Efficient Computation of New Space-Time Norms for the Wave Equation

Supervisor: Carolina Urzúa-Torres.

The numerical solution of evolution partial differential equations (PDEs) is usually based on semi-discretizations, either in space (method of lines), or in time (Rothe method). As a consequence, the resulting linear systems inherit a tensor product structure of space and time variables, which typically induces that their solution method is sequential in time. In contrast, space-time discretizations treat time just as another space variable. This allows for adaptivity and parallelization in space and time simultaneously without additional effort [?, ?]. Given today's computing capabilities, this feature is of great advantage, as it paves the way to develop algorithms that can use large numbers of cores more efficiently.

Indeed, space-time methods for evolution PDEs have been gaining popularity in recent years. We are particularly interested in numerical methods that can solve transient wave equations in an accurate and computationally efficient manner, as they may also give us insight on how to deal with other hyperbolic PDEs. Many approaches have been devised recently, but so far the only one that is amenable to FEM/BEM coupling is the one from [1, 2]. There, instead of working on standard Sobolev spaces, one looks for solutions in more general spaces that are usually denoted by $\mathcal{H}_{0,}(Q)$ and $\mathcal{H}_{0,}(\Sigma)$.

The goal of this master project is to further understand these approaches by finding an efficient implementation of the $\mathcal{H}_{0,}$-norms and measuring convergence error with respect to them.

Research description

Within this master project the following tasks are foreseen:

1. Research literature on space-time methods for the wave equation.
2. Understand the $\mathcal{H}_{0,}(Q)$ and $\mathcal{H}_{0,}(\Sigma)$.
3. Use recent characterizations of the $\mathcal{H}_{0,}$-norms to come up with an efficient implementation of them.
4. Run numerical experiments and compare the new approach with existing ones.
5. Master thesis.

No previous knowledge on boundary integral equations or BEM is needed, but the student is expected to be willing to learn some key notions, and to have basis knowledge of numerical methods, discretization schemes, finite elements, and programming.

During the project, the student will have access to unpublished ongoing work further explaining the framework and may use existing Python implementations.

References

[1] O. Steinbach, C. Urzúa-Torres: *A new approach to space-time boundary integral equations for the wave equation*, SIAM J. Mathematical Analysis 54 (2), 1370-1392 (2022).

[2] O. Steinbach, M. Zank: *A generalized infsup stable variational formulation for the wave equation*. Journal of Mathematical Analysis and Applications 505 (2023) 235457.