Relative Permeability Hysteresis in Porous Media

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Abstract

In reservoir modeling, relative permeability hysteresis in multi-phase systems has been broadly documented. The models introduced by Killough (1976) and Carlson (1981) have become the industry standard for simulating hysteretic variables in two-phase systems in porous media. These models assume relative permeability functions to be independent from time. Here, a time-dependent relative permeability problem is formulated and a corresponding model is presented.

1 INTRODUCTION

In petroleum reservoir engineering various techniques are used to enhance the oil recovery from a reservoir. Practices such as water and gas injection have the secondary effect of changing the internal configuration of the fluids inside the reservoir. Afterward the system behaves differently, depending not only on the present state of the reservoir but also on its previous history. Several variables presenting this "memory", known as hysteresis, must be integrated properly into the simulation.

Such variables are usually determined empirically, but recent analytic models are starting to predict these functions. Hysteretic variables and their modeling are the main topic of this document. In particular, a model will be sought to predict the values of the relative permeabilities of a system in porous media.

1.1 Permeability

Before describing hysteresis, let us first review the definition of permeability. It represents the capacity for flow through porous material, with higher permeability representing higher capacity. The relation between the permeability k of a fluid and its flow q (per square unit) is specified by Darcy's law:

$$q = \frac{k}{\mu} \frac{\Delta P}{L} \tag{1}$$

where μ is the viscosity of the fluid, ΔP is the pressure gradient, and L the length of the material through which the fluid moves.

Permeability in this case is determined by the material alone. However for multiple phase flow, that is, a system containing two or more fluids, the presence of one fluid affects the flow of the others. The capacity of one phase to flow with respect to the others is called the relative permeability. Hence when considering an oil and water system, the total permeability of the oil, for example, is related to both the material and the amount of water.

We call absolute or intrinsic permeability the one determined by the porous material alone. The total permeability is the product of the absolute and relative permeabilities. While the absolute term is usually considered constant, the relative part varies with the amount of the other phase. In fact, permeabilities are complex functions of the structure and chemistry of the fluids, and as a result they can vary from place to place in a reservoir. Inside a control volume, however, we may look at the relative permeabilities as functions solely of the quantity of each fluid.

The fluids in a system are often characterized by their wettability. Since water has higher wettability than oil, in a two phase system they are usually referred to as the wetting and the non wetting phase, respectively. The fraction of the pore space occupied by each phase in the control volume is called its

saturation. The water saturation and oil saturation in a two phase system are denoted s_w and s_n, respectively. If the system consists of only these two fluids, then

$$s_w + s_n = 1 \tag{2}$$

holds at all times. Since we can always recover $s_n = 1 - s_w$, it suffices to look at variables as functions of one saturation only. It is important to mention that the saturation s_w, and hence also s_n, never physically attains the values 0 nor 1, not even asymptotically. The actual range of the saturation goes from s_{wc}, called the critical saturation, to the maximum saturation s_w^{max}.

The usual shape of the water relative permeability function k_w, with respect to the water saturation s_w, is as follows
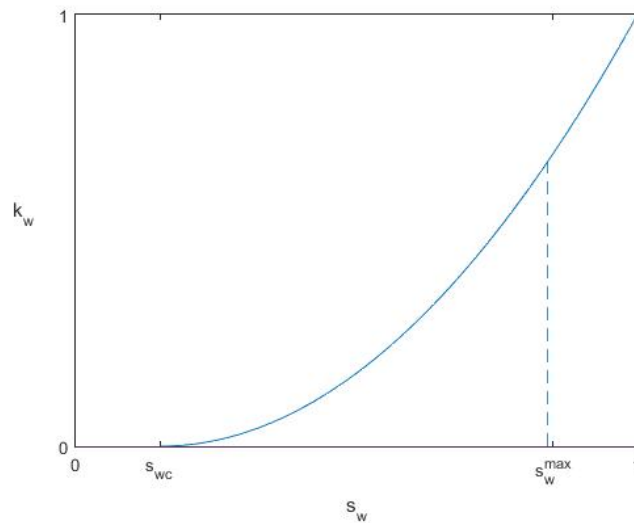

Figure 1: Relative permeability of the wetting phase vs wetting phase saturation.

This figure agrees with the intuition that the less oil there is in the volume, the easier the water will flow. Conversely, one would expect oil relative permeability k_n to increase as the water saturation drops.

This is in fact what occurs, the curve of k_n is always a decreasing function of s_w. However, a mayor anomaly is observed every time this is measured: the shape of the function k_n changes depending on whether the water saturation is increasing or decreasing. Indeed, this means we have two different shapes for this curve:

The value of k_n depends on both the saturation s_w and the direction in which it is moving. When the water saturation is decreasing, i.e. when $\frac{\partial s_w}{\partial t} < 0$, then
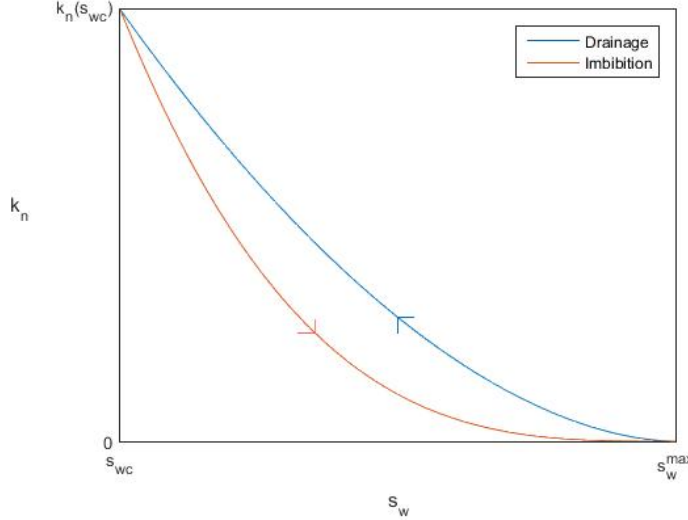
3

Figure 2: Relative permeability of the non-wetting phase vs water saturation.

k_n follows the first curve, which is called the *drainage* curve, since water is being "drained" from the volume. Conversely when $\frac{\partial s_w}{\partial t} > 0$, we are in the case of the second curve, called the *imbibition* curve.

This is explained by the fact that oil and water move differently through porous media, hence as one phase forces the other out, the distribution of the saturations inside the volume changes significantly, which in turn affects the phases capacity to flow, producing a process that is not exactly reversible.

The result is that k_n is a function not only of s_w but also of the previous state of the system, i.e. its history. This dependence on the past of the system is called hysteresis. Other variables such as capillary pressure also exhibit hysteretic behavior when plotted against changes in water saturation.

1.2 Scanning Curves

Figure 2 shows two cases where the derivative does not change sign at any time. Let us consider the alternative:

Assume water saturation is at its minimum, i.e. s_{wc}, and water starts being pumped into the reservoir, then k_n should follow the imbibition curve until the saturation reaches s_w^{max}. If, after reaching its maximum, the water starts being drained, k_n will now follow the drainage curve until $s_w = s_{wc}$ again.

However, if the draining process is interrupted before the saturation reaches s_{wc}, for instance at s_{wi}, $s_{wc} < s_{wi} < s_w^{max}$, and water starts being pumped back into the volume, now k_n needs to stop following the drainage curve and follow

4

the imbibition curve instead. But the imbibition and drainage curve do not intersect at almost any point, so for the transition from one to the other to be smooth, we need anther curve starting at point $(s_{wi}, k_n(s_{wi}))$. These transition curves are called *scanning* curves.
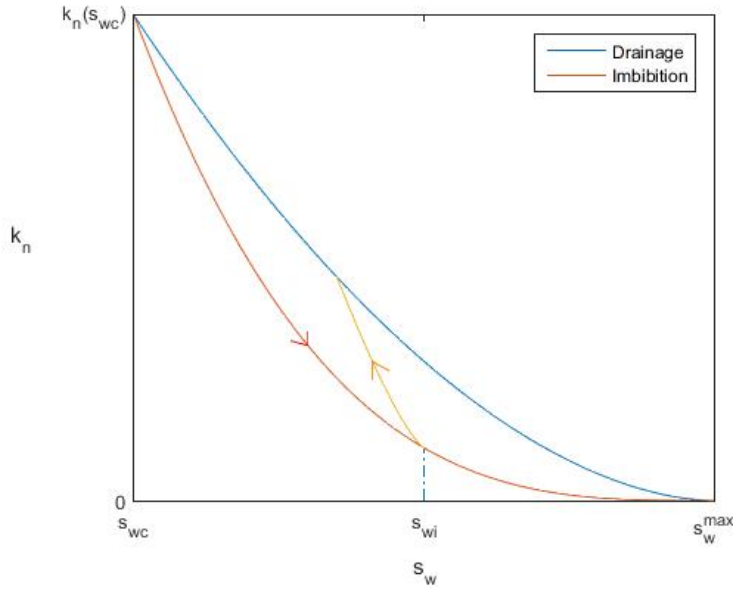

Figure 3: Imbibition process for k_n reversed at s_{wi}.

Figure 3 shows an imbibition process that has been reversed, hence the scanning curve goes in the opposite direction, i.e. the direction of decreasing wetting saturation, until it reaches the drainage curve. At this point, k_n follows the drainage curve again. Analogously, the drainage process can also be reversed at any point, which would result in a scanning curve going in the direction of increasing saturation, until it reaches the imbibition curve again.

In fact, any model that wishes to accurately describe k_n would need an infinite number of scanning curves, at any point s_{wi} where the process may be reversed.

In general the drainage and imbibition curves, also called bounding curves, are empirically known and the scanning curves are predicted based on this information. On this document, several models describing different methods of constructing these scanning curves will be examined.

5

2 TWO-PHASE SYSTEMS

As stated before, various techniques are common practice for enhanced oil recovery. In water-flooding, water is injected in one or more places (injection wells) in a reservoir under high enough pressure for the oil in the reservoir to be pushed by the injected water towards the producing wells of the reservoir (oil displacement). In water alternating gas (WAG) injection water and gas are injected in turn for the same effect.

Consider a water-flooding in one space dimension. On one end water is injected and on the other end oil and water are produced. Both oil and water are assumed to be incompressible. In one spatial dimension, the flow is described by variables depending on (x, t), the space and time coordinates. The main variables driving the model are phase saturation s_l and phase pressure p_l.

2.1 Transport Equations

The two-phase flow model of incompressible fluid flow through a porous medium in one space dimension is given by the transport equations for oil and water mass:

$$\frac{\partial}{\partial t}(\phi \rho_w s_w) + \frac{\partial}{\partial x}(\rho_w v_w) = 0. \tag{3}$$

$$\frac{\partial}{\partial t}(\phi \rho_o s_o) + \frac{\partial}{\partial x}(\rho_o v_o) = 0. \tag{4}$$

where v_l is the seepage velocity for phase l. This is not the actual velocity of a phase but its apparent velocity through the reservoir. Actual velocity is higher because of the tortuosity of the actual path of the flow through the pore space. According to Darcy's law for two phase flow in a porous medium, seepage velocity is given by

$$v_l = -K \frac{k_l}{\mu_l} \frac{\partial p_l}{\partial x} \tag{5}$$

As before, p_l represents the pressure of each phase and k_l the relative permeability. Two properties of the fluids, the mass density ρ_l and the viscosity μ_l are assumed constant. The permeability K and the porosity ϕ, i.e. the fraction of the total volume occupied by pores, are properties of the porous rock and are also taken as constants.

2.2 Convection-Diffusion Equation

The total velocity is defined $v = v_o + v_w$. Dividing equations (3) and (4) by ρ_w and ρ_o respectively, adding the resulting equations, and using the fact that $s_w + s_o = 1$, we obtain

$$\frac{\partial v}{\partial x} = 0$$

Thus v is a function solely of t and is determined by boundary conditions. For simplicity, is taken to be nonzero and independent of time.

The water and oil fractional functions are defined, respectively, by

$$f_w = \frac{k_w/\mu_w}{k_w/\mu_w + k_o/\mu_o} \quad \text{and} \quad f_o = \frac{k_o/\mu_o}{k_w/\mu_w + k_o/\mu_o}$$

Clearly $f_w + f_o = 1$. It is easy to see that

$$v f_w = -K \frac{k_w}{\mu_w} \left(f_w \frac{\partial p_w}{\partial x} + f_o \frac{\partial p_o}{\partial x} \right) \tag{6}$$

We define the capillary pressure as the difference in pressures $p_c = p_o - p_w$. Using the fact that $f_w + f_o = 1$, we obtain

$$v f_w = v_w - K \frac{k_w}{\mu_w} f_o \frac{\partial}{\partial x} p_c \tag{7}$$

Substituting v_w from equation (7) into equation (3) yields the convection diffusion equation for the water phase

$$\frac{\partial}{\partial t}(\phi s_w) + \frac{\partial}{\partial x}(v f_w) = \frac{\partial}{\partial x}\left[K \varepsilon \frac{\partial s_w}{\partial x} \right]$$

where

$$\varepsilon = -\frac{k_w}{\mu_w} f_o \frac{\partial p_c}{\partial s_w}$$

is the capillarity-induced diffusion coefficient. For $v \neq 0$, we can set

$$t = \frac{\phi K}{v^2} \tilde{t} \quad \text{and} \quad x = \frac{K}{v} \tilde{x}$$

in order to remove constants K, ϕ and v from our equation. For simplicity, we drop the tildes:

$$\frac{\partial}{\partial t}(s_w) + \frac{\partial}{\partial x}(f_w) = \frac{\partial}{\partial x}\left[\varepsilon \frac{\partial s_w}{\partial x} \right] \tag{8}$$

An analogue expression is found for the oil phase. Notice how only the terms f_w and ε are subjected to hysteresis, as they depend on k_w, k_o and p_c.

2.2 Wettability

Wetting is the ability of a liquid to maintain contact with a solid surface. The degree of wetting, known as wettability, is determined by a force balance between adhesive and cohesive forces. Adhesive forces between a liquid and solid cause a liquid drop to spread across the surface. Cohesive forces within the liquid cause the drop to ball up and avoid contact with the surface.

The contact angle θ is the angle at which the liquidvapor interface meets the solidliquid interface. The contact angle is determined by the result between adhesive and cohesive forces. As the tendency of a drop to spread out over a flat, solid surface increases, the contact angle decreases. Thus, the contact angle provides a useful characterization of wettability.
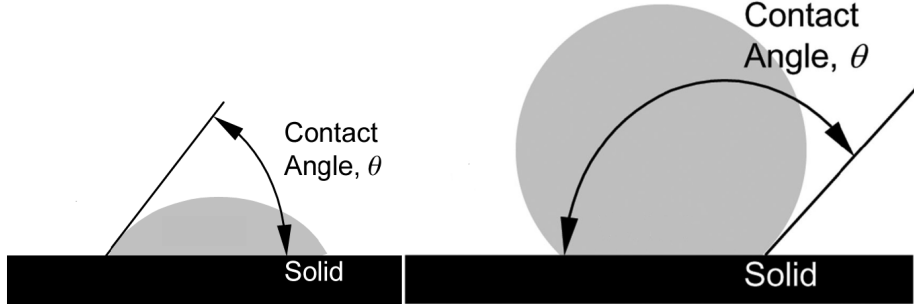

Figure 4: Different fluids exhibiting different wettability. The contact angle θ serves as an inverse measure of wettability.

Capillary pressure and relative permeabilities depend on the interaction between the phases, which in turn depend on the size and shape of the pores and the wettability of the phases. Different rocks exhibit different wetting levels for each phase, but in general water-wet systems, i.e. surface with preference to be coated with water, are far more common.

2.3 Phase Saturations

The range in which the wetting phase saturation s_w varies goes from the critical saturation s_{wc}, the saturation at which this phase starts to flow; to the maximum saturation $s_w^{max} = 1 - s_{nr}$, where the irreducible saturation s_{nr} is the saturation at which the non wetting phase can no longer be displaced.

Formally, s_{nr} is the value at which the non wetting phase can no longer be displaced by the wetting phase, while s_{nc} is the value at which the non wetting phase can no longer be displaced by any kind of pressure gradient. Because of other variables, s_{nc} and s_{nr} may be different but clearly $s_{nr} \geq s_{nc}$ at all times.

This means, although $1 - s_{nc}$ is the maximum theoretical value for s_w, it is not always attainable, so we consider $1 - s_{nr}$ instead. Furthermore, while $1 - s_{nc}$ usually remains constant, $1 - s_{nr}$ may change if parameters are modified during a process.

Analogously, the range of the non wetting phase saturation s_n goes from s_{nc} to $1 - s_{wr}$. When considering water, it can often be assumed that $s_{wc} = s_{wr}$.

8

3 HYSTERESIS MODELS

Hysteresis in relative permeability occurs whenever porous rock exhibiting a strong wettability preference for a specific phase experiences a change in saturation from a drainage to an imbibition process. When this happens the non wetting phase is subject to entrapment by the wetting phase.

In reservoir modeling gross errors can result if hysteresis is ignored. For example, drainage data used instead of imbibition data in a gas reservoir with a strong water drive could result in predicted recoveries as much as twice the amount actually observed [10].

The best known models used in industry, introduced by Killough ([8],1976) and Carlson ([5],1981), will be presented along with two more recent methods. A brief description of the physical phenomenon is also explained in this chapter.

3.1 Physical Background

Land ([9],1968) formalized the concept of hysteresis by describing the behavior of the trapping that the non wetting phase undergoes, and explaining its effect on the relative permeability. Ever since, almost every model builds upon his ideas to describe hysteresis.

In a water-wet system, water inside the pore space tends to gather close to the surface of the rock, while the oil stays further away from the rock walls. Hence in the smaller pores and pore throats, which have a larger surface/volume ratio, water is generally more present than oil and tends to flow easier.

According to Land's experiments, as the oil begins to flow into the medium, it invades first the bigger pores. As the oil saturation continues to increase, the smaller the size of the pores it starts to occupy. During this process k_n follows the primary drainage curve, until the process is reversed. When this happens, the wetting phase enters the system, pushing the main bulk of the oil phase first, and trapping a portion of the non wetting phase in the smaller pores.

Since the variables of interest are mostly dependent on the saturation, this trapped volume will play a role in describing their behavior. We will see more details of Land's work as they come up in the following different models.

3.2 Carlson

Let us now consider the model proposed by Carlson ([5],1981). It focuses on the relative permeability of the non wetting phase k_n in a two phase system and it assumes that the relative permeability of the wetting phase k_w exhibits no hysteretic behavior. Furthermore, k_n follows two bounding curves, the primary drainage curve k_n^D and the primary imbibition curve k_n^I.
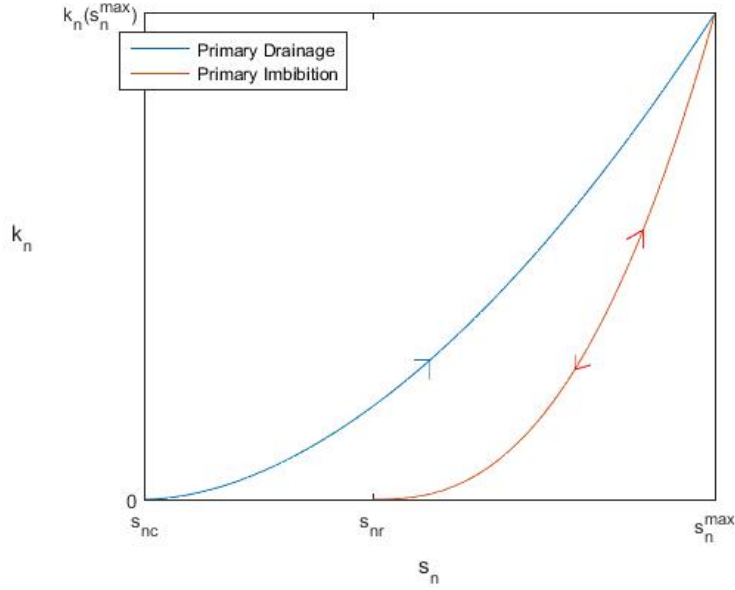

Figure 5: Relative permeability of oil vs oil saturation, according to Land's experiments.

As suggested by Land, it is assumed that trapping only occurs during imbibition, hence if the imbibition process is reversed then the imbibition curve will be retraced exactly, as shown in figures 5 and 7. Hence, instead of scanning curves, what we have is different imbibition curves k_n^I, as in figure 7.

Let s_{nt} be the "trapped" fraction of the saturation described by Land, and s_{nf} the "free" fraction, so that

$$s_n = s_{nf} + s_{nt} \tag{9}$$

Following Carlson's reasoning, we can predict the imbibition curve by using the drainage curve and adjusting for the trapping. The values of imbibition curve that k_n follows must be the values of the drainage curve evaluated on the free saturation only, equivalently

$$k_n^I(s_n) = k_n^D(s_{nf}) \tag{10}$$

10

This states that if no trapping occurred the imbibition and drainage curves would be identical.
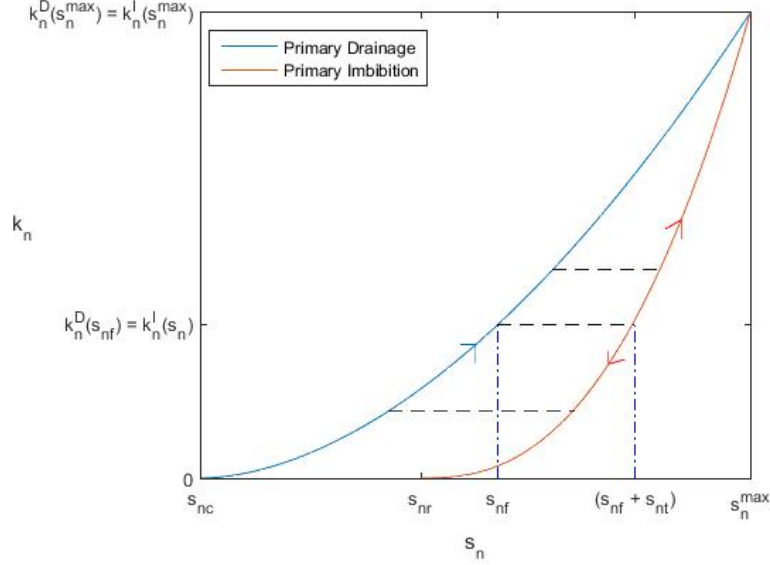

Figure 6: Relative permeability of oil vs oil saturation. Equation (10) explains the distance between the two curves caused by the trapped saturation s_{nt}.

At the beginning of the imbibition, at s_n^{max}, both curves have the same value, since no trapping has taken place yet. As the imbibition process goes on, and water starts trapping the oil, the trapped saturation s_{nt} grows and the curves drift further apart. Hence in order to predict the curve k_n^I we need to know the value of s_{nt} for any given saturation s_n.

Estimating the trapping

Since part of the non wetting fluid was trapped by the incoming wetting fluid, the irreducible saturation s_{nr} is strictly greater than the original saturation s_{nc}, as it contains all the trapped oil that could not be displaced. In fact, the later the drainage process is reversed, the more trapping occurs, which results in a larger residual saturation s_{nr}.

Land [9] formalized this through his experiments and established the relation between the saturation at which the drainage is reversed, s_{ni}, and the irreducible saturation s_{nr}:

$$\frac{1}{s_{nr}} - \frac{1}{s_{ni}} = C \tag{11}$$

where C is a constant.

11

Hence s_{nr} increases along with the historical maximum s_{ni}, as shown in figure 7. Intuitively, the more non wetting phase enters our volume before we start forcing it back out, the harder it will be for the wetting phase to push it all out, as more non wetting volume will be trapped in the smaller pores.
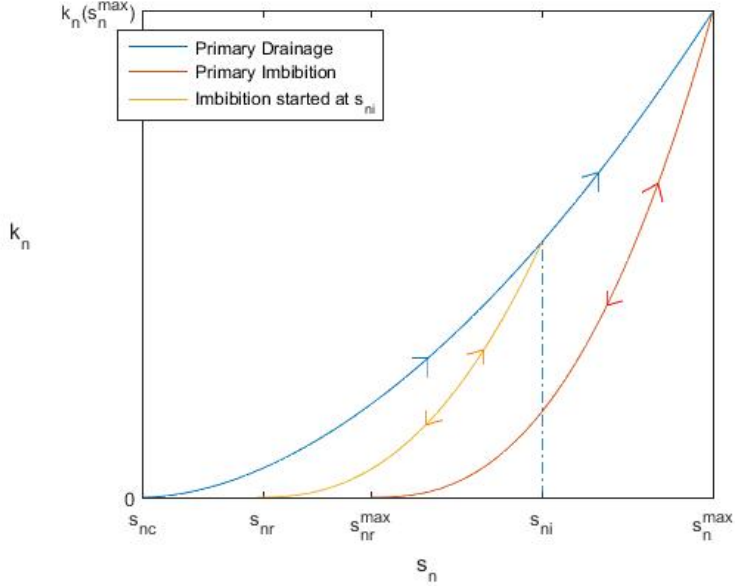

Figure 7: Relative permeability k_n vs s_n. The drainage process has been reversed at s_{ni}. Equation (11) states the relation between s_{ni} and s_{nr}.

Land computed the value of the free saturation s_{nf} as a function of s_n, s_{nr} and C. To show this, let us recall equation (9):

$$s_n = s_{nf} + s_{nt}$$

It is clear that $s_{nt} = 0$ when $s_n = s_{ni}$, and that $s_{nf} = 0$ when $s_n = s_{nr}$. At any other value s_n, $s_{ni} > s_n > s_{nr}$, it is possible, using equation (11), to determine the distribution of s_n between s_{nt} and s_{nf}.

At s_n, exactly s_{nt} has already been trapped. The free saturation s_{nf} is subject to further entrapment according to equation (11). The amount in s_{nf} that is yet to be trapped, s_{nfr}, is determined by substituting s_{nf} for s_{ni} and s_{nfr} for s_{nr} in equation (11):

$$\frac{1}{s_{nfr}} - \frac{1}{s_{nf}} = C$$

Equivalently

12

$$s_{nfr} = \frac{s_{nf}}{1 + Cs_{nf}} \tag{12}$$

Eventually s_{nt} will reach s_{nr}, but at the moment the trapped saturation is the future total trapped saturation minus the saturation yet to be trapped, i.e.

$$s_{nt} = s_{nr} - s_{nfr}$$

Substituting (12) into this last equation yields

$$s_{nt} = s_{nr} - \frac{s_{nf}}{1 + Cs_{nf}}$$

Replacing s_{nt} by $s_n - s_{nf}$,

$$s_n - s_{snf} = s_{nr} - \frac{s_{nf}}{1 + Cs_{nf}}$$

Solving for s_{nf} yields

$$s_{nf} = \frac{1}{2}\left[(s_n - s_{nr}) + \sqrt{(s_n - s_{nr})^2 + \frac{4}{C}(s_n - s_{nr})} \right] \tag{13}$$

This equation allows us to determine s_{nf} at any given moment. The value of s_{nf} can then be used in equation (10) to predict the unknown imbibition curve from the empirically known drainage curve.

However, we need the right input to compute s_{nf}. If s_{ni} is known and C can be predicted, then s_{nr} and s_{nf} can be computed using (11) and (13).

Estimating C

Let us assume s_{ni} is known exactly. In practice measurements of s_{nr} are difficult to obtain, but we can use the following procedure to compute it without experimental determination.

Let s_{n_j} be N experimental imbibition data points, and s_{nf_j} their respective free saturation fractions. Substituting equation (11) into (13) and solving for s_{nr} gives us

$$s_{nr_j} = \frac{1}{2}\left[s_{n_j} - s_{nf_j} + \left((s_{n_j} - s_{nf_j})^2 + \frac{4s_{ni}s_{nf_j}(s_{n_j} - s_{nf_j})}{s_{ni} - s_{nf_j}} \right)^{\frac{1}{2}} \right]$$

A value s_{nr_j} is computed for every experimental data point j, in order to deal with the uncertainties that may arise. An unbiased estimate of s_{nr} is then obtained by taking the average

$$\bar{s}_{nr} = \frac{1}{N}\sum_{j=1}^{N} s_{nr_j}$$

Once $\bar{s}_{nr}$ is obtained, it can be used in equation (11) to compute C. With C determined, we can use equation (11) to calculate the corresponding s_{nr} given any s_{ni}. The value of s_{nf} follows immediately from this using (13). Finally, by equation (10), the imbibition curve k_n^I will be given by the drainage curve k_n^D evaluated on s_{nf}.

The whole process requires exact knowledge of the primary drainage curve and the point s_{ni}, and at least one experimental value s_{n_j} in the imbibition curve.

3.3 Killough

Killough's model [8] was introduced before Carlson's and it uses a simpler solution but requires more input data. As in Carlson's model, assume no trapping occurs during drainage. Hence any imbibition curves, when reversed, are retraced exactly until we arrive again at the primary drainage curve.
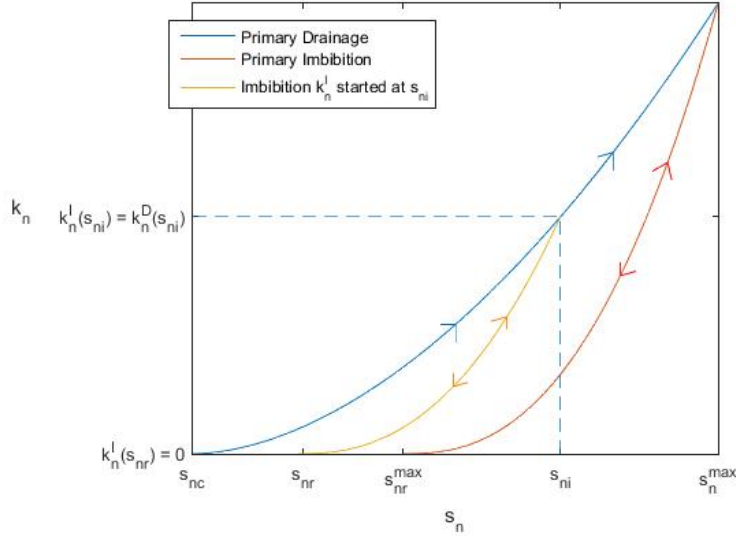

Figure 8: Relative permeability of oil k_n vs oil saturation s_n. Drainage process reversed at s_{ni}. The resulting imbibition curve (yellow) results of interpolating its two extreme values.

Assume the primary drainage process is reversed at saturation s_{ni}. At this moment it is true that

$$k_n^I(s_{ni}) = k_n^D(s_{ni}) \tag{14}$$

However we know that the new imbibition curve will reach $k_n = 0$ when saturation arrives at s_{nr}, which depends on s_{ni} by equation (11).

$$k_n^I(s_{nr}) = 0 \tag{15}$$

To predict the intermediate curve k_n^I lying between (14) and (15), Killough considered two methods: a) parametric interpolation and b) normalized experimental data.

a) Using parametric interpolation on (14) and (15) yields

$$k_n^I(s_n) = k_n^D(s_{ni}) \left(\frac{s_n - s_{nr}}{s_{ni} - s_{nr}} \right)^\lambda \tag{16}$$

15

where λ is a given parameter. Clearly it is satisfied that $k_n^I = k_n^D$ at s_{ni} and $k_n^I = 0$ at s_{nr}.

Notice that in order to obtain the corresponding value of s_{nr}, this method requires computing parameter C from equation (11), although Killough described it simply as

$$C = \frac{1}{s_{nr}^{max}} - \frac{1}{s_{ni}^{max}}$$

i.e. defined by the extreme values of the saturation, corresponding to the end of the primary drainage process and the end of the primary imbibition process. This requires measuring s_{nr}^{max}, which can be difficult in practical situations.

b) Alternatively to (16), using normalized experimental data results in

$$k_n^I(s_n) = k_n^D(s_{ni}) \left[\frac{k_n^{I^*}(s_n^*) - k_n^{I^*}(s_{nr}^{max})}{k_n^{I^*}(s_n^{max}) - k_n^{I^*}(s_{nr}^{max})} \right]$$

where $k_n^{I^*}$ is the experimental or analytical primary imbibition curve, which lies between the maximum possible s_n and s_{nr}^{max}, and s_n^* is given by

$$s_n^* = \left[\frac{(s_n - s_{nr})(s_n^{max}) - s_{nr}^{max}}{s_{ni} - s_{nr}} \right] + s_{nr}^{max} \tag{17}$$

For this last method, is clear that both boundary curves, drainage and imbibition, are assumed known at least empirically.

Wetting Phase Hysteresis

Killough also considered the effect of trapping on the wetting phase relative permeability. The solution follows the same idea, with the scanning curve ranging from $k_w^I(s_{ni}) = k_w^D(s_{ni})$ to a maximum $k_w^I(s_{nr})$. This last value is approximated using

$$k_w^I(s_{nr}) = k_w^D(s_{nr}) + [k_w^{I^*}(s_{nr}^{max}) - k_w^D(s_{nr}^{max})] \left(\frac{s_{nr}}{s_{nr}^{max}} \right)^a$$

where $k_w^{I^*}$ is defined analogous to $k_n^{I^*}$, and a is a given parameter. The interpolation between $k_w^I(s_{ni})$ and $k_w^I(s_{nr}^{max})$ is given by

$$k_w^I(s_n) = k_w^D(s_{ni}) + \left[\frac{k_w^{I^*}(s_n^*) - k_w^{I^*}(s_n^{max})}{k_w^{I^*}(s_{nr}^{max} - k_w^{I^*}(s_n^{max})} \right] (k_w^I(s_{nr}) - k_w^D(s_{ni}))$$

where s_n^* is defined as in (17).

16

3.4 The Scanning Hysteresis Model

Generally denoted SHM, the model described in ([3],2000) is based on the experimental data gathered by Gladfelter and Gupta [7] and by Braun and Holland [4]. The curves they registered for relative permeability are different that the ones used by Land:
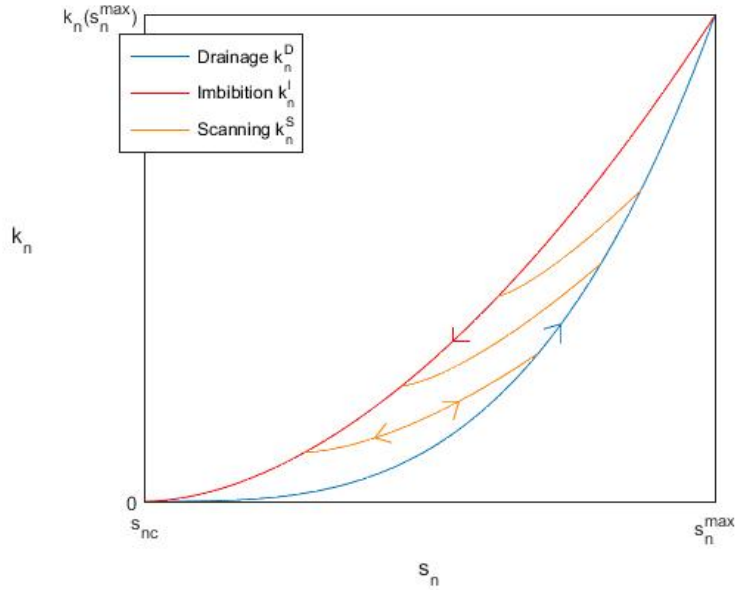

Figure 9: Relative permeability of oil vs oil saturation, as documented by [7] and [4]. In the SHM, the scanning curves are exactly reversible.

Notice how the boundary curves have inverted roles as compared to the previous models. This discrepancy in empirical data helps us understand why are there no well-established physical models around hysteresis.

In this model, whenever a primary process is reversed, a scanning curve is used to move from one primary curve to the other. If a process is reversed while on a scanning curve, the scanning curve is retraced exactly. The boundary curves are assumed known and denoted by

$$k_n^D(s_n) \qquad \text{and} \qquad k_n^I(s_n)$$

For the scanning curves, a parameter π is introduced to serve as the "memory" of the system:

$$k_n^S = k_n^S(s_n, \pi)$$

As we move along one of the boundary curves, i.e. drainage or imbibition, the memory state of the system changes, hence π changes accordingly. As soon

as we enter a scanning curve, parameter π remains constant during the duration of the scanning process, until we reach another boundary curve.

For consistency, π is different for every scanning process, which implies scanning curves never touch. Since it is only a reference parameter, π values can be chosen arbitrarily. In this case, $\pi \in [0, 1]$.

For continuity, when following a primary curve, π must be modified in such a way that

$$k_n^D(s_n) = k_n^S(s_n, \pi) \qquad \text{along the primary drainage curve}$$

and

$$k_n^I(s_n) = k_n^S(s_n, \pi) \qquad \text{along the primary imbibition curve}$$

This, together with smoothness and monotony assumptions on the relative permeability functions, uniquely determines π. Hence π can be solved as a function of the saturation s_n in any of both cases:

$$\pi = \pi^D(s_n) \quad \text{(drainage)} \quad \text{and} \quad \pi = \pi^I(s_n) \quad \text{(imbibition)}$$

Mathematical Model

Next, an expression for k_n^S must be chosen. Schaerer et al [2] use the following choices, defined as functions of $s = s_w$:

$$k_n^D(s) = (1 - s)^\eta$$

$$k_n^I(s) = (1 - s)^\theta$$

With $1 < \theta < \eta$, and

$$k_n^S(s, \pi) = \frac{(1 - \pi)^\xi}{(1 - \alpha\pi)^\zeta}(1 - \alpha s)^\zeta$$

Where ξ, ζ are also shaping parameters greater than 1. In [2], they use $\theta = 2, \eta = 3, \xi = 2$ and $\zeta = 1$.

Once π^D and π^I are defined, the convection-diffusion equation for the wetting phase (8) is modified to include the parameter π:

$$\frac{\partial s}{\partial t} + \frac{\partial}{\partial x}F(s, \pi) = \frac{\partial}{\partial x}\left[\varepsilon\frac{\partial s}{\partial x}\right] \tag{18}$$

where $s = s_w$, ε is taken as a small positive constant, and F is divided in three cases,

$$F(s, \pi) = f^D(s) = \frac{k_w(s)/\mu_s}{k_w(s)/\mu_s + k_n^D(s)/\mu_n} \qquad \text{when} \qquad \pi = \pi^D(s) \ \text{ and } \ \frac{\partial s}{\partial t} < 0$$

$$F(s, \pi) = f^I(s) = \frac{k_w(s)/\mu_s}{k_w(s)/\mu_s + k_n^I(s)/\mu_n} \qquad \text{when} \qquad \pi = \pi^I(s) \ \text{ and } \ \frac{\partial s}{\partial t} > 0$$

$$F(s, \pi) = f^S(s) = \frac{k_w(s)/\mu_s}{k_w(s)/\mu_s + k_n^S(s, \pi)/\mu_n} \ \text{ and } \ \frac{\partial \pi}{\partial t} = 0 \qquad \text{otherwise}$$

corresponding to the drainage, imbibition and scanning case, respectively.

The system is then supplied with appropriate initial and/or boundary conditions. Riemann solutions for this problem are presented in [3] and [2].

3.5 Larsen & Skauge

Hysteresis is also present during changes in saturation during three phase flow. Most three-phase systems consist of a wetting phase (water), an intermediate phase (oil) and a non wetting phase (gas). In these processes, such as water alternating gas injection (WAG), the two-phase hysteresis models will generally not be able to describe relative permeabilities reported for the reservoirs.

Larsen and Skauge ([10],1998) present a representation for relative permeability that accounts for hysteresis in a three-phase scenario.

Three-phase systems

During two-phase flow, there is only one independent saturation, hence the system can only move in two directions, drainage or imbibition. In the three-phase system however, at least two saturations are independent, meaning there is an infinite number of directions the saturation distribution can take.

For instance a DDI process consists of decreasing water saturation, decreasing oil saturation, and increasing gas saturation. In order to be compatible with the two-phase case, relative permeabilities must be defined for every trajectory, as in figure 10:
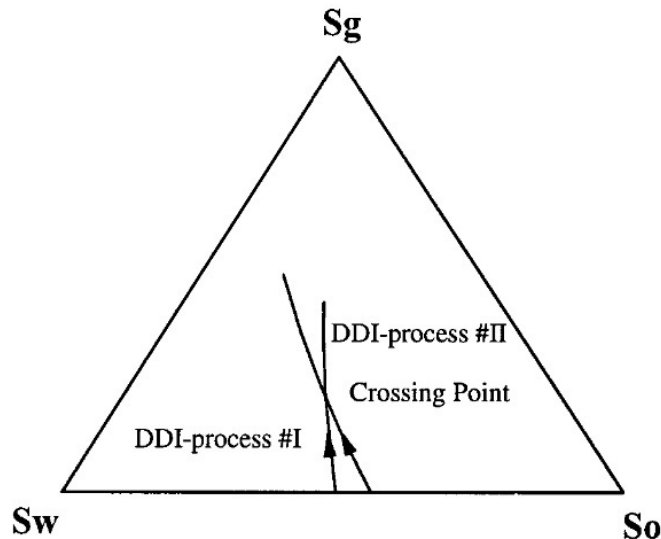

Figure 10: Two DDI process starting from different water and oil saturation. Due to hysteresis, at the crossing point relative permeabilities are in general not unique.

For consistency, trajectories must be described by the three saturation directions. That is, no phase can change saturation direction during a trajectory. These are known as constrained trajectories and the model focuses on these processes only.

Like in the previous models, relative permeability of a trajectory will be a function of saturation and the starting point of a trajectory. In the two-phase case, we used s_{ni} to denote the point at which a process was reversed, i.e. when a new trajectory was started. Hence permeabilities were usually of the form

$$k = f(s_x, s_{xi})$$

where x represents either phase. In three-phase, at least two saturations are needed to determine the third one, and two initial saturations to determine the initial point of a trajectory, hence we will have

$$k = f(s_x, s_y, s_{xi}, s_{yi})$$

The Model

Consider a water alternating gas scenario. Every time both a water and a gas injection is complete, the cycle starts again, as shown in figure 11. These will result in hysteresis "loops", each loop displaying overall less relative permeability, as trapping occurs on every cycle.

To estimate the trapping, Land's formalism will be used. Only non wetting phase (gas) hysteresis will be explained.
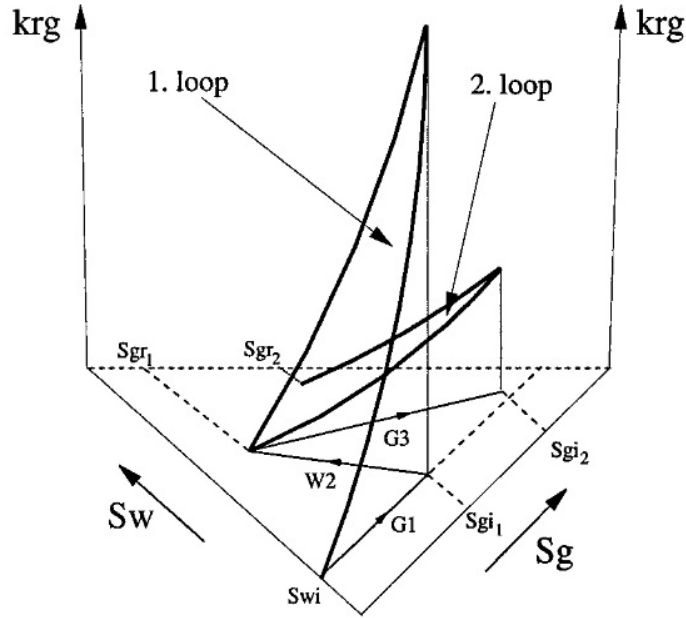

Figure 11: Gas relative permeability vs water and gas saturation, during a WAG process.

During increasing gas saturation (drainage), k_g^D for loop n is calculated by

$$[k_g^D(s_g, s_w^I, s_g^{start})]_n = \left\{ [k_g^{input}(s_g) - k_r^{input}(s_g^{start})] \left(\frac{s_{wi}}{s_w^I} \right)^\alpha \right\}_n + [k_g^I(s_g^{start})]_{n-1}$$

(19)

where $s_g \in [s_g^{start}, 1]$. The primary gas relative permeability curve, G_1, exists from $s_g = 0$ to the maximum gas saturation. The first set of parenthesis on equation (19) represents a transformation of the G_1 curve at $s_g = s_g^{start}$.

The second set accounts for reduction of gas relative permeability in presence of moving water. The last term is the stopping point of the last hysteresis loop. This term ensures continuity between hysteresis loop n and $n-1$. When $n = 1$, this term is zero.

Decreasing gas saturations (imbibition) obey the trapped gas model of Land. For every loop, this involves a small transformation of the gas saturation

$$(s_g^{trans})_n = (s_g)_n - (s_g^{end})_{n-1}$$

where $(s_g)_n \in [(s_g^{end})_{n-1}, (s_{gi})_n]$. In the same way, we have

$$(s_{gr}^{trans})_n = (s_{gr})_n - (s_g^{end})_{n-1}$$

$$(s_{gi}^{trans})_n = (s_{gi})_n - (s_g^{end})_{n-1}$$

and

$$(C^{trans})_n = \frac{1}{(s_{gr}^{trans})_n} - \frac{1}{(s_{gi}^{trans})_n}$$

Now equation (13) can be used with transformed saturations. Note that the $(s_g^{end})_{n-1}$ term cancels out in the resulting equation:

$$(s_{gf})_n = \frac{1}{2} \left[(s_g - s_{gr}) + \sqrt{(s_g - s_{gr})^2 + \frac{4}{C^{trans}}(s_g - s_{gr})} \right]_n$$

The transformed free saturation can then be calculated as

$$(s_{gf}^{trans})_n = (s_{gf})_n + (s_g^{end})_{n-1}$$

The relative permeability can now be computed using equation (10), with s_{gf} replaced by s_{gf}^{trans}

$$[k_g^I(s_g) = k_g^D(s_{gf}^{trans})]_n$$

where $s_{gf}^{trans} \in [(s_g^{start})_n, (s_{gi})_n]$.

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