Finite Element Modelling Of Thermal Processes With Phase Transitions

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Steam, Heat and Energy Production

Goal of the master's project

Finite Element Simulation to model thermal processes with phase transitions using density-enthalpy phase diagram.

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In order to:

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Finite Element Simulation to model thermal processes with phase transitions using density-enthalpy phase diagram.

In order to:

Optimize and innovate such thermal processes.

Netherlands Organisation for Applied Scientific Research

Outline

Density-enthalpy phase diagram

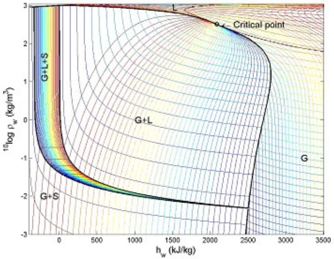


Figure: Density-enthalpy phase diagram for pure water

0D Boiler System

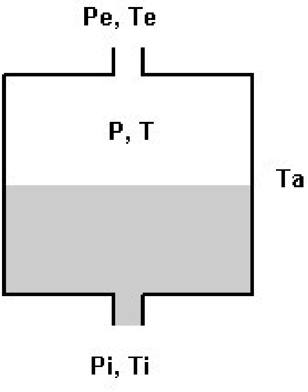


Figure: Boiler

Mass and Heat balances

$$
\begin{aligned}
V\frac{d\rho}{dt} &= \Phi_i - \Phi_e, \\
V\frac{d(\rho h)}{dt} &= \Phi_i.h_i - \Phi_e.h_e + Q + V\frac{dP}{dt}.
\end{aligned}
$$

V	Volume $[m^3]$,
ρ	Density $[Kg/m^3]$,
Φ_i	Inflow mass $[Kg/s]$,
Φ_e	Outflow mass $[Kg/s]$,
Q	Heat flow $[W]$,
h	Specific enthalpy $[J/Kg]$,
P	Pressure $[Pa]$.

Mass and Heat balances

$$\begin{aligned}\Phi_i &= A_i\sqrt{2\rho_i(P_i - P)},\\ \Phi_e &= A_e\sqrt{2\rho_e(P - P_e)},\\ Q &= AU(T_a - T).\end{aligned}$$

A_i	Inlet area $[m^2]$,
A_e	Exit area $[m^2]$,
T_a	Ambient temperature $[K]$,
U	Heat transfer coefficient $[W/m^2/K]$.

Dry Boiling

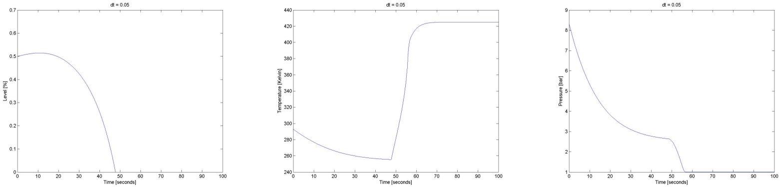


Figure: Level, Temperature and Pressure.
$A_i << A_e, P_i = 10$ bar, $P_e = 1$ bar, $T_a = 500K$

Over flow

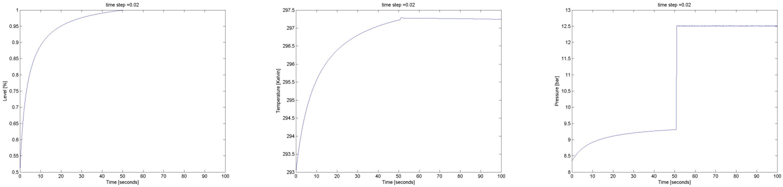


Figure: Level, Temperature and Pressure.
$A_i >> A_e, P_i = 10\text{bar},\ P_e = 1\ \text{bar}, T_a = 400K$

Poisson Equation

Poisson equation in one dimension

$$
\begin{aligned}
-k\frac{d^2T}{dx^2} &= f(x), \quad 0 \leq x \leq \pi \\
T(0) &= 0, \\
\frac{dT}{dx}(\pi) &= 0.
\end{aligned}
$$

Weak Formulation

1. Multiplying by a test function η, satisfying: $\eta(0) = 0$

$$\int_0^{\pi} \left(k \frac{d^2 T}{dx^2} + f \right) \eta dx = 0,$$

2. Integrating by parts

$$-\int_0^{\pi} k \frac{d\eta}{dx} \frac{dT}{dx} d\Omega + \int_0^{\pi} \eta f d\Omega + \left[\eta \left(k \frac{dT}{dx} \right) \right]_0^{\pi} = 0,$$

3. Application of BC, $\frac{dT}{dx}(\pi) = 0$ and $\eta(0) = 0$

$$\int_0^{\pi} k \frac{d\eta}{dx} \frac{dT}{dx} d\Omega = \int_0^{\pi} \eta f d\Omega.$$

Galerkin Approximation

Galerkin Equations

$$\sum_{j=1}^{n} \int_0^{\pi} k \frac{d\varphi_i}{dx} \frac{d\varphi_j}{dx} d\Omega = \int_0^{\pi} f \varphi_i d\Omega, \quad i = 1, \ldots, n.$$

Matrix Form

$$ST = F,$$

Stiffness Matrix

$$S_{ij} = \int_0^\pi k \frac{d\varphi_j}{dx} \frac{d\varphi_i}{dx} dx,$$

vector F

$$F_i = \int_0^\pi f \varphi_i dx.$$

Numerical Solution

If $f(x) = sin(x)$, then the exact solution is $T(x) = \frac{1}{k} sin(x) + \frac{x}{k}$

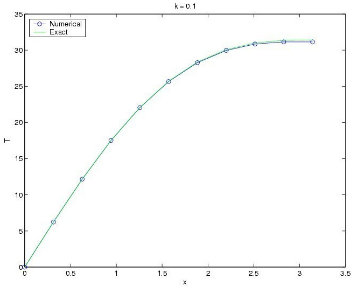


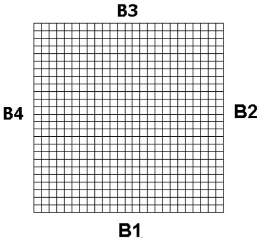
Figure: Exact and Numerical Solution of 1D Poisson Equation, k = 0.1

2D Convection Diffusion Equation

$$\frac{\partial T}{\partial t} - \vec{\nabla}.\left(k\vec{\nabla}T\right) + \left(\vec{u}.\vec{\nabla}T\right) = f$$

k diffusion term,
u convection term,
f source term.

Numerical Solution



On the boundary $B1 \cup B4$: $T|_{B1 \cup B4} = 20,$

On the boundary $B2 \cup B3$: $k\frac{\partial T}{\partial n}|_{B2 \cup B3} = 0,$

At the starting time t_0: $T|_{t_0} = 20.$

Standard Galerkin Method

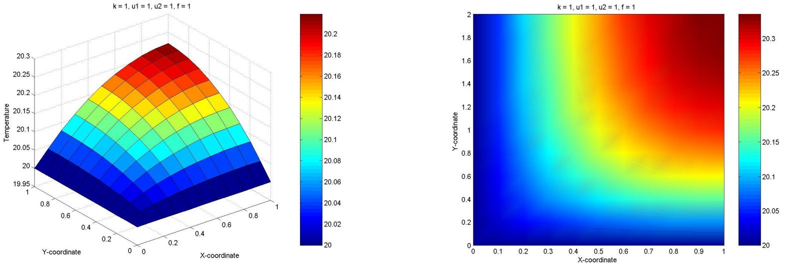
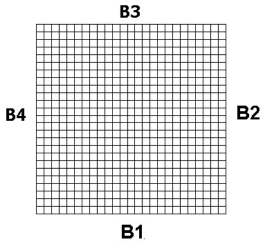


Figure: Numerical solution of 2D time dependent convection diffusion equation using SGA method and implicit scheme, dt $= 0.1$

Numerical Solution



On the boundary $B1 \cup B3$: $k\frac{\partial T}{\partial n}|_{B1 \cup B3} = 0$,
On the boundary $B4$: $T|_{B4} = 20$,
On the boundary $B2$: $T|_{B2} = 40$,
At the starting time t_0: $T|_{t_0} = 20$.

Standard Galerkin Method

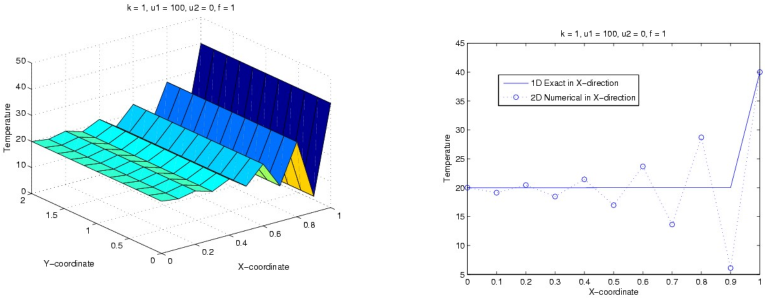
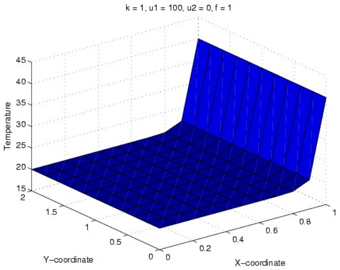


Figure: Numerical solution of 2D time dependent convection diffusion equation using SGA method and implicit scheme after 500 time steps, $dt = 0.1$

Streamline Upwind Petrov Galerkin Method



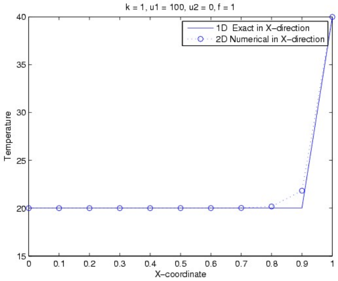


Figure: Numerical solution of 2D time dependent convection diffusion equation using SUPG method and implicit scheme after 500 time steps, $dt = 0.1$

Further Research

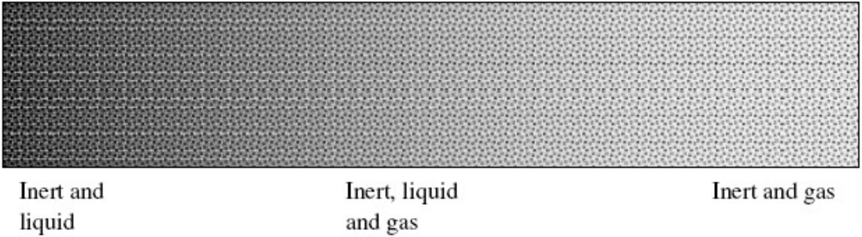


Figure: Porous media

1D Porous media

Solve two coupled nonlinear PDE:

$$\begin{aligned}
\frac{\partial \rho}{\partial t} - \frac{K}{\mu}\frac{\partial P}{\partial x}\frac{\partial \rho}{\partial x} &= \rho\frac{\partial}{\partial x}\frac{K}{\mu}\frac{\partial P}{\partial x}, \\
\rho\frac{\partial h}{\partial t} - \frac{\partial P}{\partial t} - \rho\frac{K}{\mu}\frac{\partial P}{\partial x}\frac{\partial h}{\partial x} &= \frac{\partial}{\partial x}\lambda\frac{\partial T}{\partial x} + q.
\end{aligned}$$

Initial and boundary conditions:

$$\begin{aligned}
P|_{t_0} &= P_a, \\
T|_{t_0} &= T_a, \\
\frac{K}{\mu}\frac{\partial P}{\partial x} &= k_m\left(P_a - P\right), \\
\lambda\frac{\partial T}{\partial x} &= k_h\left(T_a - T\right).
\end{aligned}$$

1D Porous media

where:

t_0	starting time $[s]$,
λ	heat conductance coefficient $[W/m/K]$,
μ	dynamic viscosity $[Pa.s]$,
ρ	density $[Kg/m^3]$,
K	permeability [Darcy],
k_h	heat transfer coefficient $[W/m^2/K]$,
k_m	mass transfer coefficient $[Kg/m^2/K]$.

Further Research

1. Set up 1D model for porous system and solve it using MATLAB.
2. Set up 2D and 3D model for porous system and solve them using SEPRAN.

Questions