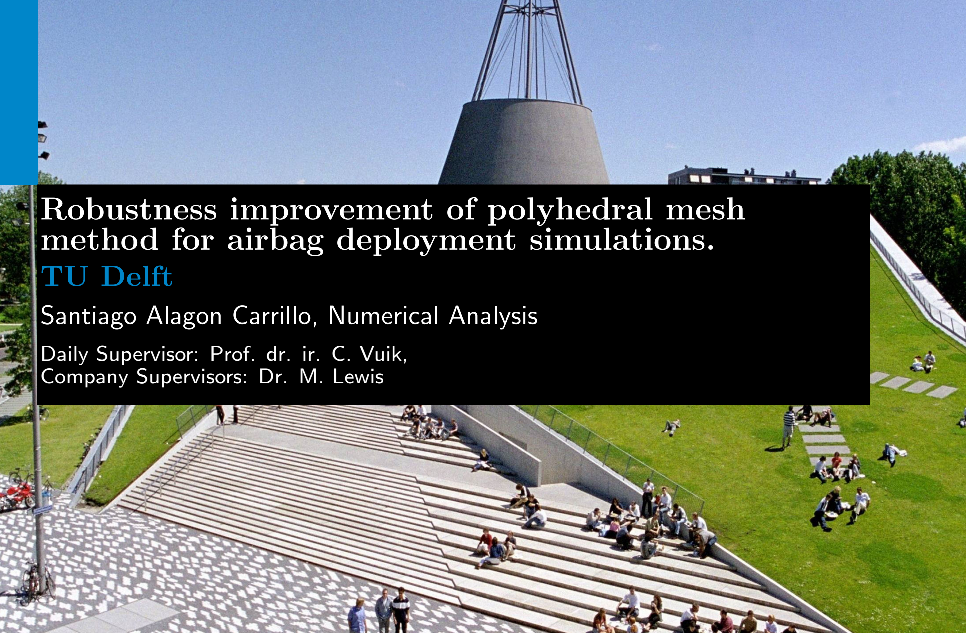


Robustness improvement of polyhedral mesh method for airbag deployment simulations.

TU Delft

Santiago Alagon Carrillo, Numerical Analysis

Daily Supervisor: Prof. dr. ir. C. Vuik,
Company Supervisors: Dr. M. Lewis

Outline

Industry Background and Problem Description

Mathematical tools and Results

Solution Methods.

Implementation and results

Contributions and Conclusion

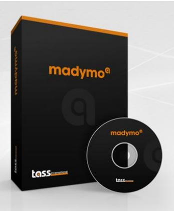
Questions

Industry Background and Problem Description

- **TASS** develops software to analyze occupant safety systems.
- **MADYMO** solver platform comprises Multi-Body, Finite-Element, Computational Fluid Dynamics and Contact Models.

MADYMO is constantly being updated.
CFD solver: **Gasflow** $\rightarrow$ **Gasflow2** $\rightarrow$ *robustness issues*

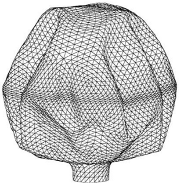
tass international
Innovation for Mobility



Industry Background and Problem Description

- Airbag → flexible membrane. Modeled by equations of elasticity, solved numerically using **FEM**
- Flow → *Euler Equations*, solved numerically using **FVM**.

Use of Cartesian Mesh Method.



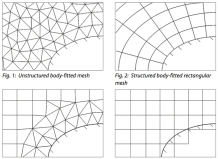
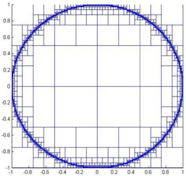


Fig. 1: Unstructured body-fitted mesh

Fig. 2: Structured body-fitted rectangular mesh

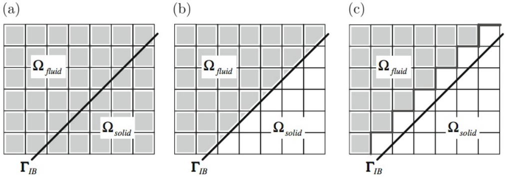
Fig. 3: Combination of structured Cartesian mesh and hybrid unstructured body-fitted mesh near the wall

Fig. 4: Structured Cartesian immersed-body mesh



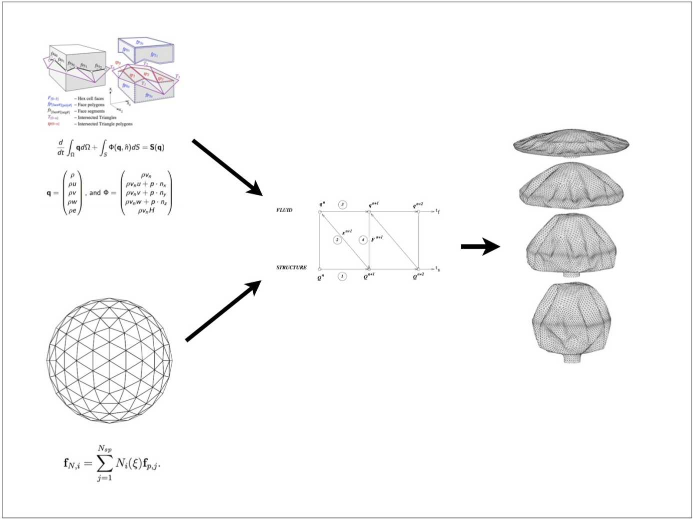
Industry Background and Problem Description

- Airbag triangulation intersects in arbitrary way the Cartesian Mesh.
- Cartesian cells classifies as active, inactive and intersected cells.



Gasflow $\rightarrow$ staircase implementation.
Gasflow2 $\rightarrow$ cutcell implementation.

Industry Background and Problem Description



Geometrical Algorithm.

CFD solver requires the geometrical and topological information of the Cartesian Cells.

Search Algorithm obtains exact geometry.

Coloring Algorithm
Classifies the cells into active cells, inactive cells and cut-cells; and determines the active and inactive regions of the cut-cells.

Coloring Algorithm and Problem Description

Project Goal: design the Coloring Algorithm for a Cartesian cut-cell method suitable for time varying flow domains defined by a Finite Element triangulation.

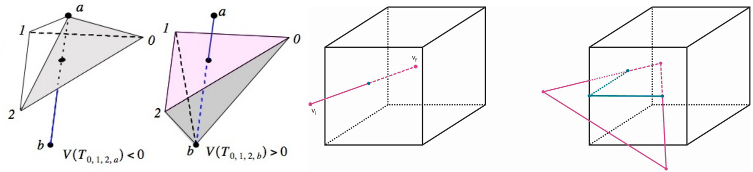
Focus of the project placed on the determination of the active and inactive regions of the cut-cells.

Search Algorithm.

Geometrical constructors too expensive!

Signed volume of a simplex

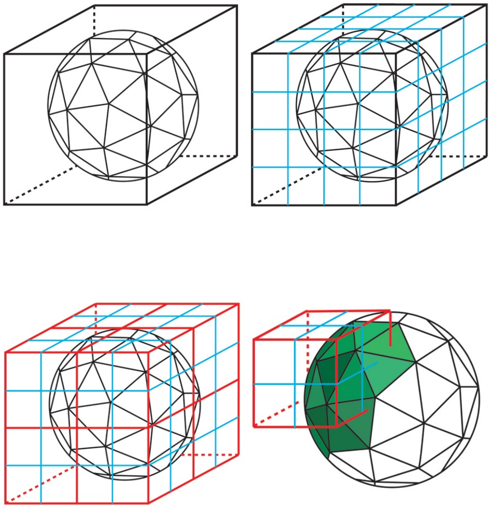
$$V(T_{v_0 v_1 \dots v_d}) = \frac{1}{3!} \begin{pmatrix} v_{a_0} & v_{a_1} & v_{a_2} & 1 \\ v_{b_0} & v_{b_1} & v_{b_2} & 1 \\ v_{c_0} & v_{c_1} & v_{c_2} & 1 \\ v_{d_0} & v_{d_1} & v_{d_2} & 1 \end{pmatrix} \qquad (1)$$



Figuur: Signed Volume Property and Topological Tests

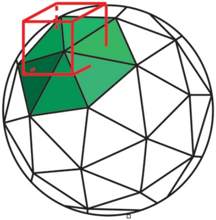
Search Algorithm.

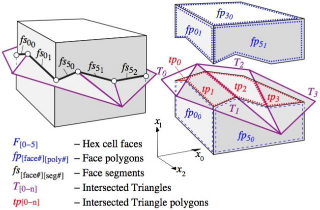
Global Search



Search Algorithm.

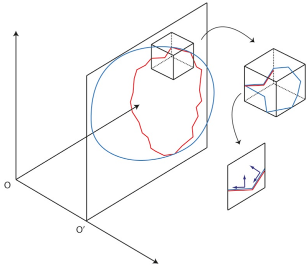
Euler Search and **Exact Geometry Search**





Mathematical tools and Results

What's the big deal?
Airbag sliced by plane results in polygon.
Cell Face is a polygon.



The intersection of the airbag polygon and cell face polygon → results in the active facepolygon.

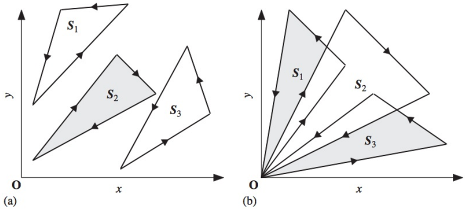
Abstract Geometrical Modeling.

Representation of polygons using Simplicial Chains.

A *simplex* is a generalization of the notion triangle to any dimension. A *d-dimensional simplicial chain* is an expression of the form

$$\lambda = \sum_{i=1}^{m} \alpha_i S_i$$

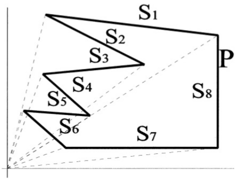
where the coefficients α_i are integers and the S_i are simple d-dimensional simplices.



Abstract Geometrical Modeling.

Theorem *Let P be a bidimensional polyhedral solid determined by n edges $e_1, e_2, \ldots\ldots e_n$ oriented in counterclockwise order. Let S_i be the simplex determined by the origin* **O** *and the edge e_i, being V_{i1} and V_{i2} their vertices and α_i its coefficients. Then*

$$P = P_\lambda \text{ where } \lambda = \sum_{i=1}^{n} \alpha_i \cdot S_i \text{ and } \alpha_i = \text{sign}(\text{Area\_sign}(T_i))$$



$\lambda = S_1 - S_2 + S_3 - S_4 + S_5 - S_6 + S_7 + S_8$

Layer 1: S_1, S_8 **Layer 3:** S_3 **Layer 5:** S_5
Layer 2: S_2 **Layer 4:** S_4 **Layer 6:** S_6, S_7

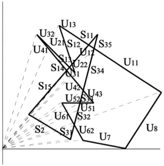
Abstract Geometrical Modeling.

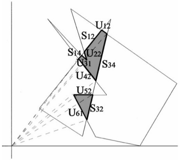
Theorem *Let P_1 and P_2 be polyhedral solids in $\mathbb{R}^d$, and χ_1 and χ_2 their associated chains respectively*

$$\chi_1 = \sum_{i=1}^{n} \alpha_i S_i \ , \ \chi_2 = \sum_{j=1}^{m} \beta_i U_i$$

Then the associated normal chain to the intersection solid $P_\chi = P_1 \cap P_2$ is

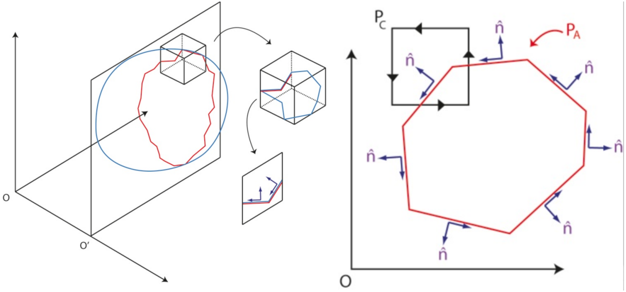
$$\chi = \sum_{i=1}^{n} \sum_{j=1}^{m} (\alpha_i \cdot \beta_j) \cdot Sim(S_i \cap U_j)$$





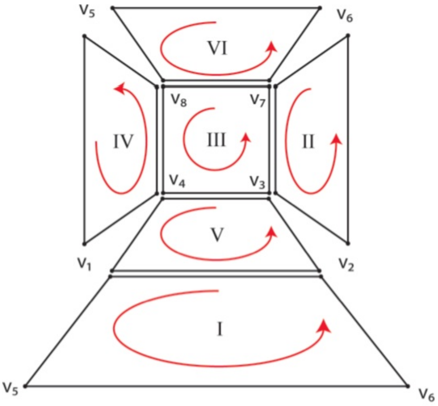
Abstract Geometrical Modeling.

Result The polygonal regions inside each cut cell face, P_C, belonging to the flow regions are those for which the counterclockwise description of their boundaries from, form the point of view of an interior point to the region is completely consistent with the general counterclockwise descriptions of the polygons P_A and P_C.



Two dimensional equivalence

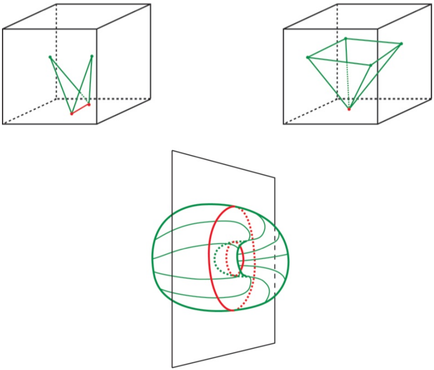
All observations are performed from the point of view of an observer inside the cut cell.



Figuur: View point for cut cell

Problematic cases

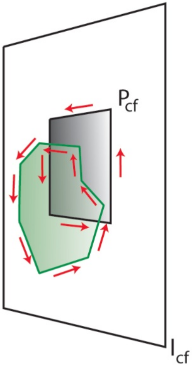
If we describe all polygons in CCW fashion, are we done then?
Not quite!



Figuur: Cases where no niCCW can be determined.

Non Local

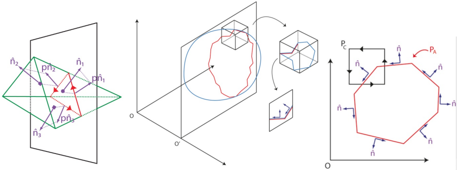
This method would require non-local information



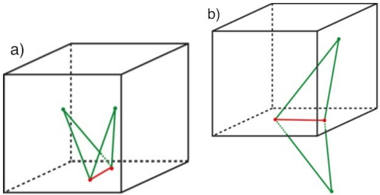
Figuur: Cell face plane regions

Problematic cases

The problem of non-local information can be solved using the outward normal of the FE-triangulation

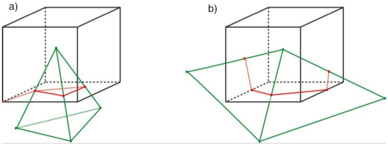


Still there are problems



Test case classification.

Test case 1, At most one zero and only s-face segments



Figuur: Test case 1

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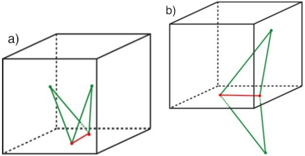
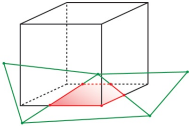
Test case 2 and 3, At most two zeros



Figuur: Test case 2 and 3

Test case classification.

Test case 4, Three zeros, coplanar case



Figuur: Test case 4

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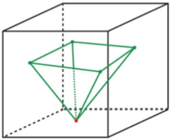
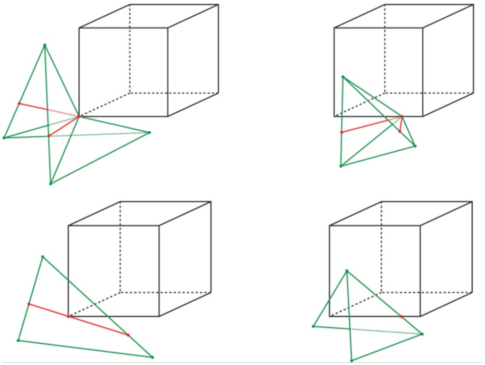
Test case 5, only one zero, cone in one side of plane



Figuur: Test case 5

Test case classification.

Test case 6, 7, 8 and 9, only one zero, no face segments



Figuur: Test case 6,7,8,9

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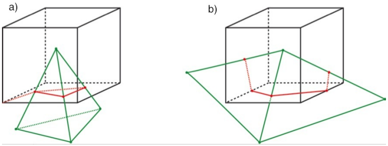
Solution methods

Extended Consistency Method, for test cases 1, 3 and 4



Figuur: Test case 1

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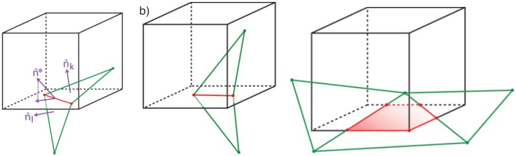
- For each cut-cell face, traverse each of the face polygons in counter-clock wise direction.
- For each of the polygons, if the counter-clock wise direction is not consistent with niCCW direction of at least one of the face segments then the face polygon is determined to be outside.

Solution methods

Extended Consistency Method, for test cases 1, 3 and 4

- Can be used to solve test cases 3 and 4 through the projection of the edge-outward pointing normal $\hat{n}^*$

$$\hat{n}^* = \frac{\hat{n}_k + \hat{n}_l}{||\hat{n}_k + \hat{n}_l||} \quad (2)$$



Figuur: Traversing direction consistency.

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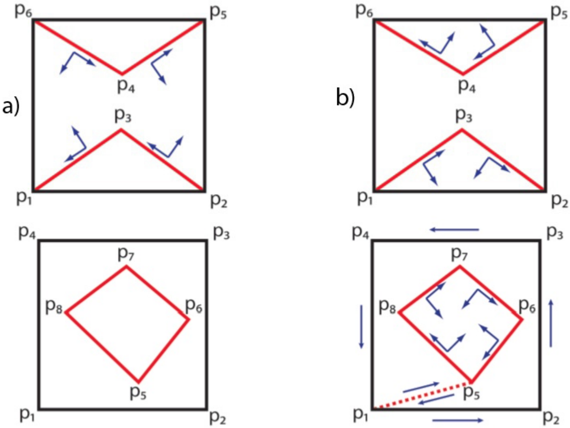
Solution methods

Extended Consistency Method, for test cases 1, 3 and 4

Application to simple polygons and contained polygons



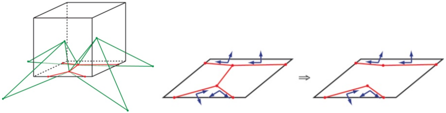
Figuur: Traversing direction consistency.

Solution methods

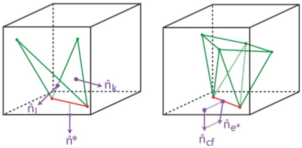
Local half-space determination method 1, treatment of Test Case 3

- Project the edge-outward pointing normal $\hat{n}^*$ onto the face outward pointing normal. Verify the sign of the projection.

Non problematic case



Problematic case

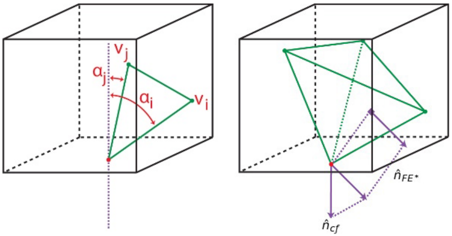


Figuur: Projection of edge-outward normal

Solution methods

Local half-space determination method 2, treatment of Test Case 5

- Project the outward pointing normal of the FE-element closes to the cell face plane onto the face outward pointing normal. Verify the sign of the projection.

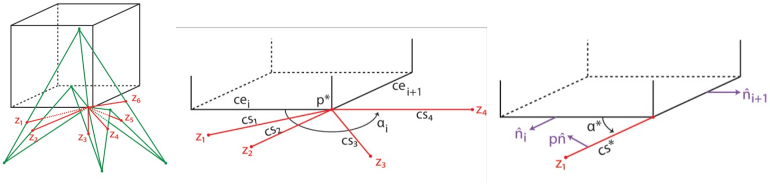


Figuur: Projection of edge-outward normal

Solution methods

Local half-space determination method 3, treatment of Test Cases 6, 7, 8 and 9

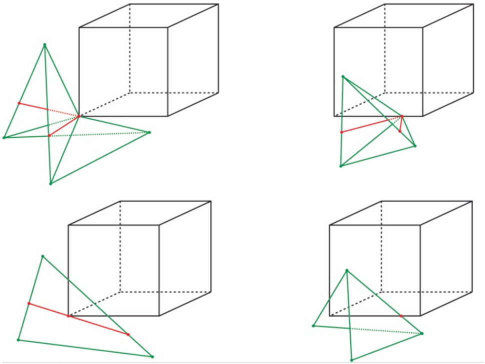
- Compute all zeros of FE-elements and cell face plane
- Build edges cs_i between the computed zeros and the polypoint
- Find closest cs_i edge to cell edge ce_i
- Project outward pointing normal of cs_i onto outward pointing normal of ce_i and ce_{i+1}
- Use the sign of these projections to determine activity of cell face



Figuur: Projection of edge-outward normal

Test case classification.

Test case 6, 7, 8 and 9, only one zero, no face segments **can be treated the same way**



Figuur: Test case 6,7,8,9

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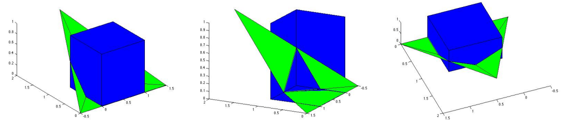
Implementation

Implementation requirements.

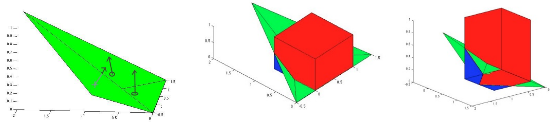
- Object oriented programming for the implementation.
- Each geometric object is defined in terms of a programing object with properties.
- Implementation using a single test cell for each solution method.
- Each cell contains all the necessary geometric information obtained from the Exact Geometry search.

Implementation

Implementation: Extended Consistency Method.



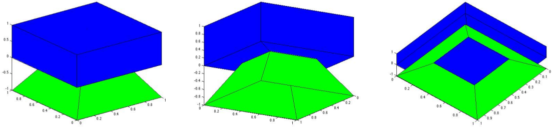
Figuur: Test case for Extended Consistency Method.



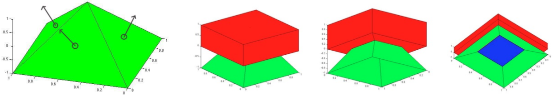
Figuur: Results for Extended Consistency Method

Implementation

Implementation: Half-space Determination Method 2.



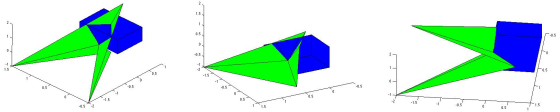
Figuur: Test case for Half-space Determination Method 2.



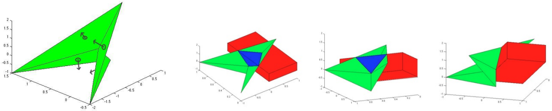
Figuur: Results for Half-space Determination Method 2.

Implementation

Implementation: Half-space Determination Method 3.



Figuur: Test case for Half-space Determination Method 3.



Figuur: Results for Half-space Determination Method 3.

Results

- Test cut cell representative of all possible geometries according to classification.
- Implementation of the different solution successfully determines the active and inactive face polygons
- Coloring algorithm capable of determining the active and inactive regions of all possible geometries of cut cells.

Conclusion

A coloring algorithm was developed which successfully determines the active and inactive regions of the cut cells. This Algorithms has the following characteristics:

- Is based on a classification of all possible geometries of the cut cells from which four types of cut cells are obtained, each treated with a different method.
- The determination of the activity of the face polygons uses only local information available to each cell.
- Suitable for parallel implementation.
- Determination of the activity of each face polygon based only on topological tests.

QUESTIONS