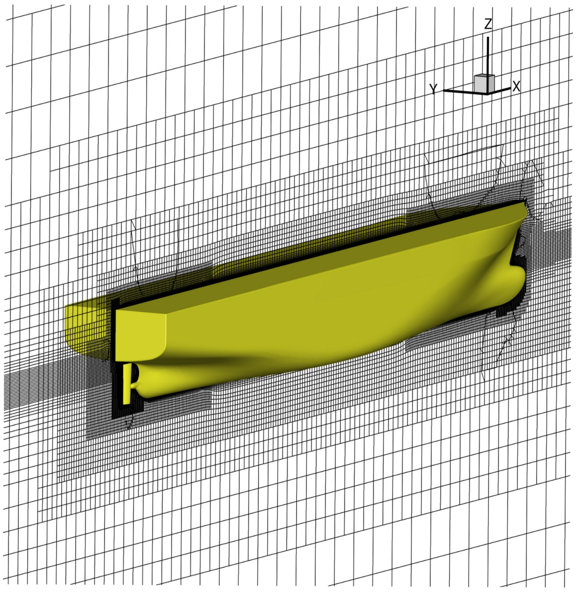
Memo

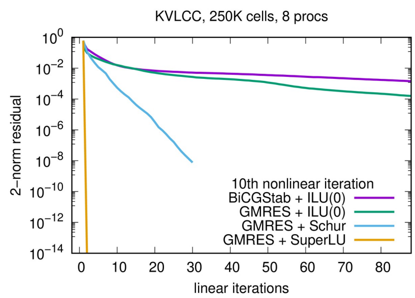
To	TUDelft (C. Vuik)
From	R&D (C.M. Klaij)
Date	November 22, 2024
Project No	–
Subject	MSc project proposal

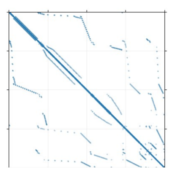
Efficient solver for linear systems in CFD

The goal of this MSc project is to find an efficient solver for the linear systems that arise in Computational Fluid Dynamics (CFD).

The CFD software package in question, ReFRESCO [1], is developed by the Maritime Research Institute Netherlands (MARIN) for applications in the maritime industry. Its research and commercial success depends on the computational speed and costs. In most CFD solvers, the discretized equations for mass and momentum conservation are solved separately. In this so-called segregated approach, the coupling between pressure and velocity is achieved by a SIMPLE-type method. However, for many maritime applications the required number of iterations with this approach is high, leading to expensive computations and large turn-around times. MARIN is therefore developing a coupled solver with higher convergence rate than the standard segregated SIMPLE-type solver. But since the coupled system is more difficult to solve, overall speed-up and cost reduction will only be achieved if an efficient solution method can be found.







$$\begin{bmatrix} A^{uu} & 0 & 0 & A^{up} \\ 0 & A^{vv} & 0 & A^{vp} \\ 0 & 0 & A^{ww} & A^{wp} \\ A^{pu} & A^{pv} & A^{wp} & A^{pp} \end{bmatrix} \begin{bmatrix} u \\ v \\ w \\ p \end{bmatrix}$$

The linear systems arise from the incompressible Navier-Stokes equations at the core of ReFRESCO. These equations are linearized and discretized with a finite volume method for unstructured meshes with variables co-located at the cell centers. The resulting matrix has 4-by-4 blocks per cell corresponding to the three velocity components and the pressure, as described in [2, Sec. 4] and shown in the figure above. The matrix and right-hand side vector can be exported from ReFRESCO and loaded into Matlab or Python for analysis.

For the MSc project, sample matrices and vectors from representative cases will be provided, together with a Python script to load and solve these systems in parallel using PETSc [3]. The systems are solved with Krylov subspace methods such as GMRES [4]. Since their efficiency highly depends on the preconditioner, a good preconditioner will have to be developed using the building blocks available in PETSc or other open-source packages.

Duration

t.b.d.

Profile

Master student in applied mathematics, or other applied science.

Department and supervisor

t.b.d.

References

[1] Maritime Research Institute Netherlands. ReFRESCO Web page. `https://www.marin.nl/en/about/facilities-and-tools/software/refresco`, 2024.

[2] M. Darwish, I. Sraj, and F. Moukalled. A coupled finite volume solver for the solution of incompressible flows on unstructured grids. *Journal of Computational Physics*, 228(1):180–201, 2009.

[3] PETSc: the Portable, Extensible Toolkit for Scientific Computation. PETSc Web page. `https://petsc.org/release/petsc4py/`, 2024.

[4] Yousef Saad. *Iterative methods for sparse linear systems*. SIAM, 2003.