilfer fractional derivatives. The main aim is to analyse the developed models, and to construct numerical methods for approximation of their solutions. In particular, you will construct *structure-preserving numerical methods* by studying the variational structure underlying the continuous problem, and developing approximation methods which preserve that structure in a discrete setting [4].



Sourse:
https://www.britannica.com/science/seismic-wave

In this project you will:

1. Derive a variational model for wave propagation on the elastic medium using the generalized Hilfer derivative.
2. Give solvability analysis of the problem; derive exact solutions, if possible.
3. Develop high-order accurate variational integrators for approximating the solutions of the problem.

References

1. Y. Zhou, Fractional diffusion and wave equations: well-posedness and inverse problems, 2024.
2. G. Oruc, H. Borluk, G.M. Muslu, The generalized fractional Benjamin-Bona-Mahony equation: analytical and numerical results, Physica D, 409 (2020), 132499; doi: https://doi.org/10.1016/j.physd.2020.132499.
3. J.A. Pava, Stability properties of solitary waves for fractional KdV and BBM equations, Nonlinearity 31, 920 (2018); 10.1088/1361-6544/aa99a2.
4. K. Hariz, F. Jimnez, and S. Ober-Blbaum. Fractional variational integrators based on convolution quadrature. arXiv preprint arXiv:2403.18362 (2024).