Full-waveform inversion using an efficient frequency-domain solver

Master thesis proposal

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A general PDE-constrained optimization problem is given by,

$$\text{Find} \quad x^* = \underset{x}{\operatorname{argmin}}\, \mathcal{J}(u(x), x) \quad \text{subject to } f(u(x), x) = 0,$$

where an objective function $\mathcal{J}$ is minimized under the constraint that a certain partial differential equation (PDE) f is fulfilled.

In the present setting, we want to minimize the misfit with measured data,

$$\min_{\rho(\mathbf{x}), c_p(\mathbf{x}), c_s(\mathbf{x})} \|\mathbf{u} - \mathbf{u}_{meas}\|, \tag{1}$$

where the optimization parameter x is the union of all physical parameters like density ρ,..., and the displacement vector $\mathbf{u}$ fulfills the time-harmonic elastic wave equation,

$$-\omega_k^2 \rho(\mathbf{x})\hat{\mathbf{u}} - \nabla \cdot \left[\lambda(\mathbf{x})\left(\nabla \cdot \hat{\mathbf{u}}\right) + \mu(\mathbf{x})\left(\nabla\hat{\mathbf{u}} + (\nabla\hat{\mathbf{u}})^T\right)\right] = \hat{\mathbf{s}}, \quad \mathbf{x} \in \Omega, \tag{2}$$

where $\hat{\mathbf{u}} \equiv \hat{\mathbf{u}}(\mathbf{x}, \omega_k) = \mathcal{F}(\mathbf{u}(\mathbf{x}, t))$, λ and μ depend on x, and suitable boundary conditions will be given. A finite element discretization of (2) exists as a Python code using the Python package `nutils`.

A master project would consist of (but is not limited to) the following milestones:

- Literature study: Familiarize with Python as a Scientific Computing language (`numpy, scipy`) and develop a sound background in Continuous Optimization algorithms (such as `BFGS`).
- Development of an iterative method in Python that minimizes the discretized objective function (1) under the contraint (2). This requires the numerical solution of (2) for multiple frequencies ω_k at every iteration.
- Investigate model order reduction techniques such as parameterized `POD` to reduce the computational work to solve (2). Therefore, the discrete version of (2) can be reformulated as a (reduced-order) shifted linear system with parameters ω_k.
- Definition of a feasible benchmark problem, and, *find oil!*

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