Spectral Methods for Fractional Difussion

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Many real-world processes exhibit anomalous diffusion that slower or faster spreading than predicted by standard diffusion equations. Examples include: Transport in heterogeneous porous media; charge movement in disordered materials; subdiffusion in biological cells; information propagation in complex networks; and viscoelastic flow and memory-dependent materials.

These systems display non-locality and memory effects — characteristics that can be accurately described using fractional-order derivatives. Fractional diffusion equations provide a richer and more realistic mathematical framework, but the increased model fidelity comes with greater analytical and computational challenges.

Analytical solutions are known only for a limited set of idealized cases. Therefore, numerical methods play a crucial role in enabling the practical application of fractional diffusion models. Advancing numerical methods for fractional diffusion contributes directly to the ability to simulate and understand complex systems where classical methods fail. As demand grows for accurate modeling in emerging fields—such as biomedical engineering, geophysics, and complex materials—this research becomes increasingly impactful.

In this project, we aim to develop a spectral method for fractional difussion (based on the integral fractional Laplacian). If you have never heard of spectral methods, you may think of them as a finite element method but that instead of using the usual (locally supported) finite element basis, you discretize with a different basis that obtains exponential convergence. Such a basis is usually composed by orthogonal polynomials or special functions, like Chebyshev polynomials or Legendre functions, which are globally supported.

Research description

Within this master project the following tasks are foreseen:

1. Research literature on numerical methods for fractional difussion.
2. Develop a novel spectral method for fractional difussion based on unpublished work by the Supervisor.
3. Run numerical experiments and compare the new approach with existing ones.
4. Master thesis.

The student is expected to have basic knowledge of numerical methods, finite elements, and programming. Previous knowledge about special functions is not required but the student is expected to be willing to learn about this.

During the project, the student will have access to unpublished ongoing work further explaining the framework.

References

[1] A. Bonito, J.P. Borthagaray, R. Nochetto, E. Otarola, and A. Salgado: *Numerical Methods for Fractional Diffusion*, Comput. Visual Sci. 19, 19–46 (2018).