Master Student Project Proposal

QuadraFEM: Higher-order finite elements for complex geometries

Ye Ji (y.ji-1@tudelft.nl) – EWI HB03.140
Matthias Möller (m.moller@tudelft.nl) – EWI HB03.040
Sham Rane (sham.rane@city.ac.uk – City St George's, University of London)
Ahmed Kovacevic (a.kovacevic@city.ac.uk – City St George's, University of London)

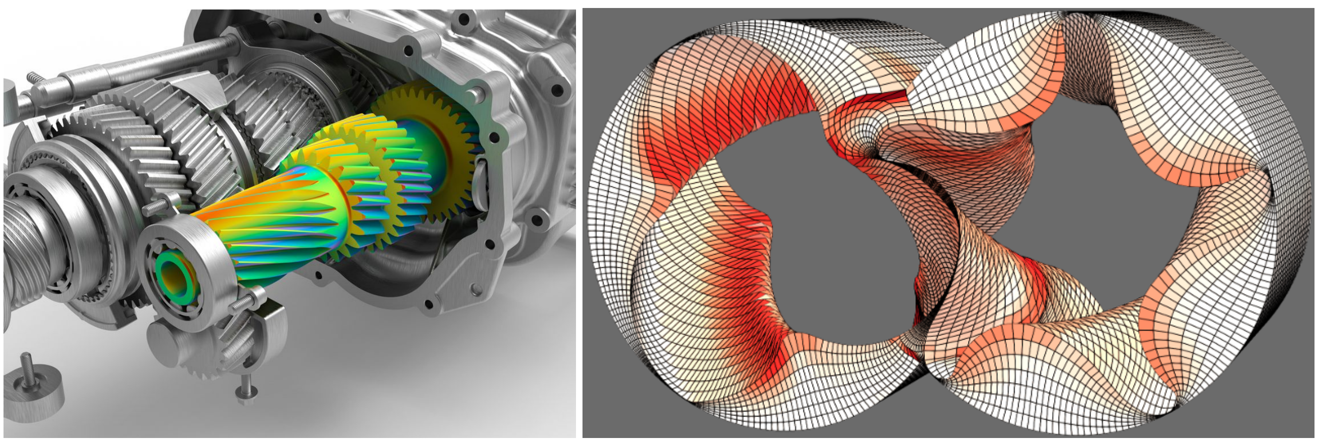

Figure 1: Left: Finite Element Simulation of a Gear Shaft (left, source: https://www.jousefmurad.com/fem/the-finite-element-method-beginners-guide/). Right: High-order structured B-spline based mesh generated by our mesh generator.

Project Description

Finite Element Methods (FEM) have long been a cornerstone in solving complex equations that describe physical systems, especially in engineering and applied sciences [1]. Recent advancements in higher-order finite element techniques have further enhanced the accuracy and efficiency of solving these systems, particularly for problems involving complex geometries and high precision requirements.

This project will focus on the development and implementation of higher-order finite element methods. The goal is to provide efficient and accurate solutions for engineering problems, with a particular focus on compressor tip leakage flows. The study will explore the application of high-order elements to improve the simulation of these flows, which are known to introduce significant challenges due to their intricate boundary conditions and complex flow patterns [2, 3].

By utilizing higher-order FEM, this project aims to achieve a more accurate resolution of flow features in compressor designs without the need for excessive computational resources. The project will also investigate the comparative performance of high-order FEM against traditional lower-order methods, evaluating both accuracy and computational efficiency.

Milestones

- **Preliminary Research and Familiarization**:
  - Deepen understanding of finite element methods, focusing on high-order elements (e.g., Lagrange, Serendipity, and Spectral elements).
  - Review literature focused on FEM in the context of compressor design and fluid flows.
- **Algorithm Development**:
  - Develop a theoretical framework for higher-order FEM in 2D and 3D, tailored to the specific requirements of compressor flow simulations.
  - Formulate appropriate basis functions and meshing strategies for complex boundary flows.
- **Implementation and Testing**:
  - Implement high-order finite elements and shock tracking techniques, possibly using the test case provided by the Cambridge study on tip leakage flows [3].
  - Perform testing on sample benchmarks to compare accuracy and computational efficiency.
- **Documentation and Dissemination**:
  - Compile all research findings, algorithms, and results into a final thesis.

Prerequisites

Basic knowledge of numerical methods and partial differential equations. Proficiency in at least one programming language, such as C/C++, Matlab, or Python, is required. Experience with finite element libraries (e.g., MFEM, deal.ii, G+Smo) is highly beneficial for this project.

References

[1] Solin, P., Segeth, K., & Dolezel, I. (2003). Higher-order finite element methods. Chapman and Hall/CRC.

[2] Patel, B. (2023). Leakage flows and Conjugate Heat Transfer in Rotary Positive Displacement Machines (Doctoral dissertation, City, University of London).

[3] Taylor, J. V., Dickens, A. M., & Simpson, H. (2024). Compressor tip leakage mechanisms. Journal of Turbomachinery, 146(5).