Adaptive Deflated Multiscale Methods
(6 months internship in Boston, USA)

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Project Description:

Preconditioning can be used to damp slowly varying error modes in the linear solver residuals, often corresponding to extreme eigenvalues. Existing multiscale solvers use a sequence of aggressive restriction, coarse-grid correction and prolongation operators to handle low-frequency modes on the coarse grid (see Figure 1). High-frequency errors are then resolved by employing a smoother on fine grid.

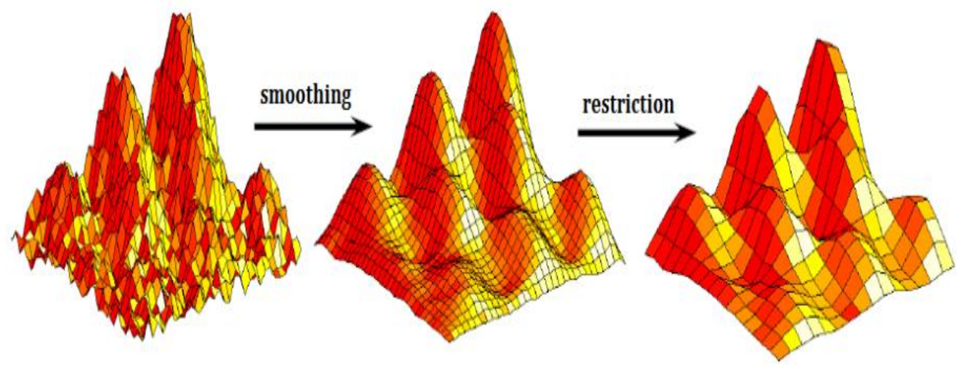

Figure 1: Typical multiscale, multigrid solution steps.

In this project, a various adaptively deflated multiscale solvers (ADMS) will be tested using challenging reservoir examples (e.g., SPE10, see Figure 2).

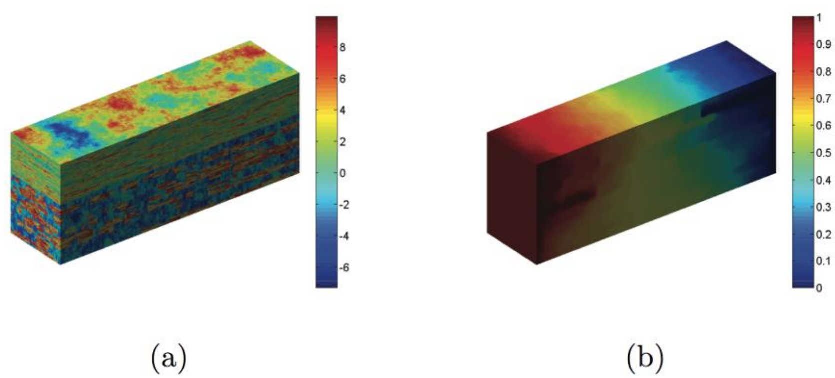

Figure 2: SPE 10 Test case, 1.1 million cells.

A particular attention will be paid to the implementation of the fully implicit ADMS methods (Fully ADMS, Decoupled ADMS, Mixed ADMS). These methods employ an enriched set of basis functions to map between fine and coarse scales. This extended set involves the conventional multiscale local basis functions and globally constructed deflation vectors using Ritz and/or Harmonic Ritz values of the coarse level system prolonged to the fine scale level by using subdomain-levelset deflation vectors (Figure 3).

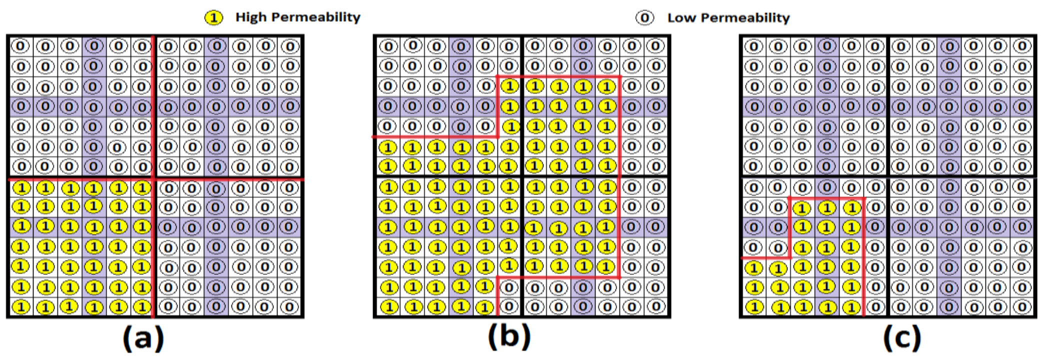

Figure 3: Subdomain (a), levelset (b) and subdomain-levelset deflation (c) .

The Ritz vectors are formed by imposing a Galerkin projection, whereas the Harmonic Ritz vectors are obtained by using the Petrov-Galerkin orthogonality conditions. The Ritz (or Harmonic Ritz) values tend to approximate the eigenvalues of the coarse system. The constraint is that the number of the additional basis functions needs to be small in order to offset the additional cost associated with their calculation. In project, the basis function set is enriched by taking a small number of approximated eigenvectors which are prolonged to the fine-scale and correspond to the smallest Ritz (or Harmonic Ritz) values.

The aim of this project is to develop a rigorous mathematical framework in order to detect the low-frequency modes, instead of relying only on the residual map. This would allow us have a clear understanding of the missing components in our today's state-of-the-art advanced iterative multiscale strategies.

The project consists of the following steps:

- Introduction to multiscale and deflation literature and C++ code.
- Analysis of the different existing enriched-strategy for coarse-scale systems.
- Implementation of the ADMS methods in the C++ code (next generation reservoir simulators).
- Systematic performance study.
- Close collaboration with Schlumberger-Doll Research Scientists.
- Writing and defence of the MSc thesis.