Msc thesis proposal:

Low Rank Tensor Approximation for Chebyshev Interpolation and Applications in Quantitative Finance

Fang Fang[[1]](#footnote-2)

**Background**

Treating high dimensionality is one of the main challenges in the development of computational methods for solving problems arising in finance, where tasks such as option pricing, calibration, and counterparty credit risk assessment need to be performed accurately and in real-time. Among the growing literature addressing this problem, Gass et al. [3] propose a complexity reduction technique for parametric option pricing based on Chebyshev interpolation. As the number of parameters increases, however, this method is affected by the curse of dimensionality.

In [1], the Chebyshev interpolation is extended to treat high-dimensional problems. By exploiting low-rank structures, [1] manages to treat parameter spaces of high dimensions up to 25 (a multi-dimensional Black-Scholes model for basket option). The core of their method is to express the tensorized interpolation in tensor train (TT) format and to develop an efficient way, based on tensor completion, to approximate the interpolation coefficients.

The starting point is the tensorized Chebyshev interpolation of conditional expectations in the parameter and state space, as introduced in [3]. Having observed for a large set of applications that these functions are highly regular, admitting sensitivities of high order or even being analytic, and that the domain of interest can be restricted to a hyperrectangular, Chebyshev interpolation is a promising choice: Its convergence is sub-exponential for multivariate analytic functions, its implementation is numerically stable, and the coefficients are simply given by a linear transformation of the function values at the Chebyshev points.

However, as the number of dimensional of the parameter space increases, the number of Chebyshev points increases exponentially. Thus, even for each dimension we use a number of Chebyshev points as low as n = 3, a problem with d = 20 parameters would need a number of Chebyshev points that is $3^{20}$, and becomes infeasible for numerical implementation.

To avoid the curse of dimensionality, [1] argues that the tensors of the Chebyshev points that are relevant to solve the problems in many financial applications tend to exhibit low-rank structures.

To exploit the low rank structure, [1] adapts a completion algorithm developed in [2] that is designed to work with tensors built and stored in tensor train (TT) format. It starts by computing a small portion of the Chebyshev grid points only. Then it uses the completion algorithm to approximate the tensor for the complete Chebyshev grid by fitting tensors of pre-specified low rank to the provided data points.

**The goal and content of this thesis**

The primary aim of this Msc thesis project is to reproduce the low rank tensor approximation method in [1]. [1] reports some preliminary but promising results for American option under the Heston stochastic volatility model in 5 parameters and basket option the multi-dimensional Black-Scholes model with 25 underlying assets.

As a bonus point, the student may investigate there exists a similar or comparable completion algorithm that works with tensors in other format such as canonical polyadic decomposition (CPD).

**Contact**

Please feel free to contact us directly if this topic is of your interest, or if you would like to learn more details: f.fang@tudelft.nl

**Reference**

1. [Low Rank Tensor Approximation for Chebyshev interpolation in parametric option pricing.](https://epubs.siam.org/doi/abs/10.1137/19M1244172) Kathrin Glau, Daniel Kressner, and Francesco Statti, SIAM Journal on Financial Mathematics, 11(3), 2020.

1. [Riemannian Optimization for High-Dimensional Tensor Completion](https://epubs.siam.org/doi/10.1137/15M1010506), Michael Steinlechner, SIAM Journal on Scientific Computing, 38(5), 2016.

1. [Chebyshev interpolation for parametric option pricing](https://link.springer.com/article/10.1007/s00780-018-0361-y), Maximillian Gass, Kathrin Glau, Micro Mahlstedt, Maximilian Mair, Finance Stochastics, 22:701-731, 2018.
1. Part-time assistant professor at the Applied Mathematics Department of TU Delft; Director of FF Quant Advisory B.V. [↑](#footnote-ref-2)