Modelling Polymer Flooding in Reservoir Simulations

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Outline

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Recovery Phases

- Primary recovery: extraction by natural mechanisms
- Secondary recovery: injection of water (waterflooding) to keep high pressure
- Tertiary recovery: any other technique to maximize recovery, based on oil displacement
- Polymer flooding: polymer is added to water

Why Polymer Flooding?

- Polymer increases water viscosity
- Results in a more favourable (lower) mobility ratio

$$M = \frac{\lambda_w}{\lambda_o} = \frac{k_{r,w}\mu_o}{k_{r,o}\mu_w}$$

- μ_α viscosity of phase α
- $k_{r,\alpha}$ relative permeability of phase α

Important Terms

- Pores: tiny empty passages in a rock
- Porosity ϕ: fraction of volume of rock which is pore space
- Saturation S_α: fraction of pore space that fluids occupy
    - $S_o + S_w = 1$
    - Irreducible water saturation S_{wir}: amount of water that cannot be displaced
- Absolute permeability k: capacity of a rock to conduct fluid
- Relative permeability $k_{r,\alpha}(S_\alpha)$: effective permeability of a phase
- $\mathbf{v}_\alpha$: interstitial velocity
- $\mathbf{u}_\alpha = \phi S_\alpha \mathbf{v}_\alpha$: Darcy (or superficial) velocity
- Fractional flow $f_\alpha(S_\alpha)$: volumetric flow rate of a phase

$$\text{in 1D:} \qquad f_w = \frac{u_w}{u_w + u_o}, \quad f_o = \frac{u_o}{u_w + u_o}$$

Fractional Flow Curves

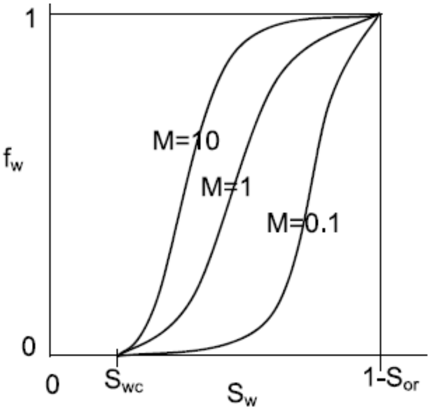


Figure: Fractional flow curve shifts to right adding polymer.

Darcy Law and Governing System

- Empirical law for two-phase flow

$$\mathbf{u}_\alpha = -\frac{k k_{r,\alpha}}{\mu_\alpha} \nabla p$$

- Governing equations from mass conservation law

$$\frac{\partial(\phi \rho_o S_o)}{\partial t} + \nabla \cdot (\rho_o \mathbf{u}_o) = 0$$
$$\frac{\partial(\phi \rho_w S_w)}{\partial t} + \nabla \cdot (\rho_w \mathbf{u}_w) = 0$$

- Additional relation

$$1 = S_w + S_o$$

Fractional Flow Formulation

- Assumptions:
  - The flow is horizontal and one-dimensional
  - Fluids are incompressible
- Total velocity $u = u_w + u_o$
- From previous assumptions and equations,

$$\frac{\partial u}{\partial x} = 0 \quad \Rightarrow \quad u \text{ is constant}$$

- Equation for water in fractional flow $f_w = u_w/u$

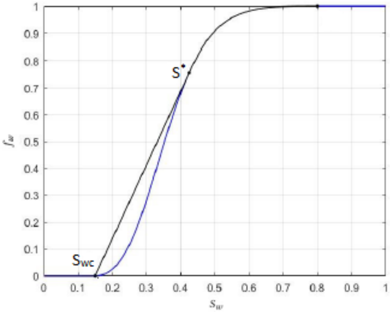
$$\phi \frac{\partial S_w}{\partial t} + u \frac{\partial f_w}{\partial x} = 0$$

- Hyperbolic equation with characteristic velocity

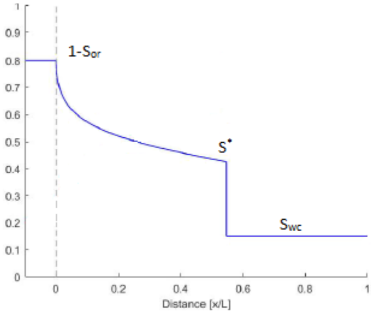
$$\left(\frac{dx}{dt}\right)_{S_w} = \frac{u}{\phi} \frac{df_w}{dS_w}$$

$\Rightarrow$ shock forms

Buckley-Leverett Front



(a) Fractional flow function and illustration of an admissible physical solution.



(b) Buckley-Leverett solution for a fixed $t > 0$.

Polymer Flood

- Extended fractional flow theory with further assumptions:
  - Polymer capillary forces and adsorption to rock are negligible
  - Polymer is present only in the aqueous phase
- Equations with polymer concentration c

$$\phi \frac{\partial S_w}{\partial t} + u \frac{\partial (f_w(S_w, c))}{\partial x} = 0$$
$$\phi \frac{\partial (c S_w)}{\partial t} + u \frac{\partial (c f_w(S_w, c))}{\partial x} = 0$$

- Two shocks arise

Buckley-Leverett Front

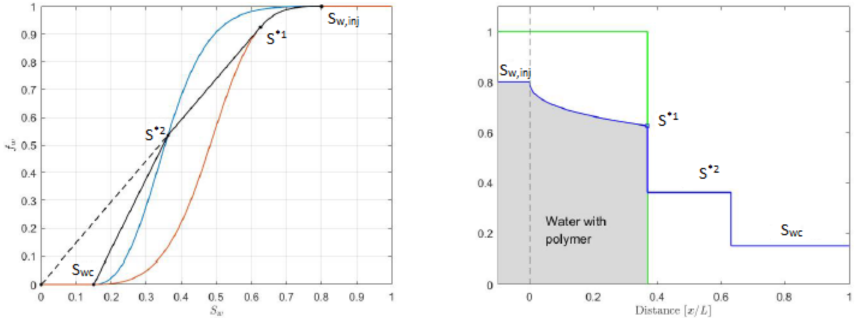


Figure: On the left, the construction of the polymer flooding solution through fractional flow function curves. On the right, water saturation profile is depicted in blue, while polymer concentration is depicted in green.

Inaccessible and Excluded Pore Volumes

- Inaccessible pore volumes (**IPV**): not all pores accessible to polymer
- Excluded pore volumes (**EPV**): layer close to the pore wall not accessible to polymer
- Due to IPV and EPV polymer travels faster than an inert tracer

⇒ Velocity enhancement effect for polymer molecules

Constant Velocity Enhancement Factor

- Model velocity enhancement through a constant factor

$$\alpha = \frac{\phi}{\phi_p} > 1$$

- Equations with velocity enhancement term

$$\phi \frac{\partial S_w}{\partial t} + u \frac{\partial f_w}{\partial x} = 0$$
$$\phi \frac{\partial c S_w}{\partial t} + \alpha u \frac{\partial c f_w}{\partial x} = 0$$

- Problem: constant factor leads to ill-posedness

Concentration Profile for Constant Factor

What is physical and what is due to ill-posedness and numerical instabilities?

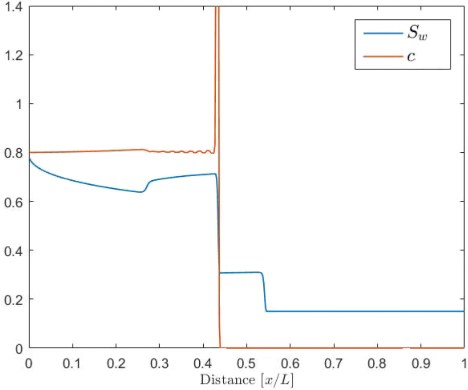


Figure: Water saturation and polymer concentration profiles with a constant velocity enhancement factor.

Percolation Model for IPV

- Pores have characteristic radius r
- Polymer cannot enter pores with $r < r^*$
- Water is assumed to enter smallest pores first, so that a threshold saturation S^* must be reached before polymer is allowed to flow
- Assume $S^* < S_{wir}$

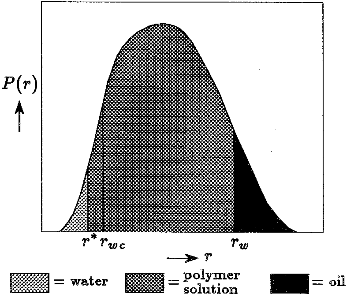


Figure: Example of a probability density function showing the critical radius r^*.

Saturation dependent factor

- Velocity enhancement factor is saturation dependent

$$\alpha(S_w) = \frac{S_w}{S_w - S^*}$$

- Restriction: $S^* < S_{wir}$
- Polymer continuity equation

$$\phi \frac{\partial c S_w}{\partial t} + u \frac{\partial \alpha(S_w) c f_w}{\partial x} = 0$$

- System is shown to be strictly hyperbolic

Concentration Profile

- No uncontrolled pile-up of polymer at the front
- Velocity enhancement model based on a physical concept, but is this the correct profile?

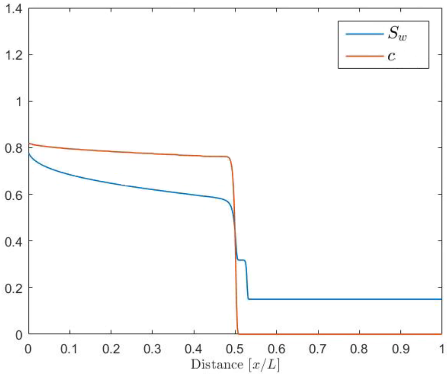


Figure: Monotone profile of water saturation and polymer concentration with percolation model.

Model Extension

- Aim: derive a robust model to relax the restriction $S^* < S_{wir}$
- Tool: derive a necessary condition on $\alpha(S_w)$ for well-posedness
- Use theory from hyperbolic conservation laws
- Focus on IPV effect

A Necessary Condition

- Assume polymer behaves as an inert component ($f_w = f_w(S_w)$)
- Discontinuity in water saturation results in a discontinuous flux for polymer concentration equation
- Riemann Problem with $z = S_w c$, $g_l \neq g_r$ are constants

$$\begin{cases} \dfrac{\partial z}{\partial t} + \dfrac{\partial (g_l z)}{\partial x} = 0 & \text{if } x < 0 \\ \dfrac{\partial z}{\partial t} + \dfrac{\partial (g_r z)}{\partial x} = 0 & \text{if } x > 0 \end{cases}$$

- Necessary condition: find values (z_-, z_+) at the discontiuity such that
  - For $x < 0$, waves travel only from right to left
  - Rankine-Hugoniot condition $g_l z_- = g_r z_+$
  - For $x > 0$, waves travel only from left to right
- Results in the condition on $\alpha(S_w)$

$$\alpha(S_w) \leq \frac{S_w}{S_w - S_{wir}}$$

Non-Uniform Polymer Diffusion

- Assume that pores are filled successively in increasing size
- IPV effect implies that there exists a value r^* such that

$$\hat{c}(\hat{r}) = \begin{cases} 0 & \text{if } \hat{r} \leq r^* \\ \bar{c} & \text{if } \hat{r} > r^* \end{cases}$$

- The model is still ill-posed for $S^* > S_{wir}$, so relax definition of inaccessibility

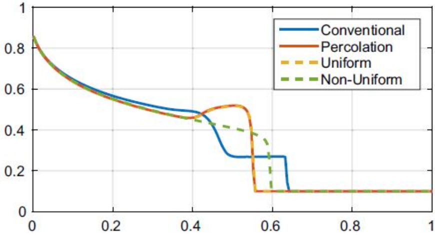
$$\hat{c}(\hat{r}) = \hat{w}(\hat{r})\bar{c},$$

$$\hat{w}(\hat{r}) = \begin{cases} \frac{\epsilon}{S^*} & \text{if } \hat{r} \leq r^* \\ 1 & \text{if } \hat{r} > r^* \end{cases}$$

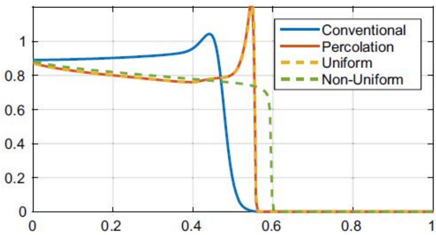
- The resulting model satisfies the necessary condition with optimal choice $\epsilon = S^* - S_{wir}$

Numerical Simulations

Results of simulations with $S^* > S_{wir}$



(a) Water saturation profiles for the four proposed models.



(b) Polymer concentration profiles for the four proposed models.

Numerical Methods: MRST Simulator

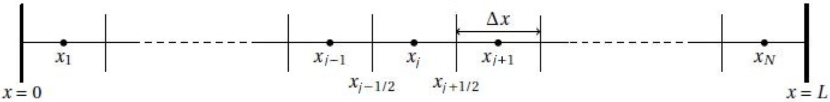


Figure: Cell-centered grid.

- Equations in conservation form

$$\frac{\partial(\phi S_\alpha)}{\partial t} - \frac{\partial}{\partial x}\left(\lambda_\alpha \frac{\partial p}{\partial x}\right) = 0$$

- Solve flow equations with finite volumes, first order upwind scheme for mobilities and implicit time

$$\phi \Delta x \frac{S_{\alpha,j}^{n+1} - S_{\alpha,j}^{n}}{\Delta t} = \left(\lambda_{\alpha,j}^{n+1} \frac{p_{j+1}^{n+1} - p_j^{n+1}}{\Delta x} - \lambda_{\alpha,j-1}^{n+1} \frac{p_j^{n+1} - p_{j-1}^{n+1}}{\Delta x}\right)$$

- Newton method for non-linear mobilities

Scheme for the Coupled Equations

- MRST simulator solves flow and polymer transport equations simultaneously with an implicit scheme
- Scheme is stable, but computationally expensive and inaccurate
- Alternative: sequential approach
  1. Solve for S_α^{n+1} using c^n and the implicit solver
  2. Solve for c^{n+1} using S_w^{n+1}

$$(cS_w)_j^{n+1} = (cS_w)_j^n - \frac{\Delta t}{\phi \Delta x}\left(F_{j+1/2}(S_w^{n+1}, c^n, c^{n+1}) - F_{j-1/2}(S_w^{n+1}, c^n, c^{n+1})\right)$$

- Need an expression for the fluxes $F_{j\pm1/2}$

High-Resolution Schemes

- To improve accuracy, use a high order flux defined as

$$F_{j+1/2} = F_{L,j+1/2} + \Phi_{j+1/2}(F_{H,j+1/2} - F_{L,j+1/2})$$

- $\Phi_{j+1/2}$: flux limiter function
- An implicit scheme preserves monotonicity, but it is inaccurate and expensive
- A semi-implicit scheme is more accurate, but conditionally monotone
- Strategy: use semi-implicit high-resolution and switch to first order implicit scheme at the discontinuity

Numerical results

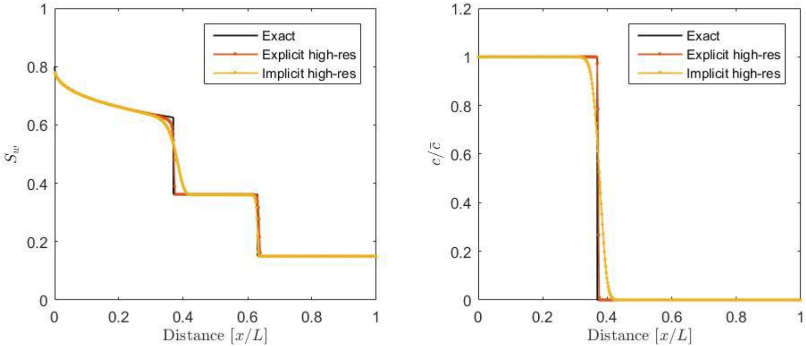


Figure: Solutions for water saturation and normalized concentration using high-resolution methods and 500 cells and time steps (No IPV effects).

Conclusions

- Polymer flooding may considerably increase performance, but the modeling presents physical and numerical challenges
- A constant velocity enhancement factor leads to ill-posedness and uncontrolled sharp peaks
- Saturation dependent models result in a monotone profile, but these models must be validated with physical experiments
- Common simulators still employ a constant factor
- Numerical schemes stability depends also on the adopted model of velocity enhancement
- Future work: acquire experimental data to validate appropriate models and derive robust numerical schemes