

Finite Element Modelling Of Thermal Processes With Phase Transitions

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Steam, Heat and Energy Production



Goal of the master's project

Finite Element Simulation to model thermal processes with phase transitions using density-enthalpy phase diagram.

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Optimize and innovate such thermal processes.

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To apply scientific knowledge with the aim of strengthening the innovative power of industry and government

Outline

- 1 Density-Enthalpy Phase Diagram
- 2 0D Boiler Simulation
- 3 Finite Element Method
- 4 Further Research

Density-enthalpy phase diagram

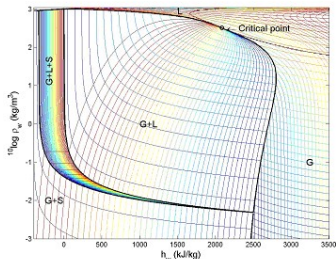


Figure: Density-enthalpy phase diagram for pure water

0D Boiler System

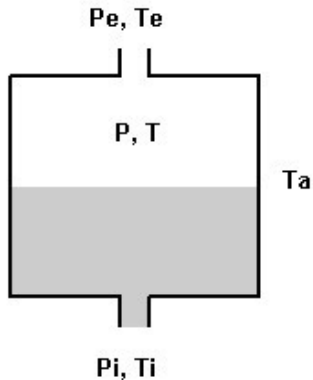


Figure: Boiler

Mass and Heat balances

$$V \frac{d\rho}{dt} = \Phi_i - \Phi_e,$$
$$V \frac{d(\rho h)}{dt} = \Phi_i \cdot h_i - \Phi_e \cdot h_e + Q + V \frac{dP}{dt}.$$

- V Volume [m^3],
 ρ Density [Kg/m^3],
 Φ_i Inflow mass [Kg/s],
 Φ_e Outflow mass [Kg/s],
 Q Heat flow [W],
 h Specific enthalpy [J/Kg],
 P Pressure [Pa].

Mass and Heat balances

$$\Phi_i = A_i \sqrt{2\rho_i(P_i - P)},$$

$$\Phi_e = A_e \sqrt{2\rho_e(P - P_e)},$$

$$Q = AU(T_a - T).$$

A_i Inlet area [m^2],

A_e Exit area [m^2],

T_a Ambient temperature [K],

U Heat transfer coefficient [$W/m^2/K$].

Dry Boiling

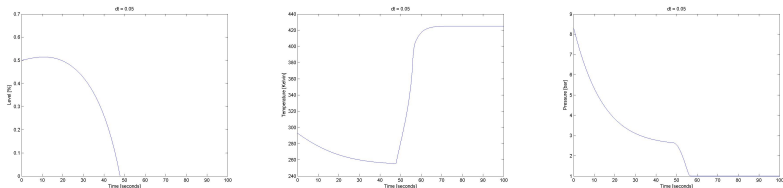


Figure: Level, Temperature and Pressure.

$A_i \ll A_e$, $P_i = 10$ bar, $P_e = 1$ bar, $T_a = 500K$

Over flow

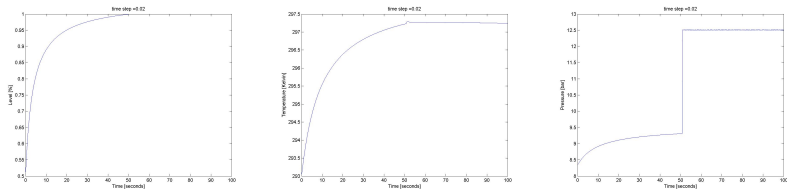


Figure: Level, Temperature and Pressure.

$A_i \gg A_e$, $P_i = 10\text{bar}$, $P_e = 1\text{ bar}$, $T_a = 400\text{K}$

Poisson Equation

Poisson equation in one dimension

$$\begin{aligned} -k \frac{d^2 T}{dx^2} &= f(x), & 0 \leq x \leq \pi \\ T(0) &= 0, \\ \frac{dT}{dx}(\pi) &= 0. \end{aligned}$$

Weak Formulation

- 1 Multiplying by a test function η , satisfying: $\eta(0) = 0$

$$\int_0^\pi \left(k \frac{d^2 T}{dx^2} + f \right) \eta dx = 0,$$

- 2 Integrating by parts

$$- \int_0^\pi k \frac{d\eta}{dx} \frac{dT}{dx} d\Omega + \int_0^\pi \eta f d\Omega + \left[\eta \left(k \frac{dT}{dx} \right) \right]_0^\pi = 0,$$

- 3 Application of BC, $\frac{dT}{dx}(\pi) = 0$ and $\eta(0) = 0$

$$\int_0^\pi k \frac{d\eta}{dx} \frac{dT}{dx} d\Omega = \int_0^\pi \eta f d\Omega.$$

Galerkin Approximation

Galerkin Equations

$$\sum_{j=1}^n \int_0^{\pi} k \frac{d\varphi_i}{dx} \frac{d\varphi_j}{dx} d\Omega = \int_0^{\pi} f \varphi_i d\Omega, \quad i = 1, \dots, n.$$

Matrix Form

$$ST = F,$$

Stiffness Matrix

$$S_{ij} = \int_0^\pi k \frac{d\varphi_j}{dx} \frac{d\varphi_i}{dx} dx,$$

vector F

$$F_i = \int_0^\pi f \varphi_i dx.$$

Numerical Solution

If $f(x) = \sin(x)$, then the exact solution is $T(x) = \frac{1}{k}\sin(x) + \frac{x}{k}$

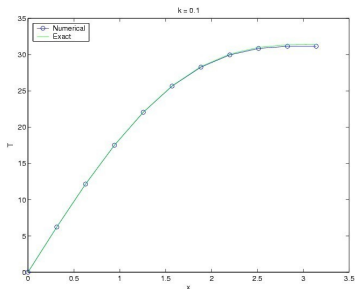


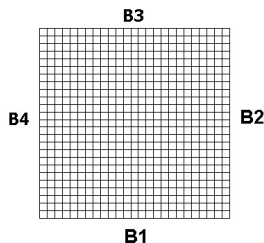
Figure: Exact and Numerical Solution of 1D Poisson Equation, $k = 0.1$

2D Convection Diffusion Equation

$$\frac{\partial T}{\partial t} - \vec{\nabla} \cdot (k \vec{\nabla} T) + (\vec{u} \cdot \vec{\nabla} T) = f$$

- k diffusion term,
- u convection term,
- f source term.

Numerical Solution



On the boundary $B1 \cup B4$: $T|_{B1 \cup B4} = 20$,

On the boundary $B2 \cup B3$: $k \frac{\partial T}{\partial n}|_{B2 \cup B3} = 0$,

At the starting time t_0 : $T|_{t_0} = 20$.

Standard Galerkin Method

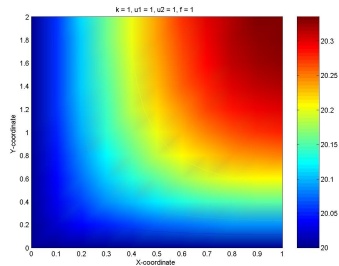
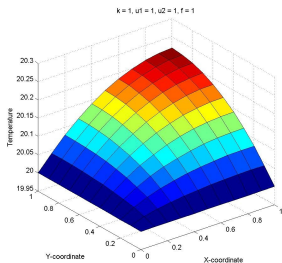
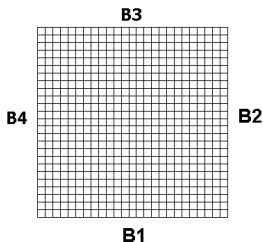


Figure: Numerical solution of 2D time dependent convection diffusion equation using SGA method and implicit scheme, $dt = 0.1$

Numerical Solution



On the boundary $B1 \cup B3$: $k \frac{\partial T}{\partial n} |_{B1 \cup B3} = 0$,

On the boundary $B4$: $T|_{B4} = 20$,

On the boundary $B2$: $T|_{B2} = 40$,

At the starting time t_0 : $T|_{t_0} = 20$.

Standard Galerkin Method

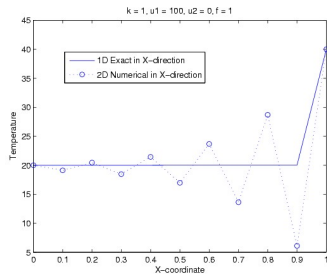
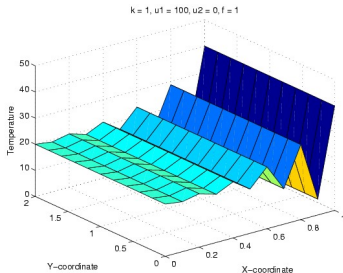


Figure: Numerical solution of 2D time dependent convection diffusion equation using SGA method and implicit scheme after 500 time steps, $dt = 0.1$

Streamline Upwind Petrov Galerkin Method

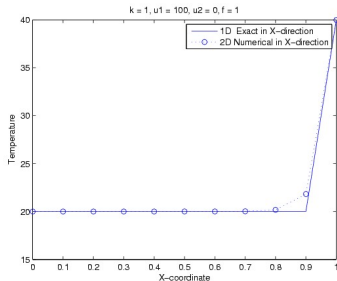
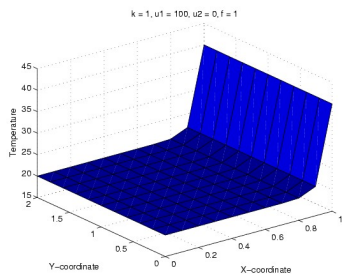


Figure: Numerical solution of 2D time dependent convection diffusion equation using SUPG method and implicit scheme after 500 time steps, $dt = 0.1$

Further Research

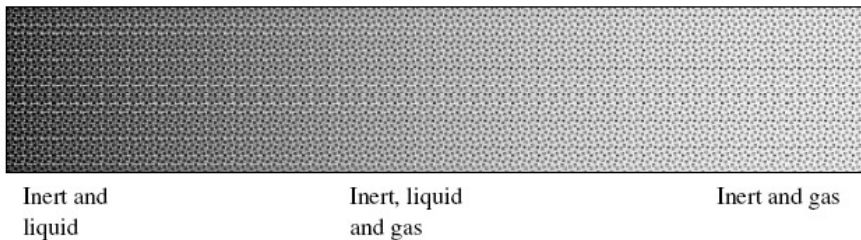


Figure: Porous media

1D Porous media

Solve two coupled nonlinear PDE:

$$\frac{\partial \rho}{\partial t} - \frac{K}{\mu} \frac{\partial P}{\partial x} \frac{\partial \rho}{\partial x} = \rho \frac{\partial}{\partial x} \frac{K}{\mu} \frac{\partial P}{\partial x},$$

$$\rho \frac{\partial h}{\partial t} - \frac{\partial P}{\partial t} - \rho \frac{K}{\mu} \frac{\partial P}{\partial x} \frac{\partial h}{\partial x} = \frac{\partial}{\partial x} \lambda \frac{\partial T}{\partial x} + q.$$

Initial and boundary conditions:

$$P|_{t_0} = P_a,$$

$$T|_{t_0} = T_a,$$

$$\frac{K}{\mu} \frac{\partial P}{\partial x} = k_m (P_a - P),$$

$$\lambda \frac{\partial T}{\partial x} = k_h (T_a - T).$$

1D Porous media

where:

- t_0 starting time [s],
- λ heat conductance coefficient [$W/m/K$],
- μ dynamic viscosity [$Pa.s$],
- ρ density [Kg/m^3],
- K permeability [Darcy],
- k_h heat transfer coefficient [$W/m^2/K$],
- k_m mass transfer coefficient [$Kg/m^2/K$].

Further Research

- 1 Set up 1D model for porous system and solve it using MATLAB.
- 2 Set up 2D and 3D model for porous system and solve them using SEPRAN.

Questions

