

Terrascope:

The atmosphere as a telescope

by

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[Literature Study Report]

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Contents

List of Figures	v
1 Introduction	1
1.1 Telescopes: Their Purpose and Limits to Current Approaches	1
1.2 Terrascope	3
1.3 Thesis Work	5
2 Geometric Optics	7
2.1 Refraction	7
2.2 Ray-Tracing.	11
2.3 Amplification	13
2.4 Angular Resolution	13
3 Atmospheric Effects	15
3.1 Absorption	17
3.2 Scattering	20
3.2.1 Clouds	21
3.2.2 Turbulence.	22
3.3 Transmission and Reflection.	26
3.4 Latent Emission: Airglow	26
3.5 Variability	27
4 A Return to the Terrascope	31
Bibliography	33

List of Figures

1.1	A protoplanetary disk, the disk of gas and dust in which planets are formed, is imaged two years apart, with the later image in higher angular resolution. Taken by the ALMA (Atacama Large Millimeter Array) radio telescope, from 2016 and 2018 [25],[1].	2
1.2	Determining the magnification of a Galilean telescope in a drawing by Sir William Herschel. Segment AB is the aperture, OQ is the focal length of the telescope, and EQ is the focal length of the eye-glass [22]	2
1.3	Diagram of the light rays inside a Newtonian reflecting telescope [28].	3
1.4	Diagram of the light rays inside a Cassegrain reflecting telescope [28].	3
1.5	Diagram of the Terrascope, with the top of the atmosphere outlined in dashed lines to show its resemblance to a lens. Figure not to scale.	4
2.1	The rectangular box structure represents an enclosed medium, through which a light ray is passing. The line perpendicular to the surface is called the "normal", and it is with respect to this line that the angles of incidence and refraction, θ_i and θ_r , respectively, are measured [15].	8
2.2	Setting sunlight enters the atmosphere at a higher altitude in order to avoid the slowing affects of the thicker atmosphere at lower altitudes. The resulting bent light makes it appear as if the Sun is higher in the sky than its true position below the horizon [15].	9
2.3	Cassini's model of atmospheric refraction. The bolder arc is the surface of the Earth and the thinner arc is the atmosphere. C is the center of the Earth, R is the radius of the Earth, O is the observer, P is where the light ray from the star enters the atmosphere, h is the height above the surface where the ray enters, and r is the ray itself. Line segment CP is the normal to the surface and atmosphere boundary. The light ray enters the atmosphere at an angle z_2 to the normal, but exits and is viewed by the observer at angle z_1 . Note that $z_1 < z_2$ because the atmosphere is denser than the vacuum of space [47].	10

2.4	Similar to Figure (2.3, but with two atmospheric layers, although more are necessary to closely approximate the gradual change in atmosphere with increasing altitude. Here, O is the center of the Earth, a is the radius of the Earth, P_e is the observer, P_n is where the light ray from the star enters the atmosphere, P_{n+1} is another atmospheric shell, r_n is the height above the surface where the ray enters, and r_{n+1} is the height of the other atmospheric shell. The light ray enters the atmosphere at angle i_n , bends to e_n , enters the next shell at i_{n+1} , bends to $Z_0 - Z$ [29].	10
2.5	Diagram for calculating how a ray of light is refracted through many layers of the Earth's atmosphere.	11
2.6	Light that passes through a single aperture causes a distinct pattern of different intensities. The distance between the maxima is determined by the size of the aperture and the distance between the aperture and the final position of the light. [18].	14
2.7	Rayleigh Criterion for angular resolution. If the angular separation of the sources is too small, one point is seen. If the angle equals the criterion exactly, the sources are just separable. If the angle is larger, the sources are clearly distinct and resolved [12].	14
3.1	Relationship between altitude, temperature, and pressure. Density scales as pressure. Pressure units shown are millibars; one bar is defined as the pressure at sea-level. [38].	16
3.2	Absorption spectrum for the Earth's entire atmosphere. Gamma rays, X-rays, UV rays, mid-IR, far-IR, and long radio waves are all blocked out in the upper atmosphere. Nitrogen gas, for example, is responsible for much of the UV absorption in the 80-100nm range [42]. There is a "window" where visible and near-IR light get through and are observed from Earth: see Figure (3.3) [40] . .	18
3.3	Absorption spectra for gases found in Earth's atmosphere, with a focus on the near-IR part of the electromagnetic spectrum. The vertical axis indicates percentage of radiation absorbed [7].	19
3.4	Different types of clouds, their altitudes, and the temperature at that altitude [27].	21
3.5	Undisturbed, or laminar, flow comes off the ocean and becomes turbulent once it encounters the mountain. To decrease seeing, observatories are placed not only on high altitudes (in order to look through less atmosphere), but on the first mountain ridge near the ocean (in order to get the undisturbed ocean winds) [35].	23
3.6	From the largest scale eddy, $L_0 \sim 10m$, down to the smallest, $l_0 \sim 1mm$ [34]. .	23
3.7	The top image shows the transmission spectrum; the bottom shows the transmission and reflection spectra, with the former in blue (or gray, if not being viewed in color) and the latter in black.	27

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- 3.8 Earth between the Sun and detector, perfectly aligned. This is the worst case scenario because the Earth's atmosphere is effectively amplifying the Sun's light into the detector. This completely overwhelms information from any other celestial object, if not destroying the detector's sensitive light-gathering equipment. However, if the detector is moved even slightly off-center, the Sunlight amplification effect is greatly reduced. 28
- 3.9 Detector between the Sun and the Earth, perfectly aligned. This configuration also poses a problem because the Sunlight reflects off the Earth and into the detector. The reflected light is quite strong and in general, the detector should face as little of the Earth's day side as possible. 28
- 3.10 Detector at a 90° angle to the Earth and Sun. In this configuration, no direct or amplified Sunlight reaches the detector and only half of the Earth appears on the day side. Although it is not shown, some Sunlight can reflect off the Earth and into the detector, but the effect is much smaller than in Figure (3.9). Thus, light from a celestial object is able to lens into the detector relatively undisturbed. However, it is not completely undisturbed; not shown in this 2D image are the Sunlight and object light both being lensed "underneath" and "over" the plane of the page. 28

Introduction

1.1. Telescopes: Their Purpose and Limits to Current Approaches

Since people first pointed the telescope to the sky in the 17th century, they have been making increasingly advanced instruments with which to observe celestial objects [21]. The first "spyglass" is credited to Hans Lippershey in 1608, but it is Galileo Galilei who used the lensing properties of glass to observe outer space. Of chief importance when considering new devices are the telescopes' ability to amplify (increase light collection), resolve (distinguish different parts of the object), and magnify (make the object appear bigger compared to the naked eye). These are dependent upon the telescope's aperture; the diameter of the light gathering apparatus. Amplification is measured as the ratio of light intensity received when using the telescope to the light intensity without the telescope. The observed intensity depends on the number of photons (light) received, which in turn depends on the telescope's light receiving area. Of course, area is determined by diameter (the aperture). A telescope's angular resolution is proportional to one over the aperture [26]. The concept of angular resolution is illustrated in Figure (1.1). Lastly, the magnification is equal to the ratio of the focal length of the telescope to the focal length of the eye-piece. The focal length of a lens depends on the material of the lens and its surroundings as well as the radius of curvature of the lens. An illustration of this is seen in Figure (1.2), where the aperture is AB and the magnification is $\frac{OQ}{EQ}$ [22].

The first type of telescope was called a "refracting telescope", due to its ability to refract light rays which passed through it. Many improvements were then made upon these refractors, including: switching from spherical to hyperbolic lenses to focus rays at a single point; using achromatic lenses to avoid wavelength-dependent light splitting; and developing the tubeless aerial telescope to prevent long telescopes from collapsing in the wind [44]. The size of a refractor is inherently important, since we desire a large aperture and long focal length. However, in the late 19th century, the size of the glass lens became a limiting factor for producing

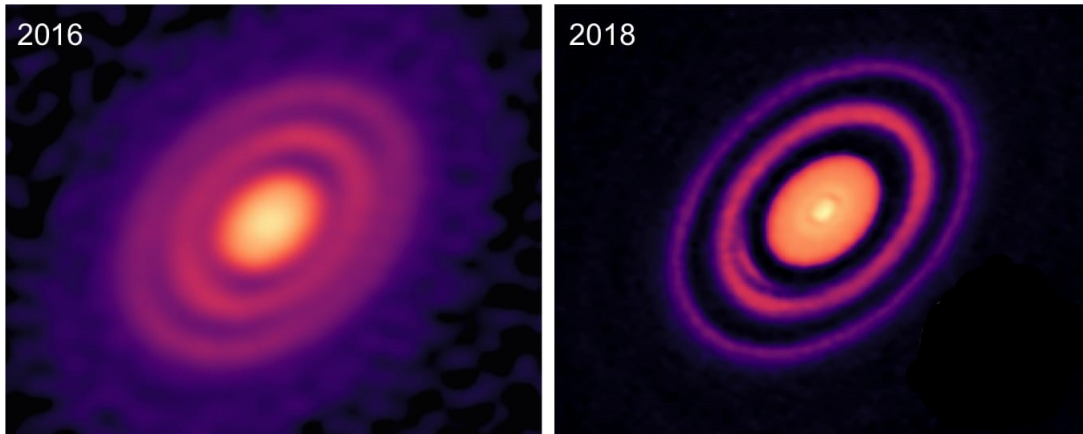


Figure 1.1: A protoplanetary disk, the disk of gas and dust in which planets are formed, is imaged two years apart, with the later image in higher angular resolution. Taken by the ALMA (Atacama Large Millimeter Array) radio telescope, from 2016 and 2018 [25],[1].

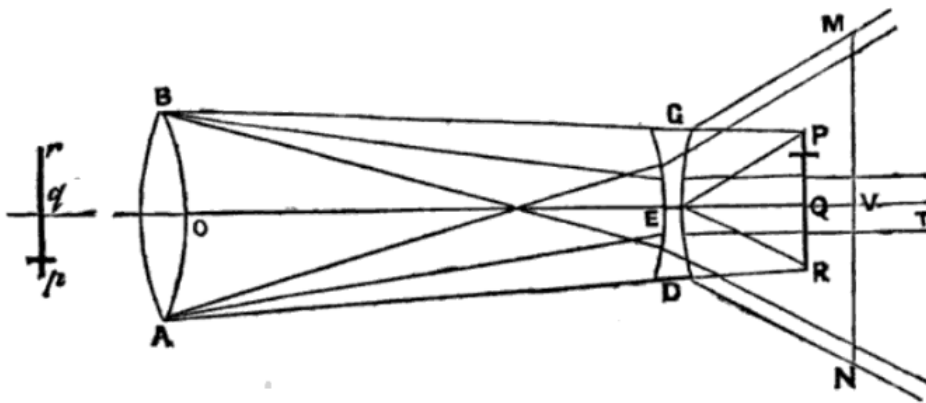


Figure 1.2: Determining the magnification of a Galilean telescope in a drawing by Sir William Herschel. Segment AB is the aperture, OQ is the focal length of the telescope, and EQ is the focal length of the eye-glass [22]

more powerful telescopes. Making, polishing, and supporting extremely pieces of glass became too difficult. The largest refractor in existence was built in 1897 at Yerkes Observatory in Wisconsin; it measured 18.9 meters long and 1.02 in diameter [19].

Reflecting telescopes offer an alternative to refracting telescopes and are called as such because they reflect light rays from objects to the observer. Reflectors were developed almost concurrently with refractors; the first one was designed by Niccolo Zucchi in 1616, but faced technical implementation problems. These issues were mostly associated with the material (it was before the time of glass mirrors) such as difficulty shaping and polishing, and the material absorbing certain colors. The Newtonian telescope (Figure (1.3)), invented in 1668 by Sir Isaac Newton, and the Cassegrain telescope (Figure (1.4)), invented in 1672 by Laurent Cassegrain, are the most widely used designs today. Over time, reflectors became the telescope of choice because multiple mirrors could be used to bounce light multiple times, yielding a longer telescope focal length without a longer containing tube [44].

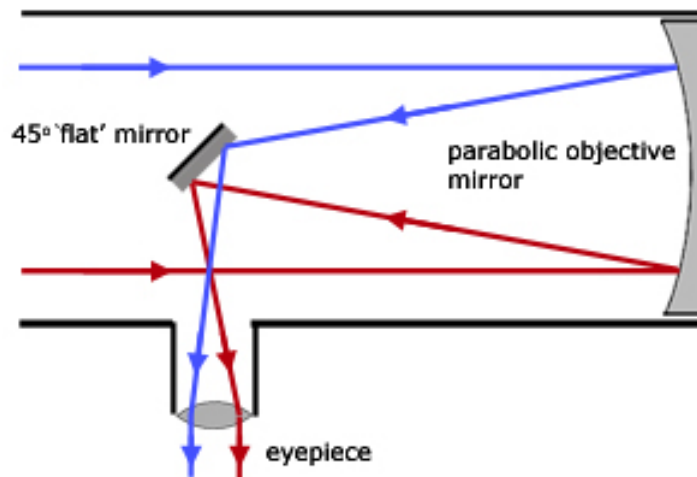


Figure 1.3: Diagram of the light rays inside a Newtonian reflecting telescope [28].

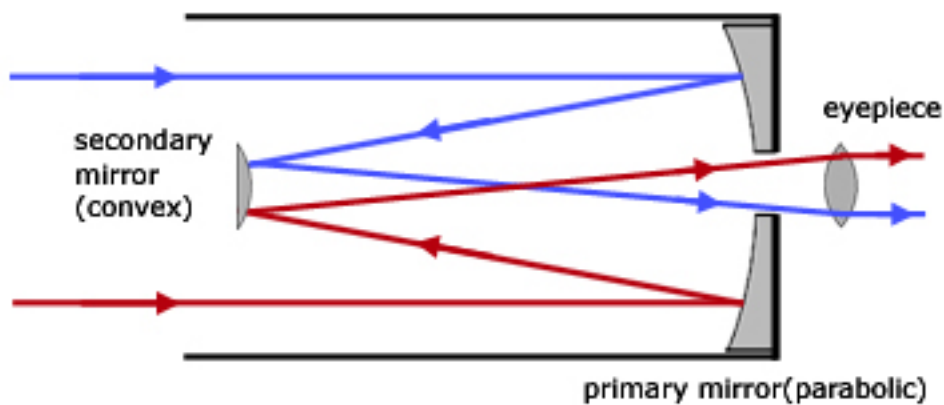


Figure 1.4: Diagram of the light rays inside a Cassegrain reflecting telescope [28].

Hundreds of years later, when humans entered the era of space travel, (reflecting) telescopes were put into space. This solved the problem of the atmosphere distorting the image of Earth-based telescopes, called atmospheric "seeing" [10]. However, larger telescopes are more costly both to make and to launch into space. The cost does not scale linearly with aperture length; rather, it scales with aperture to the 2.5 power. Segmented mirror designs— those which employ multiple small mirrors joined together rather than a single giant mirror— have a slightly less drastic power law cost behavior, estimated to be to the 2.0 power [41]. This scaling will soon yield telescopes with prices in the tens of billions, and thus scientists are always searching for an alternative.

1.2. Terrascope

In 2019, inspired by ideas to use the sun's gravitational field as a gravitational lens, David Kipping of Columbia University put forth the idea of using the Earth's gaseous atmosphere as an optical refracting lens [39], [26]. The premise is based on the atmosphere resembling

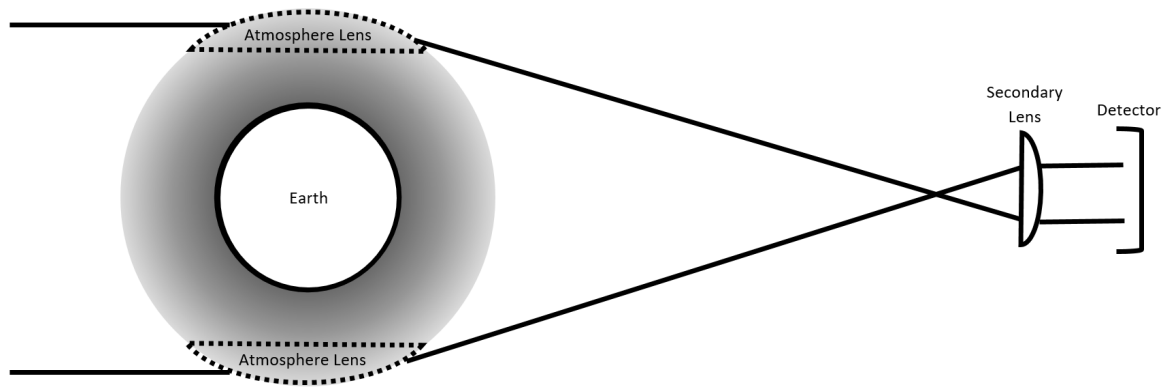


Figure 1.5: Diagram of the Terrascope, with the top of the atmosphere outlined in dashed lines to show its resemblance to a lens. Figure not to scale.

a convex lens, thicker on the "bottom" and thinner at the "top", causing light rays which pass through to converge. A detector would then be placed at the convergence point to collect the light data from the celestial objects which pass behind the Earth, and whose light is lensed. Since the concept uses the Earth as a natural lens, Kipping calls it the "Terrascope". A representation of the set-up is seen in Figure (1.5), where a second lens is used to aim the light into a detector.

If it works, the possible benefits of the Terrascope are: (1) the amplification would be unparalleled because a lens this large has never been used, (2) it is relatively cheap because amplification is achieved by using a pre-existing lens, and thus nothing has to be created and launched into space other than a light-gathering detector, (3) it opens doors to other solar system objects being used for similar endeavors, and (4) it has potential for use as an amplifier for sending signals into space, in addition to receiving them.

In his paper, Kipping outlines his plan for the Terrascope and walks through a proof of concept. He begins with splitting the Earth's atmosphere into concentric shells of equal height that are homogeneous and constant in climate. Six different temperature-pressure profiles are applied to the shells; all profiles are averages. Rays of light pass through the shells, refracting (bending) as they hit each consecutive shell boundary. The minimum depth each ray penetrates is recorded, along with the total bending angle, and how much material it travels through (air-mass). These variables heavily depend upon where the ray first hits the atmosphere— called the "impact parameter"— and so a critical impact parameter is calculated. It ranges in value from 1.7-2.3km depending upon the temperature-pressure profile and light wavelength Kipping uses. He then sets about determining the focal point. Interestingly, because there is an upper and lower impact parameter for the top and bottom of the detector, respectively, there exists a focal line. The inner (closer to Earth) point along this line has the highest potential amplification: closer and there is no image, farther and the quality diminishes. The inner focus ranges in value from 200,000-350,000km, again depending upon the atmospheric profile and wavelength used. For comparison, the moon is at a distance of 384,400km [13]. The

image along the focal line is a ring of lensed light. It is a ring rather than a complete image because the atmosphere is also only a ring; it is comparable to refracting light through only the top part of a lens and blocking the center. This ring is circular symmetric if the celestial object is directly in line with the Earth and collecting detector, called "on-axis". If the celestial object is "off-axis", then the ring is oval shaped. Kipping takes into account the absorption of the particles in the atmosphere by using a transmittance and radiance model. He determines that the amount of light blocked in this manner is roughly equivalent to that of a telescope on a high mountain with low seeing (not much atmospheric disturbance). He accounts for clouds by using effective cloud fraction data and noting that above a certain altitude, clouds are mostly absent from the atmosphere. He determines that if the detector is placed at one Hill-sphere radius (1,500,000km), the rays only penetrate to a height of 13.7km and only 10% of light is lost. Thus, Kipping proposes to put the Terrascope at this distance. Before calculating the final amplification, he cuts his estimate in half to account for half of the light being unusable due to the Sun's position. When the Sun is in direct view of the detector with the Earth in between, light from the Sun scattering through the Earth's upper atmosphere will cause background information that cannot be removed, thus rendering the observations during that period unusable. The final number at which Kipping arrives is 22,500 times the amplification of the object when using a 1m detector. For comparison, this is equivalent to using a 150m telescope in space. This potentially unprecedentedly powerful telescope is the basis for this thesis work.

1.3. Thesis Work

The research in this thesis will explore Kipping's Terrascope in more detail, using advanced models of the atmosphere. The main research question and sub-questions are as follows:

- Can the Terrascope be a useful telescope, and if so, to what extent?
 - What assumptions underlie the simplified model of using the Earth's atmosphere as a telescopic lens?
 - What effects will diminish the quality of the Terrascope?

The research behind how these questions will begin to be answered is presented in the remainder of this work. This chapter served as an introduction to telescopes and the Terrascope. Chapter 2 explores the optics of light and how to trace it from an object being observed into an amplified image in a telescope. Chapter 3 investigates the effect that the Earth's atmosphere will have on the light, and ultimately what must be accounted for when modeling the Terrascope. Chapter 4 returns to the research questions and presents a plan for answering them.

2

Geometric Optics

Essential to tracing light from a celestial object, through the Earth's atmosphere, and into the detector is understanding how that light will travel. Geometric optics is a model describing light as rays or beams [8]. This model is the most useful for this research because light will be traced through a series of different surfaces, and the "line" of light will bend at each boundary. As Richard Feynman said in one of his lectures:

If one has an actual, detailed problem in lens design, including analysis of aberrations, then he is advised to read about the subject or else simply to trace the rays through various surfaces (which is what the book tells us how to do), using the law of refraction from one side to the other, and to find out where they come out and see if they form a satisfactory image. People have said that this is too tedious, but today, with computing machines, it is the right way to do it [14].

Today, even more so than when this was said in 1961, computers are best suited to performing geometrical optics calculations. In the following section, the principles behind these optics will be explored.

2.1. Refraction

Refraction is the phenomenon of light bending when it goes from one medium (environment) to another. Observations going back thousands of years recorded that the amount of bending is determined by the two mediums and the angle at which the light ray strikes their interface. This striking angle is called the "angle of incidence", denoted θ_i , and the bent angle is called the "angle of refraction", denoted θ_r . Simple refraction is shown in Figure (2.1). In 1621, Willebrord Snell deduced a law predicting how light would bend, and which took into account the angles and the medium [15]. Snell's law is as follows:

$$n_i \sin(\theta_i) = n_r \sin(\theta_r) \quad (2.1)$$

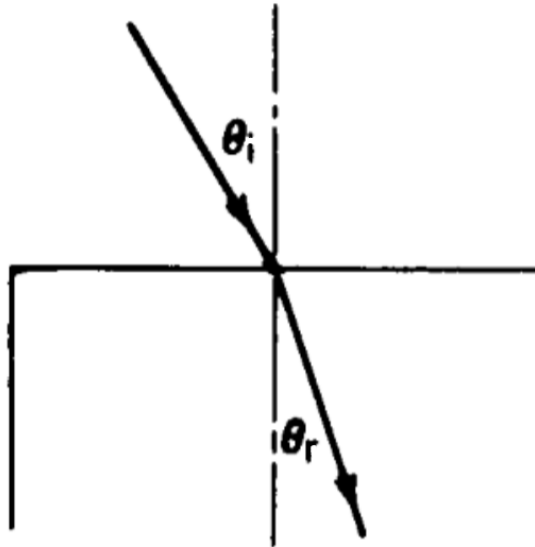


Figure 2.1: The rectangular box structure represents an enclosed medium, through which a light ray is passing. The line perpendicular to the surface is called the "normal", and it is with respect to this line that the angles of incidence and refraction, θ_i and θ_r , respectively, are measured [15].

Where

- θ_i : angle of incidence, the angle inside the incident/first medium at which the light ray strikes the surface
- θ_r : angle of refraction, the angle inside the refracted/second medium at which the light ray leaves the surface
- n_i : incident medium refractive index, the number describing how fast light travels in the incident/first medium (equal to the ratio of the speed of light in a vacuum to the speed of light in the medium)
- n_r : refractive medium refractive index, the number describing how fast light travels in the refracted/second medium (equal to the ratio of the speed of light in a vacuum to the speed of light in the medium)

The practical implications of this law are as follows: when light travels from a thinner medium to a thicker medium (from where light moves faster to where it moves slower), the angle of incidence is larger than the angle of refraction, i.e. the light ray bends inward, toward the normal. When light travels from a thicker medium to a thinner medium (from where light moves slower to where it moves faster), the angle of incidence is smaller than the angle of refraction, i.e. the light ray bends outward, away from the normal. The refraction phenomenon can be explained by Fermat's Principle of Least Time, which states that light travels between points such that it takes the least possible times. When materials change, the speed of light changes accordingly, and so the light changes its path to traverse the new medium as quickly as possible [15]. But how does refraction occur in the atmosphere?

An instance of refraction seen in everyday life is that of the setting Sun appearing higher in the sky than it actually is, shown in Figure (2.2). The Earth's atmosphere is thicker at lower alti-

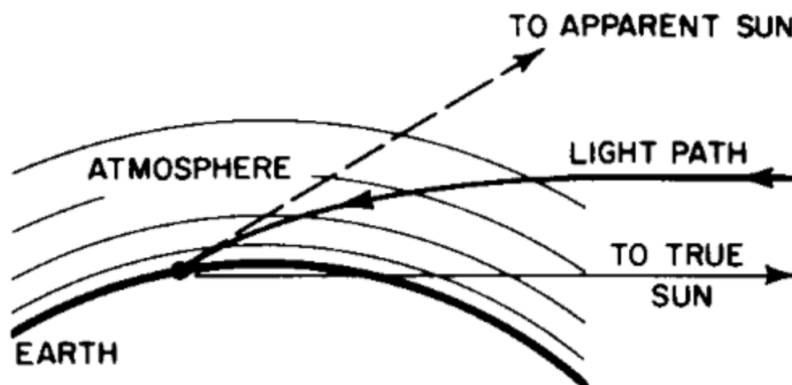


Figure 2.2: Setting sunlight enters the atmosphere at a higher altitude in order to avoid the slowing affects of the thicker atmosphere at lower altitudes. The resulting bent light makes it appear as if the Sun is higher in the sky than its true position below the horizon [15].

tudes and becomes thinner with increasing altitude. Thus, when the Sun is low on the horizon (close to setting), the light rays must travel through many layers of the thickest atmosphere at a shallow angle to arrive at someone observing the setting Sun. The speed of light is slower in thicker mediums, so to comply with the Principle of Least Time, the light instead enters the atmosphere at a higher altitude, where the atmosphere is thinner, and then travels at a steeper angle to arrive at the observer. This steeper angle at which light arrives makes it appear as if the Sun is higher in the sky than is really is. Thus, when one sees the Sun set below the horizon, it has actually already dropped behind the true horizon [15].

Astronomical refraction refers to the angular displacement of astronomical objects from their expected position due to refraction in the Earth's atmosphere. [46] This effect was discovered because the apparent positions of celestial objects differed from what they were expected to be via trigonometric computations. In 1587, Tycho Brahe used the difference in position of the Sun at the summer and winter solstices to measure astronomical refraction and calculate it for different apparent zenith angles. In 1656, Cassini improved upon this by using Snell's law; a drawing of this can be is seen in Figure (2.3).

However, they both assumed that refraction only happened in one place— at the top of the atmosphere of fixed height. Later, atmospheric measurements were taken and it was discovered that the pressure and temperature change with increasing altitude. In 1669, Picard discovered that the astronomical refraction depends on temperature and in 1708, Hawksbee discovered that it depends on density. These new insights lead to the development of the concentric spherical shell model for astronomical refraction, seen in Figure (2.4).

These shells are local regions of constant pressure and temperature, and therefore constant refractive index [29]. As the atmospheric altitude increases, the density, and therefore the re-

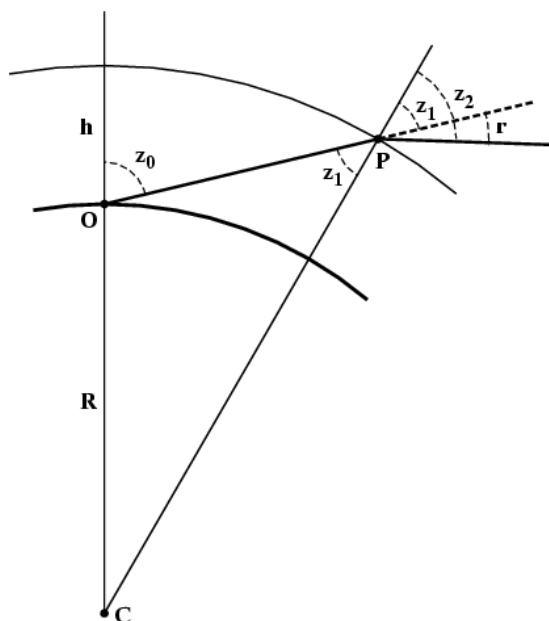


Figure 2.3: Cassini's model of atmospheric refraction. The bolder arc is the surface of the Earth and the thinner arc is the atmosphere. C is the center of the Earth, R is the radius of the Earth, O is the observer, P is where the light ray from the star enters the atmosphere, h is the height above the surface where the ray enters, and r is the ray itself. Line segment CP is the normal to the surface and atmosphere boundary. The light ray enters the atmosphere at an angle z_2 to the normal, but exits and is viewed by the observer at angle z_1 . Note that $z_1 < z_2$ because the atmosphere is denser than the vacuum of space [47].

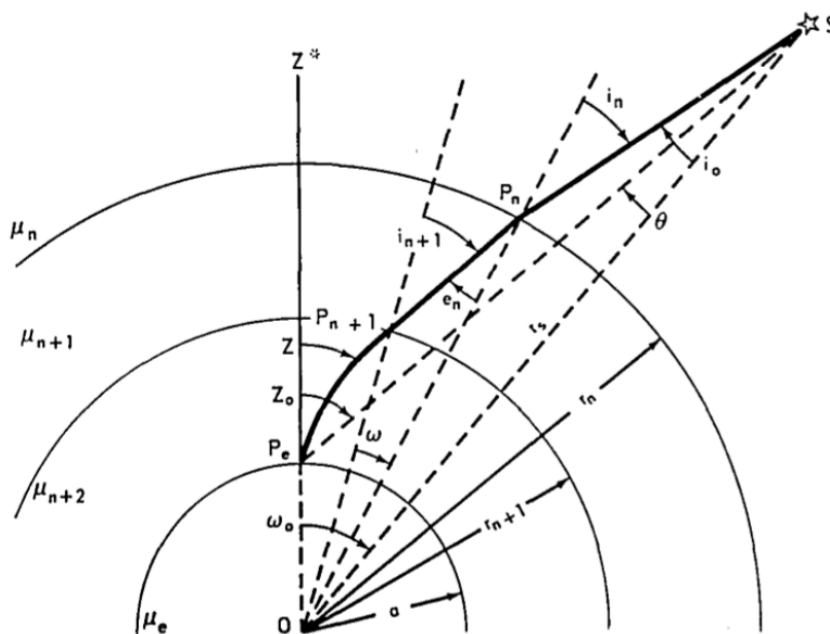


Figure 2.4: Similar to Figure (2.3), but with two atmospheric layers, although more are necessary to closely approximate the gradual change in atmosphere with increasing altitude. Here, O is the center of the Earth, a is the radius of the Earth, P_e is the observer, P_n is where the light ray from the star enters the atmosphere, P_{n+1} is another atmospheric shell, r_n is the height above the surface where the ray enters, and r_{n+1} is the height of the other atmospheric shell. The light ray enters the atmosphere at angle i_n , bends to e_n , enters the next shell at i_{n+1} , bends to $Z_0 - Z$ [29].

fractive index, decreases. Thus, each concentric shell has a smaller value of n . This is exactly the theory upon which Kipping's calculations are based. As was mentioned in the Terrascope section of the Introduction, it is expected that light rays from celestial objects will penetrate the atmosphere and refract through consecutive layers, where the bending angle will depend on the physical make-up and the thickness of the atmosphere at each layer. In the following section, these rays will be traced.

2.2. Ray-Tracing

Kipping traces rays through the atmosphere by successively using basic trigonometry and Snell's law. He draws the diagram shown in Figure (2.5).

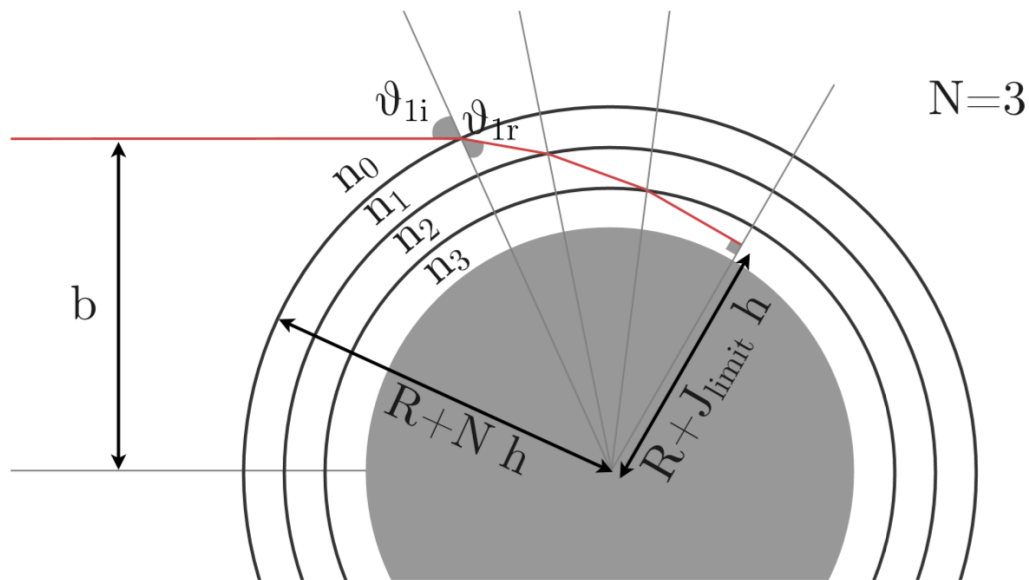


Figure 2.5: Diagram for calculating how a ray of light is refracted through many layers of the Earth's atmosphere.

Where:

- b : impact parameter; perpendicular distance between the incoming ray and the Earth's center
- N : number of shells (larger N for higher accuracy)
- n_i refractive index of the i th shell
- θ_{1i} : angle of incidence for 1st shell
- θ_{1r} : angle of refraction for 1st shell
- R : radius of the Earth
- h : thickness of each shell, defined as the ratio of the total altitude of the atmosphere to the number of shells
- J_{limit} : deepest shell penetrated by the light ray

The goal from this diagram is to find the total angle, Δ , that the light ray is deflected if and when it exits the atmosphere. This quantity can be calculated by summing the deflection angle from

each shell, α_j , which is the difference between the incident angle and the refracted angle. To calculate each of these angles, a triangle is drawn with the impact parameter b , opposite the incident angle. The hypotenuse is then $R + Nh$:

$$\sin(\theta_{1i}) = \frac{b}{R + Nh} \quad (2.2)$$

Using Snell's Law, Equation (2.1), the sine of the refracted angle is calculated:

$$\sin(\theta_{1r}) = \frac{b}{n_1 R + Nh} \quad (2.3)$$

Continuing to draw triangles in this manner, using the sine rule, and Snell's law, the equations for the j^{th} shell are acquired:

$$\sin(\theta_{ji}) = \frac{b}{n_{j-1} R + (N - j + 1)h} \quad (2.4)$$

$$\sin(\theta_{jr}) = \frac{b}{n_j R + (N - j + 1)h} \quad (2.5)$$

To solve for the deflection angle at each boundary:

$$\alpha_j = \theta_{ji} - \theta_{jr}$$

$$\sin(\alpha_j) = \sin(\theta_{ji} - \theta_{jr})$$

Using the sine addition rule on the right hand side of the equation and substituting in Equations (2.4) and (2.5), an equation for the total deflection angle is obtained:

$$\Delta = 2 \sum_{j=1}^{J_{\text{limit}}} \alpha_j \quad (2.6)$$

The factor two in front of the sum is because the deflection angles of each ray are only calculated while traveling deeper into the Earth's atmosphere; the path out is assumed to be symmetric, but in reverse. Using these equations, ray-tracing simulations are run on a grid of impact parameters and wavelengths. For each run, the total deflection angle, lowest altitude (the depth), and airmass (material traveled through) are calculated. Once this is complete, the inner focus and focal line are found (for various impact parameters and wavelengths). The focal distance, F , is given by the following equation:

$$F = b_{\text{crit}} \cot(\Delta_{\text{crit}}) \quad (2.7)$$

Where:

- b_{crit} : the impact parameter that causes the light ray to reach the deepest atmospheric shell, grazing the Earth's surface; any smaller b will not emerge from the atmosphere

- Δ_{crit} : the total deflection angle corresponding to the critical impact parameter

Kipping supposes that a detector is placed along this focal line and gathers the signal from the celestial object. A key question in examining the usefulness of the Terrascope is determining by how much the celestial object's signal is amplified.

2.3. Amplification

Amplification of a telescope, or a similar optical instrument, is defined as the ratio of the intensity of the light from the viewed object with and without the telescope. Since a telescope receives light proportional to its area and the light emitted from the object is also calculated per area, one can also define this ratio in terms of area. Kipping begins with the equation:

$$A = \epsilon \frac{\pi (b_+^2 - b_-^2)}{\pi ((W/2)^2)} \quad (2.8)$$

Where:

- A : amplification
- ϵ : loss parameter due to extinction from absorption and scattering in the atmosphere
- b_+ : impact parameter for a light ray that hits the top of the detector
- b_- : impact parameter for a light ray that hits the bottom of the detector
- W : diameter/aperture of the detector

Thus, in Equation (2.8), the numerator represents the the area of the circular ring of lensed light which the Earth's atmosphere produces; the denominator is the area of the detector. After a number of approximations, Kipping arrives at his final amplification equation:

$$\frac{A}{\epsilon} \sim \frac{55,000}{W} \quad (2.9)$$

Even when the amplification is halved to account for extinction and other effects, the results for a 1m Terrascope detector are equivalent to that of a 150m reflecting telescope in space [26]. Kipping's initial investigation of the Terrascope seem extremely promising. However, can the Terrascope merely collect and enhance light? Is it possible to resolve the celestial objects whose light is being lensed and then collected?

2.4. Angular Resolution

The wave-nature of light exists concurrently with geometric optics, the viewing of light as rays. As such, it must be understood and accounted for in calculations. One wave phenomenon of light is diffraction, the bending of light waves when they reach a corner, opening, or other type of obstacle. When light waves move through a circular opening (also called an aperture—note the similarity to telescopes) they spread out in a pattern of minima and consecutively smaller maxima, as shown in Figure (2.6) [4]. The angular resolution can be described as the diffraction limit: the minimum angle possible between two distinct incoming sources traveling

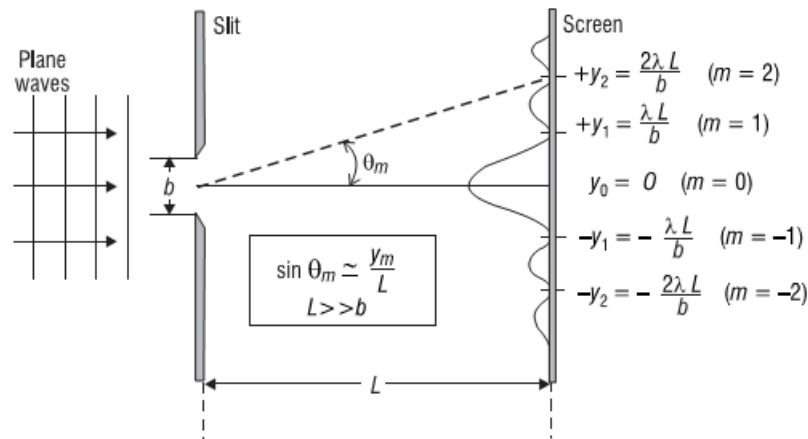


Figure 2.6: Light that passes through a single aperture causes a distinct pattern of different intensities. The distance between the maxima is determined by the size of the aperture and the distance between the aperture and the final position of the light. [18].

through an aperture such that the sources can be distinguished as separate.

Every light source through an aperture causes the diffraction pattern; if two light sources close together cause similar patterns, there must be a method to determine when the patterns overlap and the sources cannot be identified as distinct. In his paper of 1879, Lord Rayleigh said "two images are just resolvable when the center of the diffraction pattern of one is directly over the first minimum of the diffraction pattern of the other" [17]. As seen in Figure (2.6), and assuming the case where $L \gg b$, the first minimum is at $\frac{\lambda L}{b}$ [14]. Thus, that is the minimum required angular separation of two light sources (in this case, celestial objects) for the sources to be distinguishable. Call this the angular resolution angle, θ_{ar} , and define it as follows, including an observed constant:

$$\theta_{ar} = 1.22 \frac{\lambda}{b} \quad (2.10)$$

This criterion is shown in Figure (2.7), along with an unresolved image (smaller angle than the criterion) and well-resolved image (larger angle than the criterion).

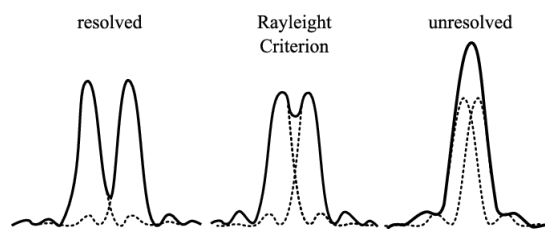


Figure 2.7: Rayleigh Criterion for angular resolution. If the angular separation of the sources is too small, one point is seen. If the angle equals the criterion exactly, the sources are just separable. If the angle is larger, the sources are clearly distinct and resolved [12].

The angular resolution of telescopes is also affected by atmospheric distortion, so-called "seeing"; this will be covered in Section 3.2.2, on turbulence.

3

Atmospheric Effects

The premise of the Terrascope lies in the propagation of light through the Earth's atmosphere; thus, an understanding of the make-up and properties of the atmosphere is essential. Earth's atmosphere is about 100km thick and consists of mostly N_2 and O_2 . Other substances include: Argon (Ar), Carbon Dioxide (CO_2), Neon (Ne), Helium (He), Methane (CH_4), Krypton (Kr), and water vapor (H_2O) [20]. In the above section on Refraction, it was stated that the atmosphere becomes less dense as it increases in altitude. Density is an important parameter for determining how light behaves in the atmosphere— two others are temperature and pressure. Air pressure is caused by the weight of the molecules above it and decreases with altitude. Density has a similar scaling to pressure because they are proportional [6]. The lower pressure at higher altitudes is partially the reason for it being colder at higher altitudes. The decrease is also due to the ground being the main absorber of sunlight and then emitter of heat energy via infrared (IR) light back into the air. With one exception (the stratosphere), the farther from the ground, the colder it is. Figure (3.1) shows more detail about how temperature, pressure, and height scale in the atmosphere.

Note that the atmosphere is divided into subsections called "spheres" with the boundaries called "pauses". These are briefly described below [36].

- **Troposphere:** It is warmed by visible light from the Sun, convection, and IR light bouncing back from the ground, but temperature decreases with increasing altitude. Where weather occurs and where most clouds are, due to the highest proportion of water vapor. The maximum height of this section varies with location, with it increased over warmer areas and lower over colder ones. The tropopause holds the jet stream and is the highest point for weather.
- **Stratosphere:** Visible and ultraviolet (UV) light from the Sun reach here. Temperature increases with increasing altitude in the lower part and decreases with increasing altitude in the upper part. This is due to the UV light being absorbed, causing the "ozone layer".

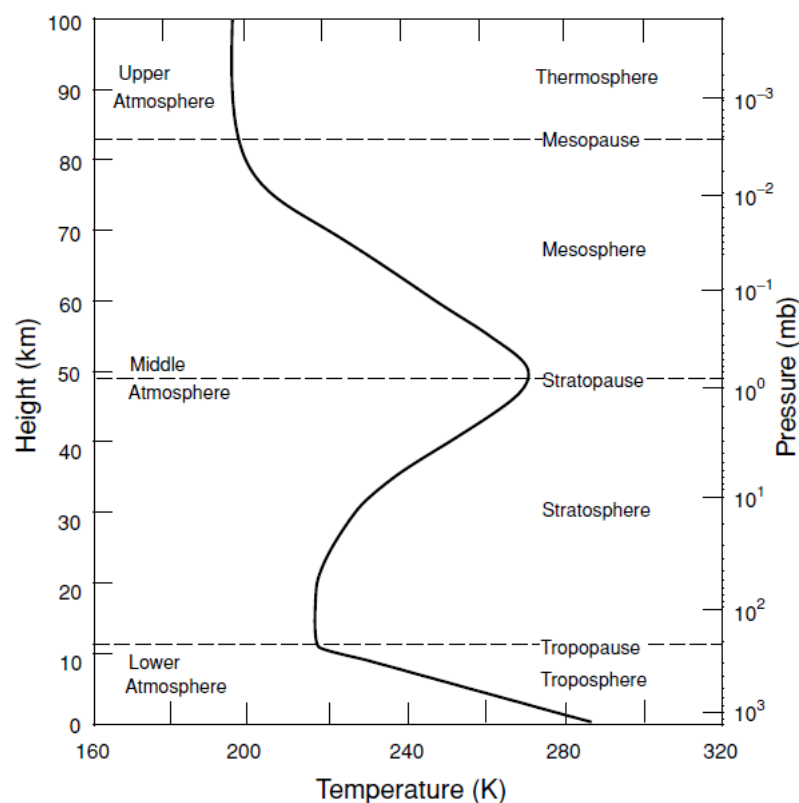


Figure 3.1: Relationship between altitude, temperature, and pressure. Density scales as pressure. Pressure units shown are millibars; one bar is defined as the pressure at sea-level. [38].

- This layer holds ozone molecules (O_3) which absorb UV light and turn it into heat. The stratosphere is less turbulent than the troposphere, so airplanes fly in the lower part.
- **Mesosphere:** Visible, UV, and X-Ray light reach here. Temperature decreases with increasing altitude. The coldest atmospheric temperatures are at the top of this layer. Most meteors burn up in this layer.
 - **Thermosphere:** Visible, UV, and X-Ray light reach here and ionize gases. The temperature increases with increasing altitude because it is heated by the Sun's light. Temperature varies with the light (energy) coming from the Sun. The auroras occur here.
 - **Exosphere** (not pictured): Similar to the thermosphere in terms of light and temperature. Where the Earth's atmosphere gradually turns into outer space.

These layers are important to keep in mind in the following sections because light from celestial objects enter, travel through, and emerge from the Earth's atmosphere at different altitudes. The properties of the atmosphere vary with these heights, and thus the lights' passage and interaction will also vary.

On a smaller scale, there are interactions with atoms, molecules, and particles in the atmosphere. While the majority of these components are completely gaseous, others are aerosols, liquids and solids suspended in gas. It is important to know the chemical make-up, size, con-

centration, and other properties of these materials in order to determine how they influence the propagation of light in the atmosphere [5]. Table (3.1) lists the gases by percentage in the atmosphere; Table (3.2) lists the gases by atmospheric height.

Name	Chemical Formula	Concentration (% by volume)
Nitrogen	N ₂	78.08
Oxygen	O ₂	20.95
Water Vapor	H ₂ O	$2 \times 10^{-6} - 3 \times 10^{-2}$
Argon	Ar	9.34×10^{-3}
Carbon Dioxide	CO ₂	3.45×10^{-4}
Neon	Ne	18.2×10^{-6}
Helium	He	5.24×10^{-6}
Methane	CH ₄	1.72×10^{-6}
Krypton	Kr	1.14×10^{-6}
Hydrogen	H ₂	5.0×10^{-7}

Table 3.1: List of ten gases in the atmosphere with the highest volumetric concentration.

Altitude (km)	Concentration (% by volume)					
	N ₂	O ₂	O	He	Ar	H
100	77	19	3.4	<0.05	0.8	<0.05
150	61	5.6	24	<0.05	0.1	<0.05
200	42	3.0	55	0.01	<0.05	<0.05
300	17	0.8	81	0.8	<0.05	<0.05
400	6.0	0.2	91	2.7	<0.05	<0.05
500	1.9	<0.05	90	8.2	<0.05	0.2
700	0.1	<0.05	55	43	<0.05	1.6
1000	<0.05	<0.05	5.7	88	<0.05	6.7

Table 3.2: How the concentration of certain gases in the atmosphere vary with altitude.

The chemical composition of the Earth's atmosphere determines how light is absorbed, scattered, transmitted, and reflected; these phenomena are outlined in the following sections.

3.1. Absorption

First, consider how much radiation is absorbed as it passes through layers of the atmosphere. Beer's absorption law gives such a relation:

$$\frac{I}{I_0} = e^{-\int \sec(\theta) a_{\lambda} dm} = e^{-\int \sec(\theta) d\tau} \quad (3.1)$$

Where:

- I : intensity of beam of radiation which has passed through a layer of the atmosphere
- I_0 : intensity of incident beam of radiation
- θ : angle at which incident beam is inclined to the vertical
- a_λ : co-efficient of mass absorption (wavelength dependent)
- dm : element of mass of the atmospheric layer through which light passes
- $d\tau$: element of optical thickness of absorbing layer

The Beer-Lambert law then gives the dependence of absorption on the radiation [23]:

$$A = \log_{10} \left(\frac{I_0}{I} \right) \quad (3.2)$$

Notice that the intensity, and therefore the absorption, is wavelength dependent, which is due to quantum effects. When materials absorb electromagnetic energy, i.e. light, their electrons are excited to higher energy levels, where the change in energy is proportional to the frequency of the light (inversely proportion to wavelength). The exact relationship is given by the Planck-Einstein equation:

$$E = h\nu = \frac{hc}{\lambda} \quad (3.3)$$

Where:

- E : energy
- h : Planck's constant, defined by this equation
- c : speed of light in a vacuum
- ν : frequency
- λ : wavelength

The energy of the incoming photon must exactly match the difference in energy level in the material in order for the electron to become excited [23]. As seen in Equation (3.3), this energy excitation corresponds to a specific wavelength. Each material has a unique absorption spectrum in which the energy dips at these specific wavelengths. Figure (3.2) shows the accumulated absorption spectrum of the atmosphere; Figure (3.3) shows the IR window of the absorption spectrum for specific gases. As can be seen in Figure (3.3), the absorption lines

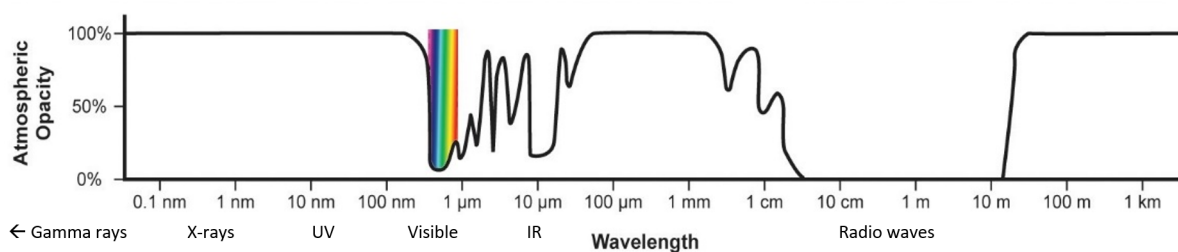


Figure 3.2: Absorption spectrum for the Earth's entire atmosphere. Gamma rays, X-rays, UV rays, mid-IR, far-IR, and long radio waves are all blocked out in the upper atmosphere. Nitrogen gas, for example, is responsible for much of the UV absorption in the 80-100nm range [42]. There is a "window" where visible and near-IR light get through and are observed from Earth: see Figure (3.3) [40]

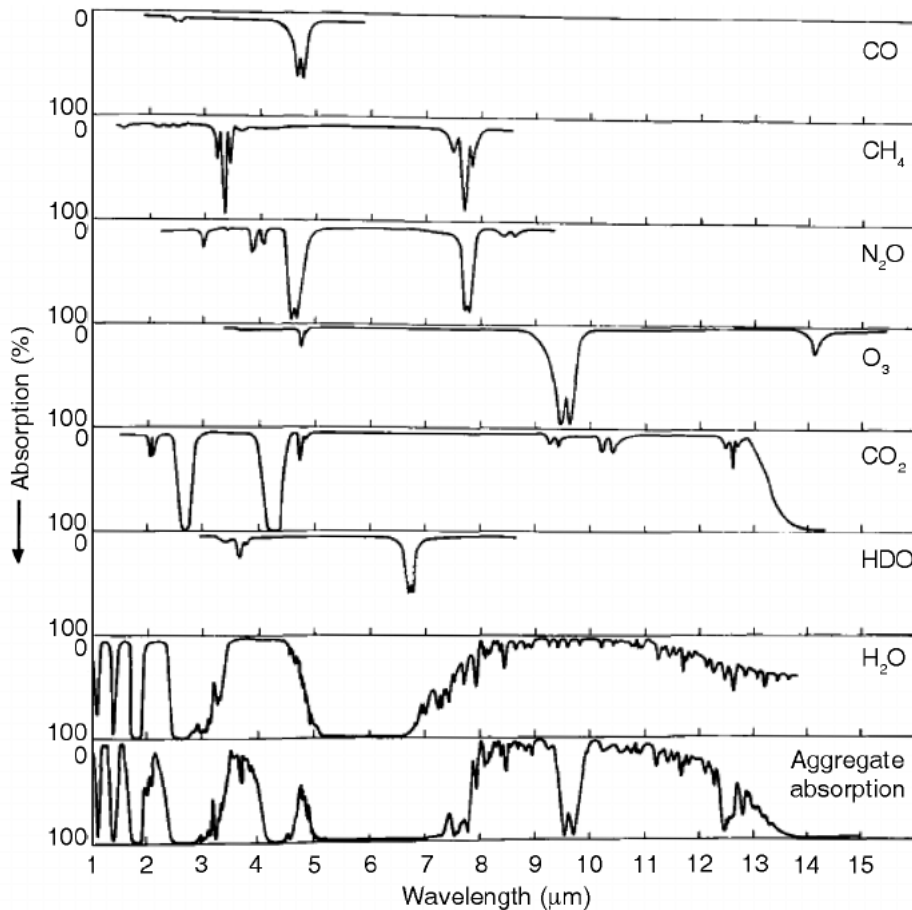


Figure 3.3: Absorption spectra for gases found in Earth's atmosphere, with a focus on the near-IR part of the electromagnetic spectrum. The vertical axis indicates percentage of radiation absorbed [7].

are not strictly lines of infinitesimal width, rather, they are wider due to "broadening" affects. These are listed below [23]:

- **Natural line broadening:** due to the uncertain nature of quantum energy states; homogeneous widening in Lorentzian shape; the broadening (defined as change in frequency) is:

$$\Delta\nu \geq \frac{32\pi^2\nu^3}{(4\pi\epsilon_0)3hc^3}|R^{nm}|^2, \quad R^{nm} \text{ is the transition moment}$$

- **Doppler broadening:** due to the Maxwell velocity distribution of the gas atoms/molecule relative to the detector; homogeneous widening in Gaussian shape; the broadening is:

$$\Delta\nu = \frac{\nu}{c} \left(\frac{2kT \ln 2}{m} \right)^{1/2} \quad m \text{ is the mass of the atom/molecule}$$

- **Pressure broadening:** due to collisions between atoms/molecules; homogeneous widening in Lorentzian shape, except at low frequencies; the broadening is:

$$\Delta\nu = (2\pi\tau)^{-1} \quad \tau \text{ is the mean time between collisions}$$

- **Power/saturation broadening:** due to increased intensity of incident light; homogeneous widening in Lorentzian shape [43]

3.2. Scattering

Scattering is the phenomenon of light interacting with an obstacle and being redirected onto new paths which can go in different directions. This process has two parts: (1) the excitation of the charges in the obstacle by the photon and (2) the subsequent reradiation of a photon by the charges [2]. The type of scattering depends upon the ratio of the radius, r , of the molecule to the incoming light's wavelength, λ [24]. We will call this parameter x , where:

$$x = \frac{2\pi r}{\lambda} \quad (3.4)$$

Scattering can be broken down into the following categories:

- **Elastic Scattering:** the photon's kinetic energy is conserved, thus the wavelength of the scattered light is the same as the wavelength of the incident light
 - **Rayleigh Scattering:** Light wavelength is much larger than the particles, $x \ll 1$. Occurs mostly in the upper atmosphere. Examples of the particles are small dust particles and nitrogen and oxygen molecules. Scattering intensity is inversely proportional to the wavelength's fourth power.
 - **Mie Scattering:** Light wavelength is same size as the particles, $x \sim 1$. Occurs mostly in the low atmosphere and when clouds are abundant. Examples of the particles are dust, pollen, smoke, and water vapor. Scattering intensity is mostly independent of wavelength, but dependent on particle size (the larger the particle, the more forward scattering).
- **Inelastic Scattering:** the photon's kinetic energy is not conserved, thus the wavelength of the scattered light is different than the wavelength of the incident light

Additionally, there is a distinction between single scattering and multiple scattering. The former is defined as a photon scattering once and is common in optically thin media; the latter is defined as photon scattering more than once and is common in optically thick media. Single scattering is an important departure point for understanding the mechanics of scattering. However, the atmosphere is optically thick enough to produce multiple scattering, and so it too will be discussed.

In the following sections on Clouds and Turbulence, two prominent sources of scattering in the atmosphere, the final result of the scattered light is the most important consideration. Scattering changes the original path, and therefore the final destination, of the light from a celestial object. Regardless of the nature of the scattering— what type and how many times a particle is scattered— the question remains: does the final scattering angle place the light in or out of the detector? Furthermore, are the wavefronts coherent enough to form an image or trace the light to its origin point?

3.2.1. Clouds

Liquid and solid matter contained in the atmosphere are considered cloud particles, aerosol particles, or falling hydrometeors. For simplicity, in this research, they will be referred to as cloud particles, but they are technically distinguishable by size, chemical composition, water content, and fall velocity. Clouds form when air rises, expands, and cools. The initial rising air can be caused by air heated by the Earth's surface, air forced upward by elevation, air forced upward by an area of low pressure, and weather fronts (when large masses of cold and hot air collide). This cooled air reaches and falls below the dew point, the water vapor in the air condenses—changes into a liquid—on condensation nuclei [5]. These nuclei are actually a particle of dust, pollen, metal or other material which makes it easier for the water vapor to turn into a water droplet. In addition to liquid water, ice crystals can form on the particles. The cloud particles are small enough and therefore have a small enough mass to remain suspended [3]. As stated in the previous section, light interaction with clouds can be described by Mie scattering. It is also important to note that different types of clouds have different properties and will therefore affect light differently. These properties include: mass/amount of particles, temperature, ratio of liquid to solid particles, and separation of particles. Figure (3.4) shows a graph of clouds by altitude and temperature. The higher the altitude, the more ice crystals in

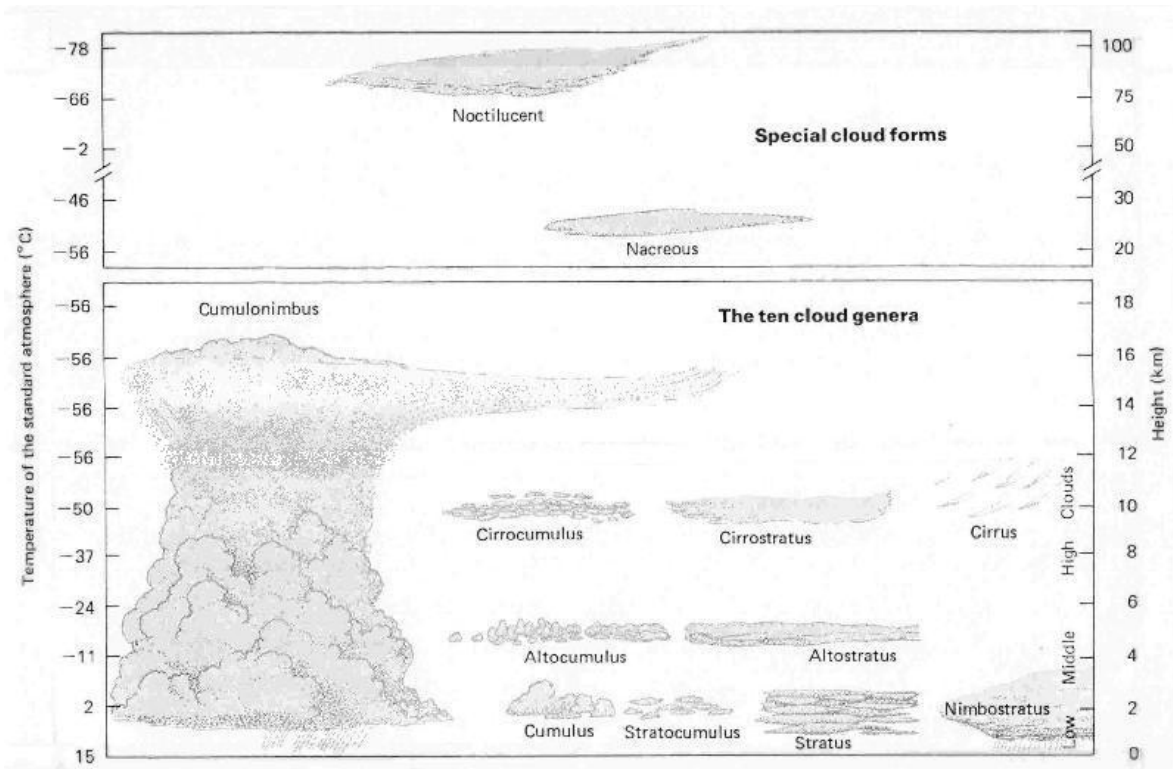


Figure 3.4: Different types of clouds, their altitudes, and the temperature at that altitude [27].

the cloud, while clouds at lower altitudes have more liquid water droplets. The transparency of the cloud reveals how full of moisture it is; the more transparent, the less moisture. In particular, cirrostratus, altostratus, stratocumulus, and nimbostratus hold large amounts of moisture.

3.2.2. Turbulence

Greek philosopher scientist Aristotle noticed the effects of turbulence on the appearance of stars, and referred to their twinkling as stellar "scintillation". Newton noticed that the air through which he looked with his telescope was shaking and that his telescopic observations were more distorted (less clear) than theoretical calculations predicted. When observations were made at certain locations, such as mountain tops, these detrimental affects decreased. Tycho Brahe in the 16th century was the first person to suggest that the cause was at least partly atmospheric. However, it was not until 1665 that Robert Hooke introduced the refraction theory of scintillation, citing "moving regions of atmosphere [with] different refracting powers which act like lenses" as the cause of the turbulent motions which had been observed for thousands of years [10].

Turbulence can have a multitude of effects on the appearance of an object, including: varying brightness, displacement from actual position, smearing out, continuous motion about an approximate center, long lasting/far moving oscillations, changing size, and pulsating irregular changes in illumination [30]. They are caused by fluctuations in the index of refraction of the air, which causes the light to bend differently, and chaotically. Turbulence is the cause of these fluctuations, and can be viewed as instability in a fluid flow which occurs at a high Reynold's number, Re , defined as [16]:

$$Re = \frac{L_0 u}{\nu} = \frac{L_0 u \rho}{\mu} \quad (3.5)$$

Where:

- L_0 : characteristic length, which defines the scale of the system
- u : characteristic velocity, which defines the flow speed of the fluid
- ν : kinematic viscosity, the ratio of viscosity to density $\left(\frac{\mu}{\rho}\right)$
- ρ : density
- μ : viscosity

Additional properties of turbulence include: (1) irregularity and randomness, (2) diffusivity (causes increased momentum, heat, and mass transfer rates), (3) fluctuating vorticity (spinning motions), and (4) dissipation if not continually supplied with outside energy (kinetic energy of turbulence is transferred to the internal energy of the fluid) [37].

Turbulence in the atmosphere is caused by wind over obstacles and by differences in pressure, temperature, humidity, and velocity in the air. Laminar—"smooth"—air becomes turbulent, characterized by swirling eddies, as seen in Figure (3.5) [35]. These eddies are at first large, on the scale of tens of meters, but then break down into smaller and smaller eddies, on the scale of millimeters, seen in Figure (3.6). The larger scale, L_0 , is called the outer scale, while the smaller scale, l_0 , is called the inner or damping scale. The varying scales in the middle are called spatial scales, l [34]. The entire phenomena of this progressive down-scaling is called a "turbulent cascade" [11]. The kinetic energy of the large scale motions transfers into the energy of the many small scale motions, until the viscosity of the fluid prevents successive

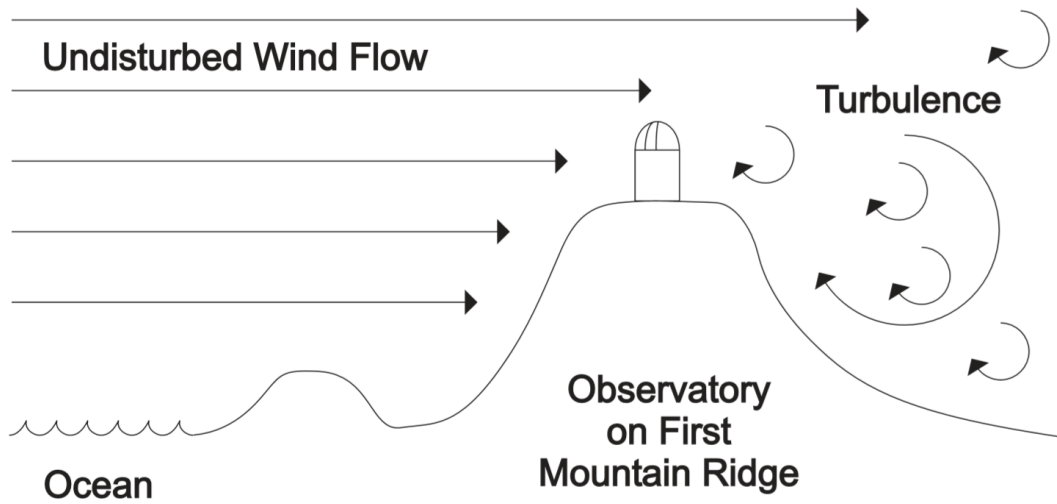


Figure 3.5: Undisturbed, or laminar, flow comes off the ocean and becomes turbulent once it encounters the mountain. To decrease seeing, observatories are placed not only on high altitudes (in order to look through less atmosphere), but on the first mountain ridge near the ocean (in order to get the undisturbed ocean winds) [35].

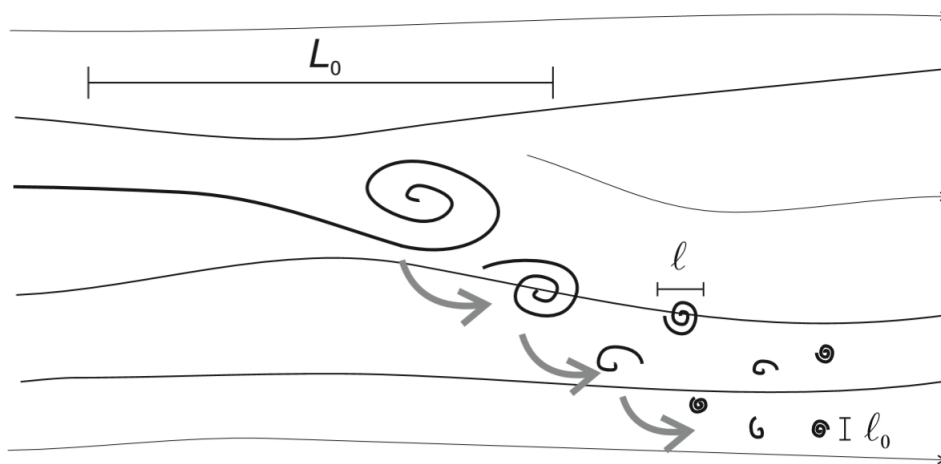


Figure 3.6: From the largest scale eddy, $L_0 \sim 10m$, down to the smallest, $l_0 \sim 1mm$ [34].

break-downs. At this length, called the dissipation scale L_v and where $R = 1$, since the eddies cannot break down further, the energy begins converting into internal energy. The cascade of eddies can be defined in terms of an energy dissipation or flow rate, called Kolmogorov’s law for turbulence:

$$E(k) = C\varepsilon^{2/3}k^{-5/3} \tag{3.6}$$

Where:

- $E(k)$: energy stored in each k-mode per gram of gas
- k : in units of $\frac{1}{\text{length}}$, this defines a new space, called k-space, which enables describing the eddy cascade from small to large k
- C : constant, approximately equal to one
- ε : energy flow

This definition stems from Kolmogorov's theory of turbulence which will be used in the formalism that follows. From Equation (3.6), it can be determined that the time scale of eddy motions is proportional to the size of the eddies, with the relationship:

$$\tau_{eddy} = \varepsilon^{-1/3} l^{2/3} \quad (3.7)$$

Additionally, the equation for the dissipation scale becomes:

$$L_v = \left(\frac{v^3}{\varepsilon} \right)^{1/4} \quad (3.8)$$

To continue into the theory of how turbulence affects light waves (the effects are, in fact, due to the wave characteristics of light), the notion of a structure function must be introduced. These are used to describe the spatial structure of a random process. In general, a structure function, $D_x(R_1, R_2)$, is defined as:

$$D_x(R_1, R_2) = \langle |x(R_1) - x(R_2)|^2 \rangle \quad (3.9)$$

Where:

- R_1 : position one
- R_2 : position two
- x : variable which is being considered and measured at the two positions

This equation yields the expected value of the difference between x as it is measured at R_1 and at R_2 . The fluctuating variables considered here in turbulence calculations are velocity and refractive index. Their structure functions are given in Equations (3.10) and (3.11).

$$D_u(R_1, R_2) = C_u^2 \cdot |R_1 - R_2|^{2/3} \quad (3.10)$$

$$D_n(R_1, R_2) = C_n^2 \cdot |R_1 - R_2|^{2/3} \quad (3.11)$$

Where:

- C_u^2 : velocity structure constant defining the turbulence strength, $C_u^2 = k\varepsilon^{2/3}$
- C_n^2 : refractive index structure constant, $C_n^2 = \frac{7.8 \times 10^{-5} P}{T^2}$

Turbulent differences in temperature alter the density, and thus the refractive index. When light waves travel through turbulent regions, specifically one of thickness δh , they undergo a phase shift. This can be defined using a structure function as well:

$$D_\phi(R_2 - R_1) = 2.914 \left(\frac{2\pi}{\lambda} \right)^2 \delta h C_n^2 (R_2 - R_1)^{5/3} \quad (3.12)$$

Note that the phase structure function depends on the refractive index structure constant. A useful way to define how turbulence affects an image is using the Fried parameter, r_0 :

$$r_0 \equiv \left(0.423 \left(\frac{2\pi}{\lambda} \right)^2 \sec \xi \int_0^\infty C_n^2(z) dz \right)^{5/3} \quad (3.13)$$

Where:

- ξ : zenith angle
- z : altitude
- $C_n^2(z)$: altitude dependent refractive index structure constant

This definition of r_0 describes the length for which errors in the wave phase are on the order of one radian. If the seeing conditions are characterized by r_0 and a long exposure image is taken, the image quality is approximately equal to the image taken by a telescope with diameter r_0 . For the purposes of Terrascope research, the definition of r_0 would have to be adjusted to integrate along a light ray, rather than its current form along the entire atmosphere at some angle to the zenith. Adjustments include: taking out the secant of the zenith angle, changing z to be the position (independent variable for the structure constant), and dz to the path length. These changes are necessary because the Fried parameter is an important variable in turbulence calculations and the remainder of the definitions rely on it. The phase structure function can be redefined in terms of the Fried parameter, as seen in Equation (3.14). Additionally, the coherence function of the wave front is introduced as Equation (3.15), also in terms of the Fried parameter. Wave coherence is essential for production of a clear image, and many times, turbulence is the cause of decoherence [35], [11].

$$D_\phi(R_2 - R_1) = 6.88 \left(\frac{R_1 - R_2}{r_0} \right)^{5/3} \quad (3.14)$$

$$B_\psi(R_1 - R_2) = \exp \left[-3.44 \left(\frac{R_1 - R_2}{r_0} \right)^{5/3} \right] \quad (3.15)$$

Equation (3.13) suggests that the Fried parameter is wavelength dependent. It has the following scaling law:

$$r_0 \propto \lambda^{6/5} \quad (3.16)$$

Thus, longer wavelengths yield better quality images. Lastly, this analysis only holds for turbulent layers smaller than the Fresnel length, d_F :

$$d_F = \frac{r_0^2}{\lambda} \quad (3.17)$$

When the layers are larger, which typically occurs at short wavelengths, large zenith angles, and poor observing sites, the light is heavily diffracted. One consequence of this diffraction is scintillation, the twinkling of stars that was initially observed and led to theories of turbulence.

3.3. Transmission and Reflection

Transmission is when light passes through a material without being scattered or absorbed. However, even in the absence of these two effects, when the light passes through to the other side, it contains the fingerprint of the material through which it passed. Thus, the Earth's atmospheric composition determines its transmission spectrum. Light that reaches the detector from a celestial object contains information from the Earth's transmission spectrum because the light passed through the atmosphere before arriving at the detector. Thus, this light and its spectral signature must be accounted for (subtracted) from the light that reaches the detector.

Sunlight reflects off the Earth resulting in a phenomenon called Earthshine. It can be seen as the unlit part of the Moon which is dark, but still visible; this is due to the Earth bouncing light onto the Moon [32]. While this research will not take into account this doubly-bounced light, it must account for the light reflected by the Earth because it could reach a Terrascope detector. Similar to transmission, the reflection depends on the composition of the Earth's atmosphere, but unlike transmission, it also depends the Earth's surface. The reflection spectrum has a unique signature and can be subtracted from the final light that reaches the detector.

During a lunar eclipse, the transmission and reflection spectra were measured. Thus, they can be subtracted from light received by the Terrascope detector [33]. The reflection and transmission spectra are shown in Figure (3.7).

Note that the relative positions of the Earth, detector, and Sun must also be taken into consideration. If the Earth is between the detector and the Sun, then the transmission of Sunlight through the Earth's atmosphere will reach the detector, but the reflection will not. However, if the Sun is directly behind the Earth, shining through the atmosphere, its light will overwhelm (and possibly break) the detector, making this configuration unusable for observations. If the detector is between the Earth and the Sun, Sunlight reflected off the Earth's surface and atmosphere will also overwhelm the detector. It is at points between these two extremes where the Terrascope can be used, although transmission and reflection spectra must still be accounted for. Figures () shows three different configurations of the Earth-Sun-Detector system.

3.4. Latent Emission: Airglow

The light emitted by the Earth's atmosphere is called airglow. The human eye can detect it faintly in the air, but its signal, an emission line in the atmosphere, was first identified in 1868 by Anders Ångström. However, it was then considered the same phenomenon as aurorae. It was discovered later that although airglow is the result of atomic and molecular excitations occurring during the evening, the cause of the excitations began during the day. Ultraviolet light from the Sun separates oxygen molecules, O_2 , which cannot recombine quickly; thus they remain until the evening, when they interact with other atoms and molecules. These interactions cause radiation (light) and corresponding emission lines in the near-UV and near-

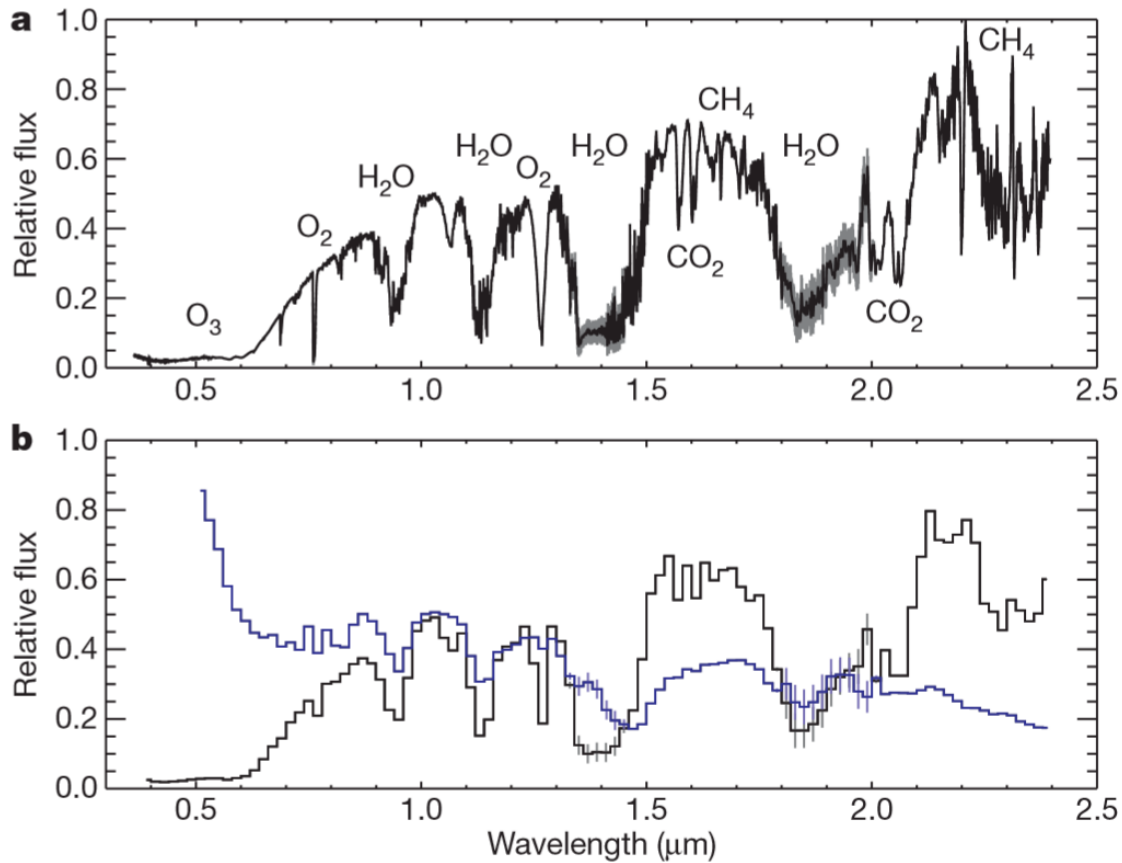


Figure 3.7: The top image shows the transmission spectrum; the bottom shows the transmission and reflection spectra, with the former in blue (or gray, if not being viewed in color) and the latter in black.

IR range. The lines are characterized by their clustered structure and narrow width. Although colors cannot be seen with the naked eye, but long-exposure photographs reveal the airglow to be red and green. The atoms and molecules involved in the production of this phenomena include O, O₂, Na, and OH. The light varies with time of night, location, and solar activity [9]. Models have been made to account for this variability when making observations from Earth-based telescopes [31]. Since the Terrascope's lensed light would go through the atmosphere, airglow contributes to the detected signal.

3.5. Variability

As suggested in the previous section, the Earth and its atmosphere are prone to variability based on time of day, location, and solar activity. There are other variabilities which have different time and length scales, all of which could affect the Terrascope. Among the temporal fluctuations are [45]:

- **Turbulence:** on the order of seconds and minutes, this phenomena, covered in more detail in a previous section, produces constantly changing wind, temperature, density, and refractive index
- **Gravity Waves:** on the order of hours, these affect temperature, wind, and ozone loss

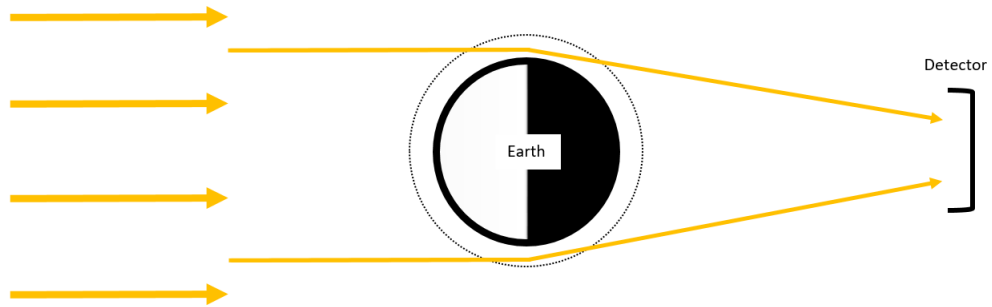


Figure 3.8: Earth between the Sun and detector, perfectly aligned. This is the worst case scenario because the Earth's atmosphere is effectively amplifying the Sun's light into the detector. This completely overwhelms information from any other celestial object, if not destroying the detector's sensitive light-gathering equipment. However, if the detector is moved even slightly off-center, the Sunlight amplification effect is greatly reduced.

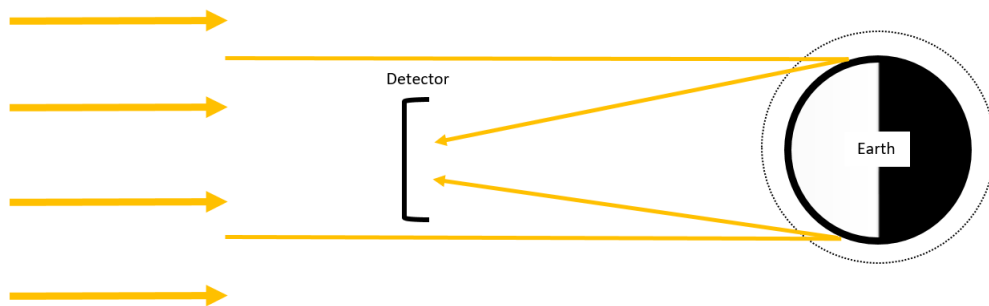


Figure 3.9: Detector between the Sun and the Earth, perfectly aligned. This configuration also poses a problem because the Sunlight reflects off the Earth and into the detector. The reflected light is quite strong and in general, the detector should face as little of the Earth's day-side as possible.

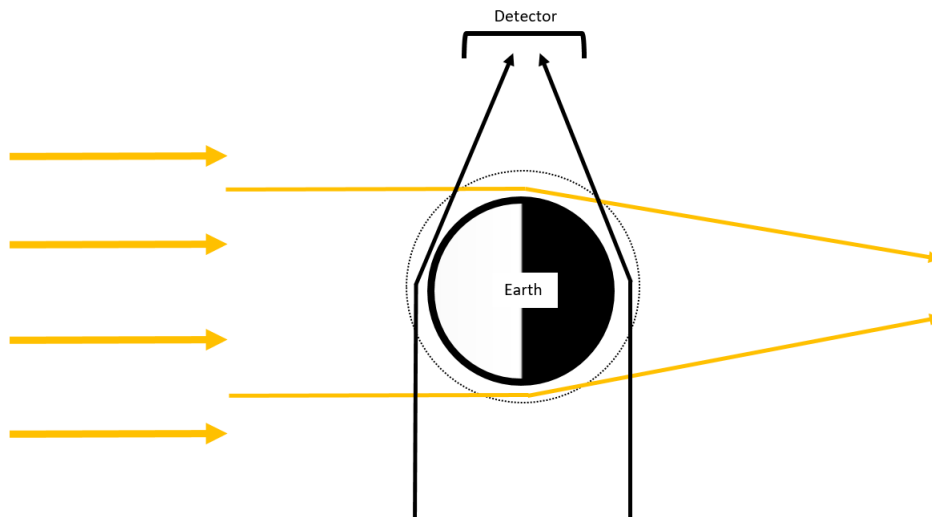


Figure 3.10: Detector at a 90° angle to the Earth and Sun. In this configuration, no direct or amplified Sunlight reaches the detector and only half of the Earth appears on the day side. Although it is not shown, some Sunlight can reflect off the Earth and into the detector, but the effect is much smaller than in Figure (3.9). Thus, light from a celestial object is able to lens into the detector relatively undisturbed. However, it is not completely undisturbed; not shown in this 2D image are the Sunlight and object light both being lensed "underneath" and "over" the plane of the page.

- **Weather Systems:** on the order of days, these are the generalized name for the colloquial "weather" and dictate temperature, wind, dryness, and precipitation
- **Atmospheric Blocking:** on the order of weeks, these are also large in spatial scale and are high-pressure areas which cause long-lasting, extreme weather conditions such as heat waves, cold spells, poor air quality, and superstorms

These and other variables affect convection, clouds, temperature, and other integral parts of the atmosphere, in turn, affecting the light which passes through the atmosphere. In order to make an effective Terrascope, the light which eventually hits the detector must be analyzed for these effects. It is comparatively simple to account for a storm which passes over an Earth-based telescope. However, the Terrascope will receive light from all parts of the Earth, and thus the large- and small-scale variabilities must be accounted for.

4

A Return to the Terrascope

Kipping's initial research shows that the Terrascope could have enormous potential; this research sets out to rigorously test that theory using complex models. Those models will use the geometric optics theory introduced in Chapter 2 and will take into account the effects described Chapter 3. Kipping only employed basic extinction (the combined affects of absorption) models used for ground-base telescopes. He then used an average effective cloud fraction to calculate a cut-off point below which light rays would not be considered, necessitating placing the detector much farther from Earth. The final amplification was halved to account for the Sun's location and direct light into the detector. This research aims to take into account the complex structure of the atmosphere and determine if the Terrascope is still possible. Furthermore it seeks to model what the detector would see from different types of celestial sources. Table (4.1) gives a detailed comparison of Kipping's research and the research that will be performed as part of this thesis.

Kipping	This Research
Start with source, end with detector	Start with detector, end with source
On- and off-axis calculated separately	Trace all rays to account for both on- and off-axis
Assume symmetry for on-axis	Don't assume symmetry
Off-axis amount denoted by offset distance, Q	Off-axis denoted by offset angle
Use global average atmospheric temperature and pressure profiles	Model small-scale, local temperature and pressure profiles
Accounts for clouds by averaging	Model local cloud affects on scattering, absorption, and transmission
No turbulence	Include turbulence
No atmospheric transmission and reflection	Include atmospheric transmission and reflection
No airglow	Include airglow
Approximate amplification	Numerically calculate amplification from light rays
Calculate shape of lensing	Render image and spectrum of source

Table 4.1: On the left, Kipping's original Terrascope research; on the right, the research planned for this thesis. The right column includes differences and additions.

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