Modelling and imaging of growing plants

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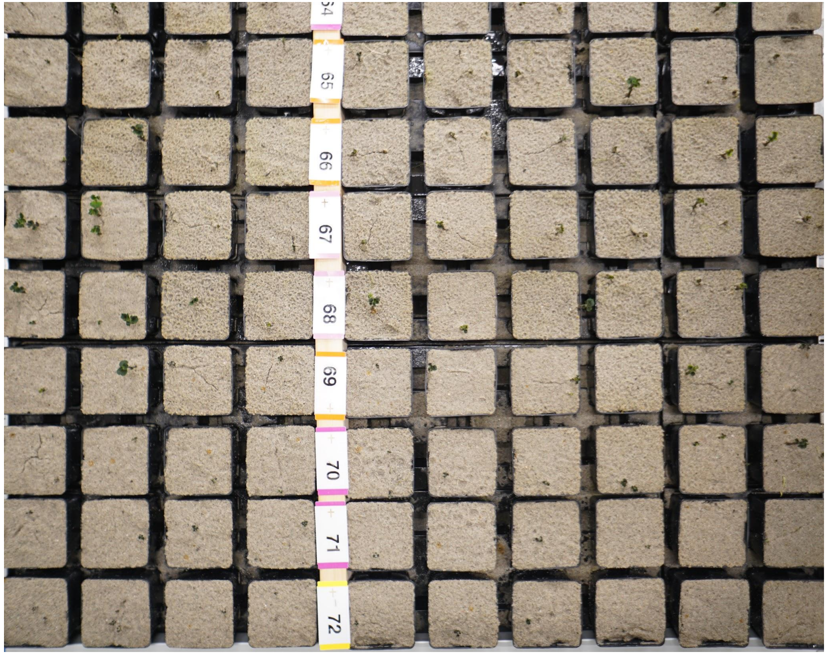
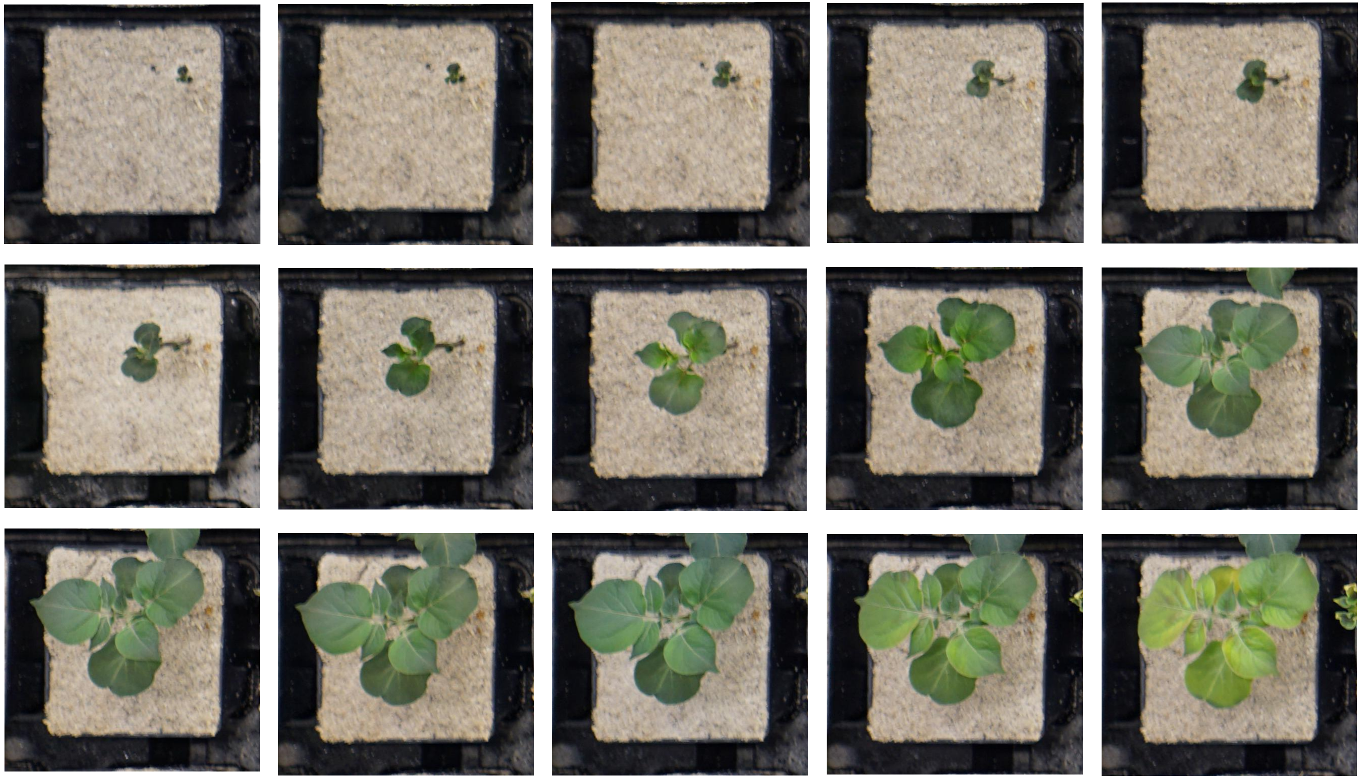
TUDelft

Problem

- The problem of quantifying the growth dynamics of plants from time-lapse images is important for determining variety-specific characteristics and subsequent breeding.
- Amount of data:
  - Six potato varieties
  - Two climate rooms
  - Dry and wet sections

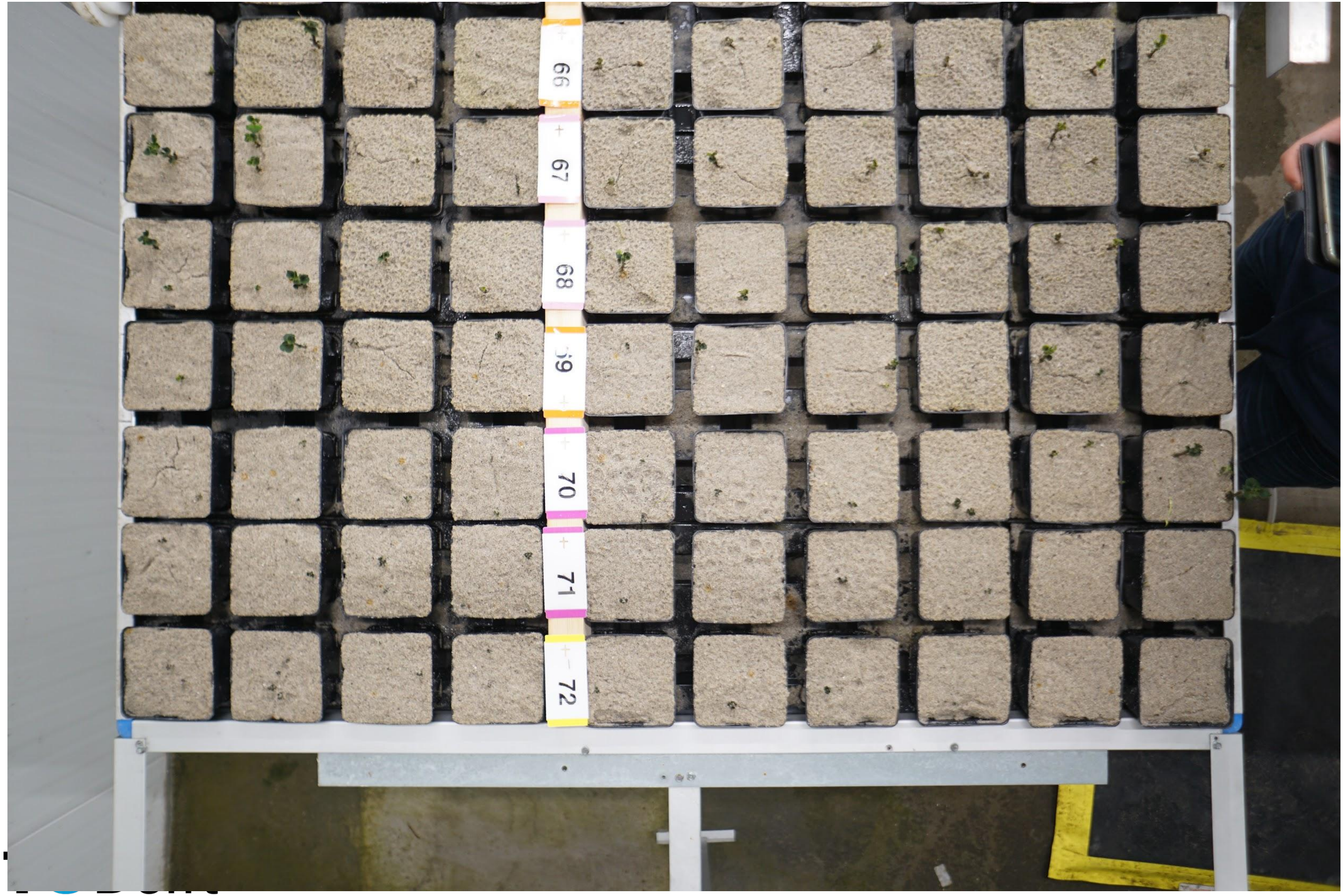


Problem



Features of interest

- Green
- Height
- Day of appearance
- Number of leaves
- Contour of leaves
- Number of plants



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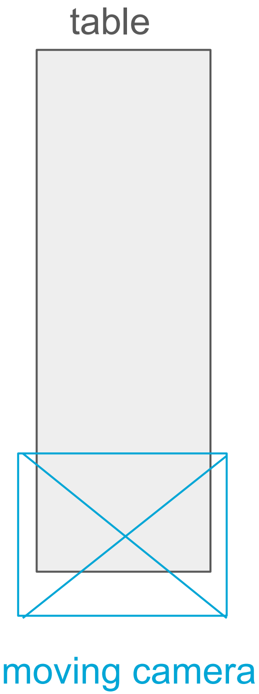

table
moving camera


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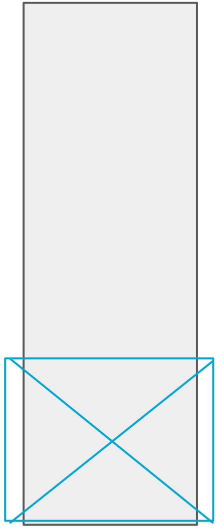


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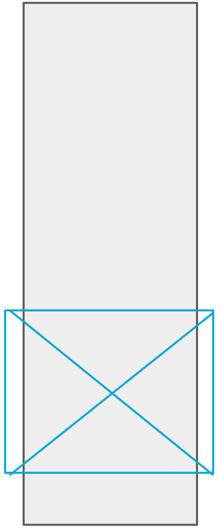
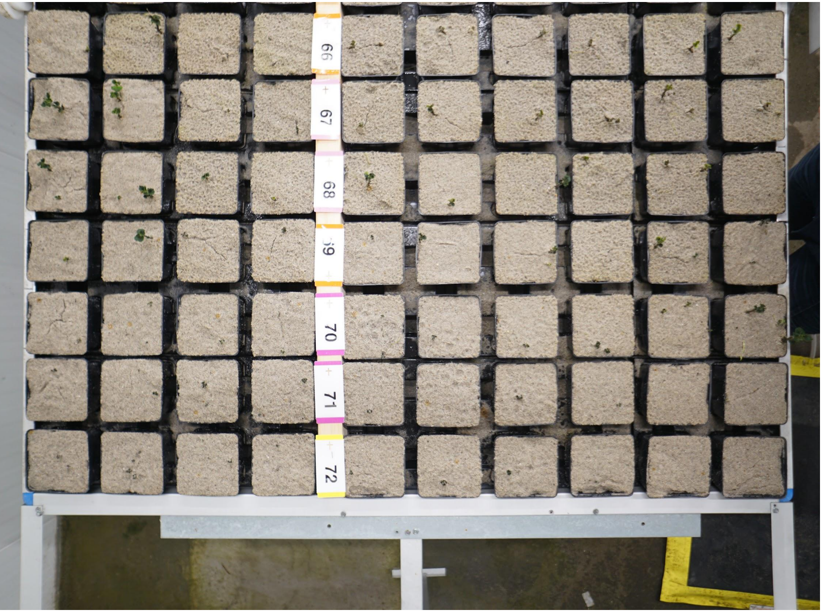
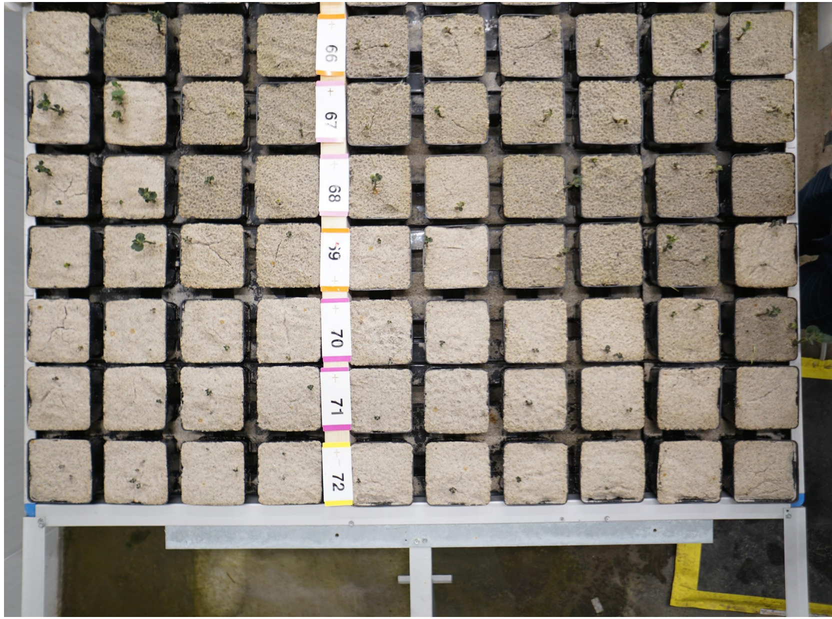


2-4 days between snapshots

Snapshot n



Snapshot n+1



Focus

- Plant outline detection
- Matching points for 3 different camera views over time
- 3D point-reconstruction

Plant outline

- Traditional Snakes algorithm with Gradient Vector Flow force field
- Kass, Michael, Andrew Witkin, and Demetri Terzopoulos. "Snakes: Active contour models." *International journal of computer vision* 1.4 (1988): 321-331.
- Prince, Jerry L., and Chenyang Xu. "A new external force model for snakes." *Proc. 1996 Image and Multidimensional Signal Processing Workshop*. Vol. 3. No. 31. 1996.

Traditional Snakes

- Energy functional of a traditional snake parameterized by $\mathbf{x}(\mathbf{s}) = (x(s), y(s))$

$$E^*_{\text{snake}} = \int_0^1 E_{\text{snake}}(\mathbf{x}(s))\, \mathrm{d}s$$
$$= \int_0^1 E_{\text{int}}(\mathbf{x}(s)) + E_{\text{ext}}(\mathbf{x}(s))\, \mathrm{d}s$$

- α and β control parameters for amount of stretch and curvature

$$E[\mathbf{x}] = \int_0^1 \frac{1}{2}\left(\alpha \left\|\mathbf{x}'(s)\right\|^2 + \beta \left\|\mathbf{x}''(s)\right\|^2\right) + E_{\text{ext}}(\mathbf{x}(s))\, \mathrm{d}s$$

- Assume local minimum of E in $\mathbf{x}$ to derive Euler-Lagrange equations

$$\alpha\mathbf{x}'' - \beta\mathbf{x}'''' - \nabla E_{ext}(\mathbf{x}) = 0$$

Gradient Vector Flow

- Define new external force field $\mathbf{v}(x,y) = (u(x,y), v(x,y))$ which minimizes the functional

$$\mathcal{E}[\mathbf{v}] = \iint \mu\left(u_x^2 + u_y^2 + v_x^2 + v_y^2\right) + |\nabla f|^2 |\mathbf{v} - \nabla f|^2 \, \mathrm{d}x\mathrm{d}y$$

μ smoothing term and f edge map derived from image.

- Again assume minimum in $\mathcal{E}$ and derive Euler-Lagrange equations

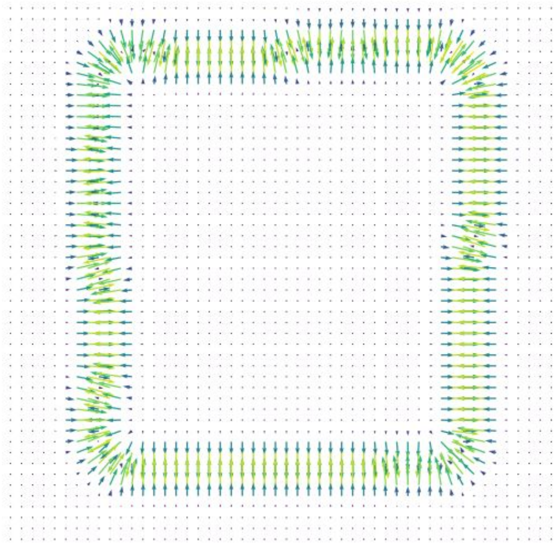
$$\mu\nabla^2 u - \left(f_x^2 + f_y^2\right)(u - f_x) = 0$$

$$\mu\nabla^2 v - \left(f_x^2 + f_y^2\right)(v - f_y) = 0$$

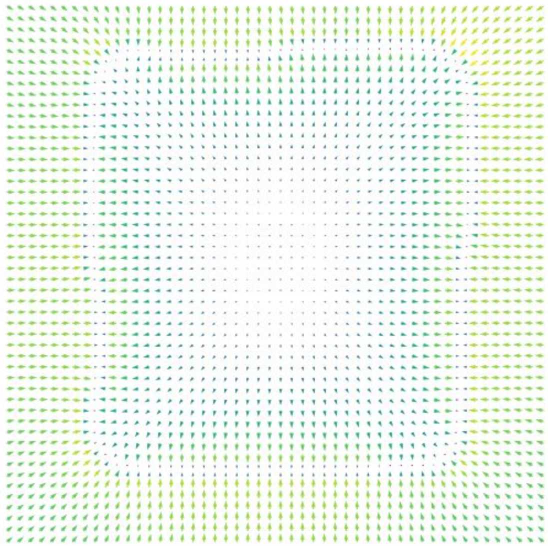
- GVF snake

$$\alpha\mathbf{x}'' - \beta\mathbf{x}'''' + \mathbf{v} = 0$$

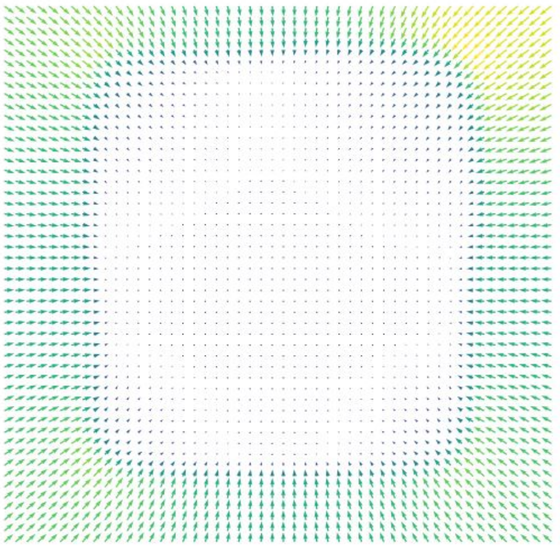
External force field



μ=0



μ=1e-3

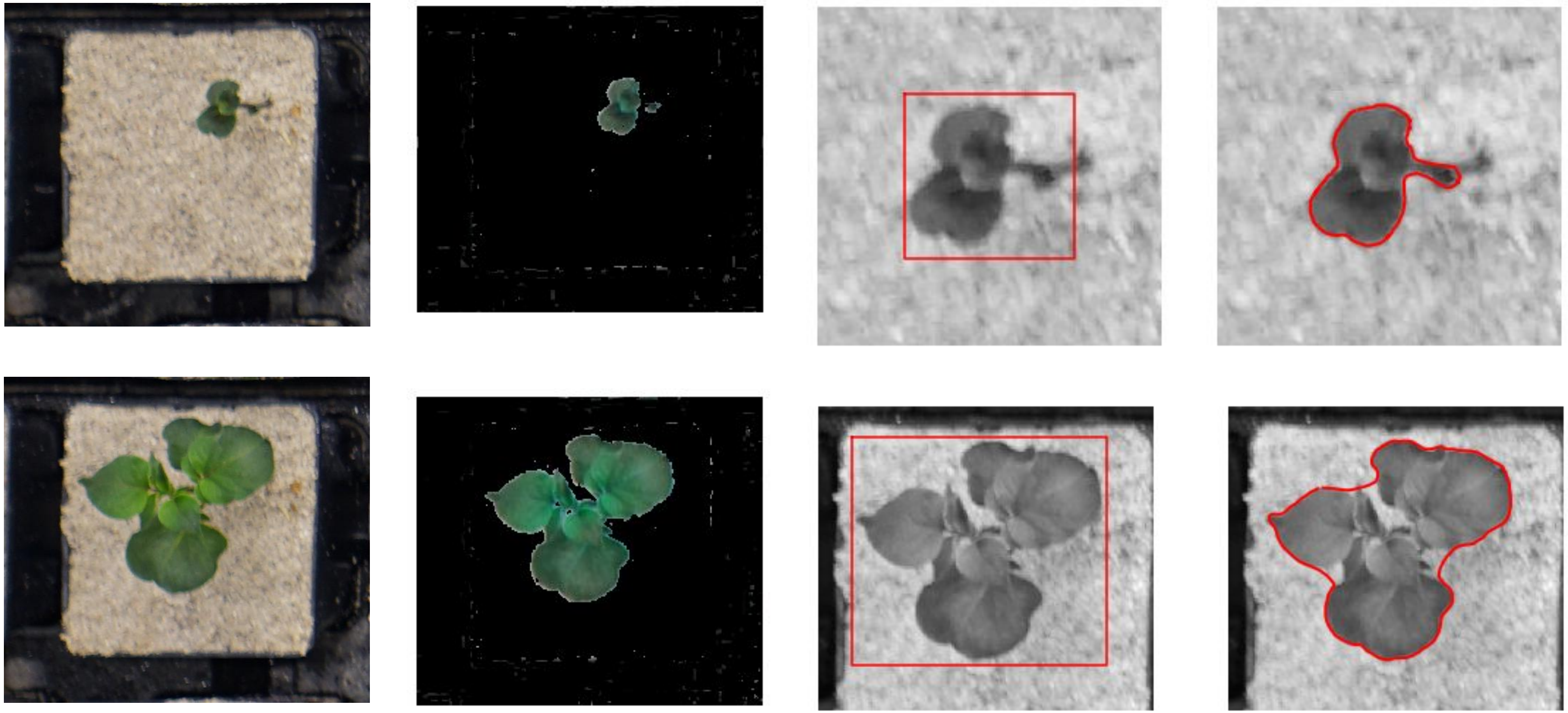


μ=0.1

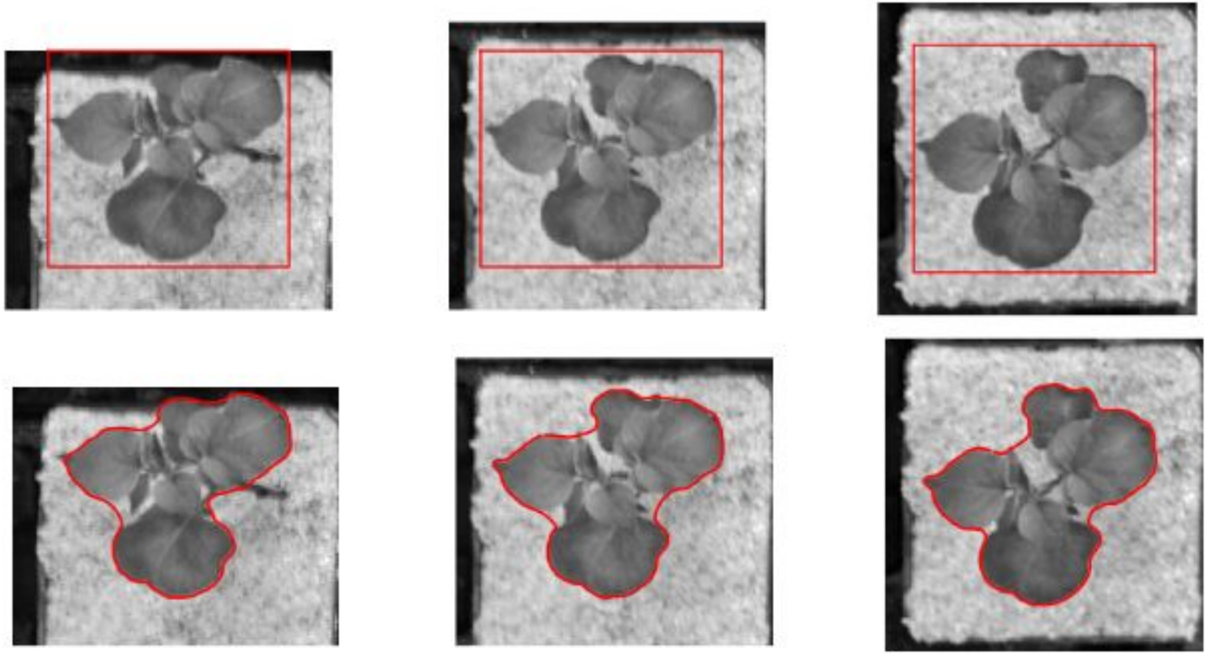
Discretization

- Dynamic snake equation $\frac{\partial \mathbf{x}}{\partial t} = \alpha \frac{\partial^2 \mathbf{x}}{\partial s^2} - \beta \frac{\partial^4 \mathbf{x}}{\partial s^4} + \mathbf{v}(\mathbf{x})$
  discretize with central differences and solve with Euler Forward
- Solve for $\mathbf{v}$ by considering BVP
  $$\begin{cases} \mu \nabla^2 u - g u &= -g f_x, \quad \text{in } \Omega \\ \frac{\partial u}{\partial n} &= 0, \quad \text{on } \partial\Omega \end{cases}, \qquad \text{where } g = f_x^2 + f_y^2$$
  discretize with central differences and solve with MINRES

Initial and final snake



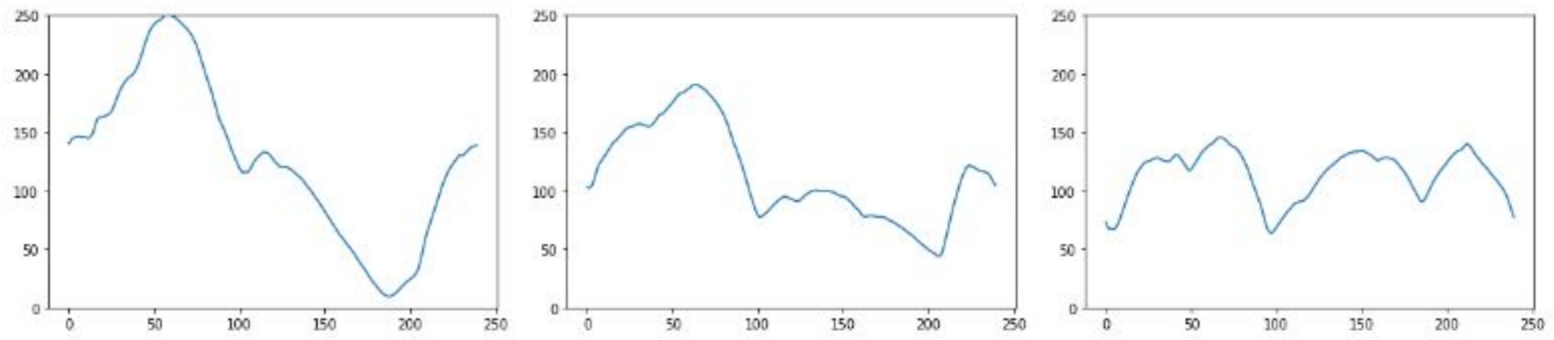
Plant outline for 3 camera views



(a) Angle -1 (b) Angle 0 (c) Angle +1

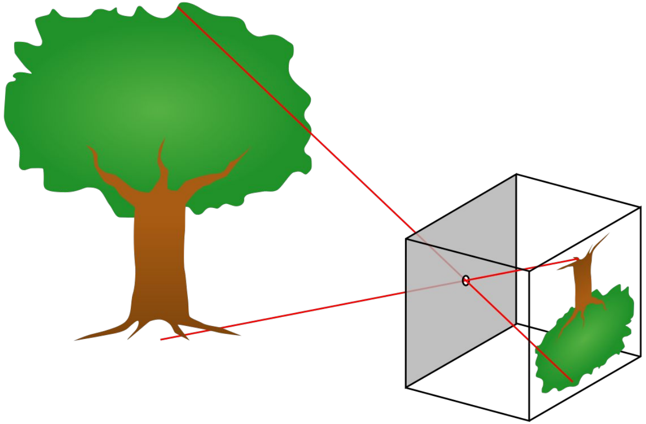
Matching points on curve

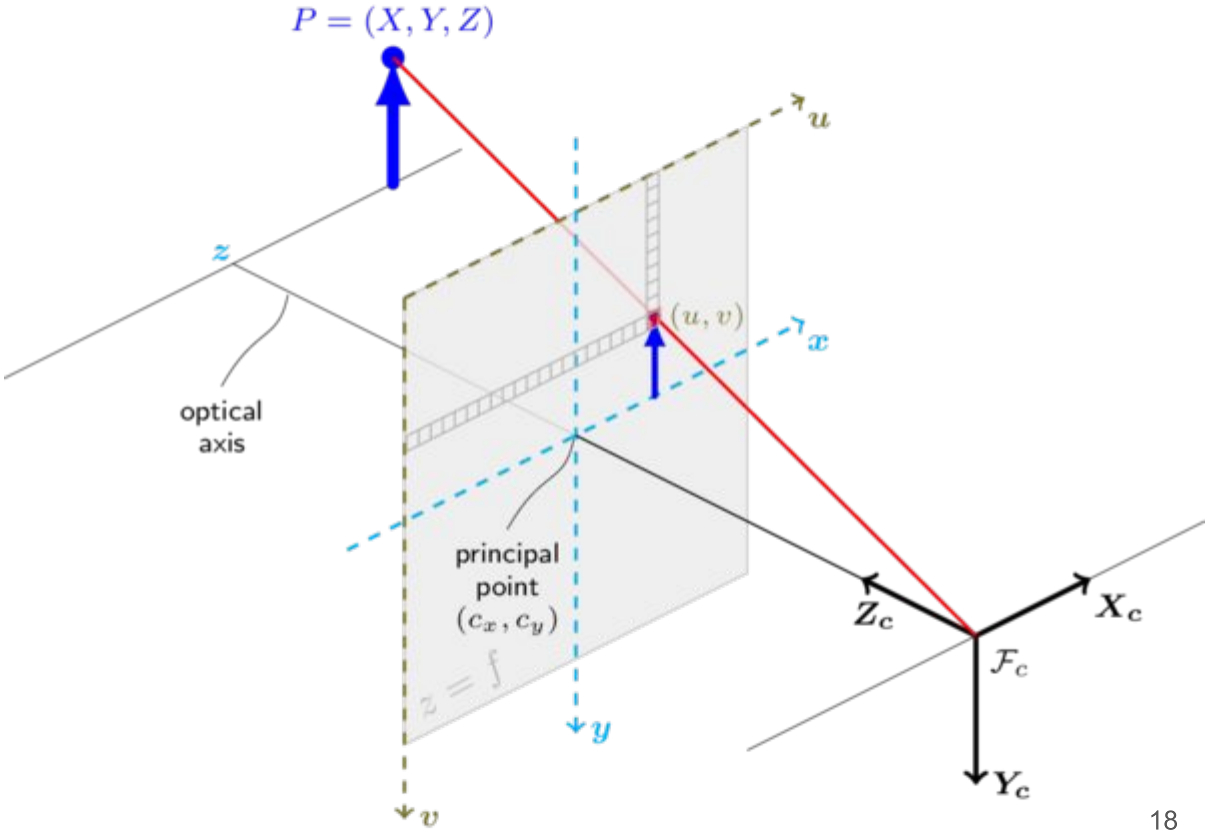
- Radius with respect to centre point plotted against curve



(a) Angle -1 (b) Angle 0 (c) Angle +1

Pinhole camera model





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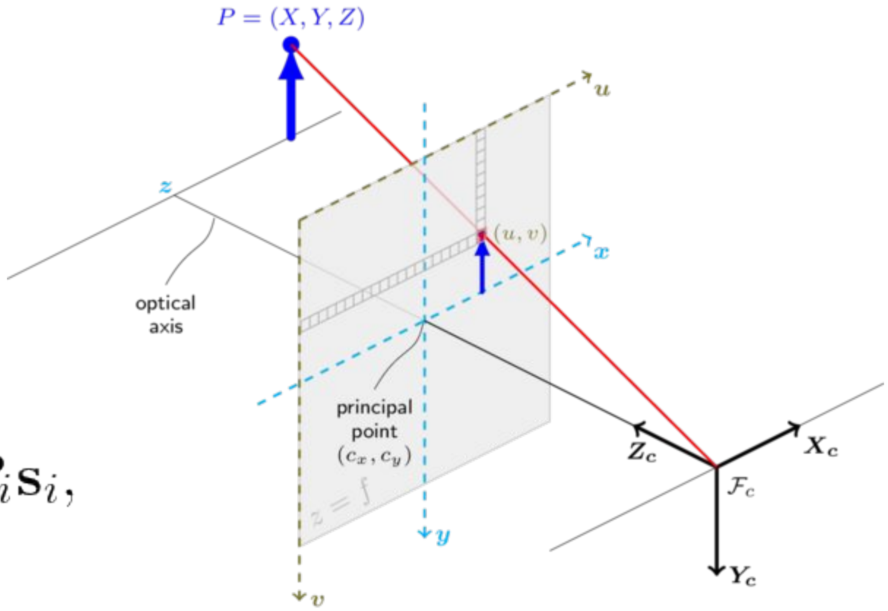
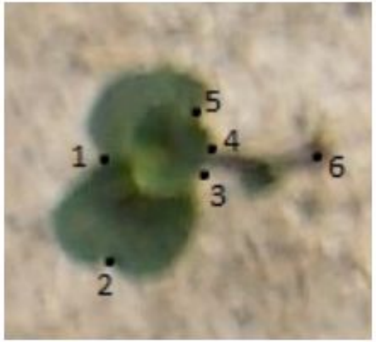
Structure from motion

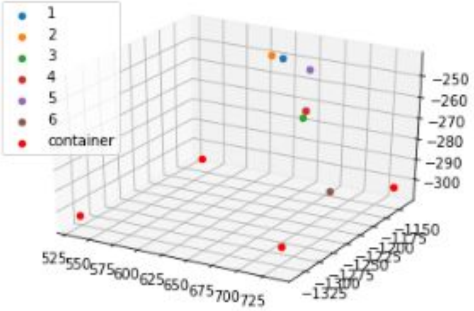
- $$\begin{bmatrix} u \\ v \end{bmatrix} = \frac{f}{Z} \begin{bmatrix} X \\ Y \end{bmatrix}$$
- $P\mathbf{r}' = \mathbf{0}$
- $\mathbf{r}' = \mathbf{s} + R\mathbf{r}$
- if we assume translation $\mathbf{s}$ and rotation $\mathbf{R}$ to be known

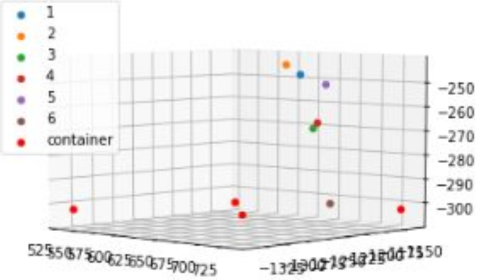
$$R^T \left(\sum_{i=1}^{3} P_i^T P_i \right) R\mathbf{r} = -R^T \sum_{i=1}^{3} P_i^T P_i \mathbf{s}_i,$$

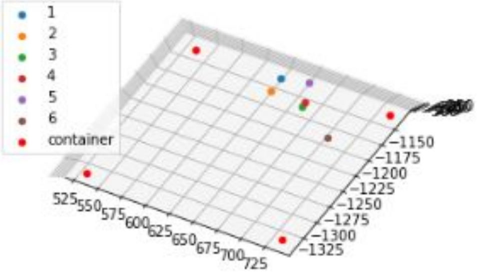


3D point-reconstruction



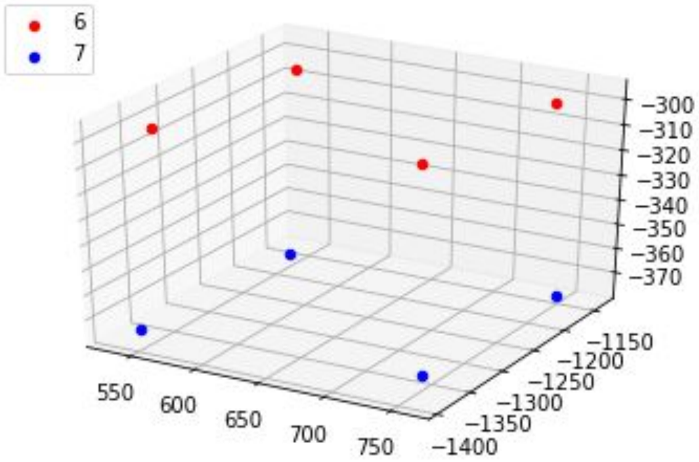






3D point-reconstruction

- Error in invariant points like container
- Caused by small perturbations in camera's positioning
- Translation and rotation not as assumed



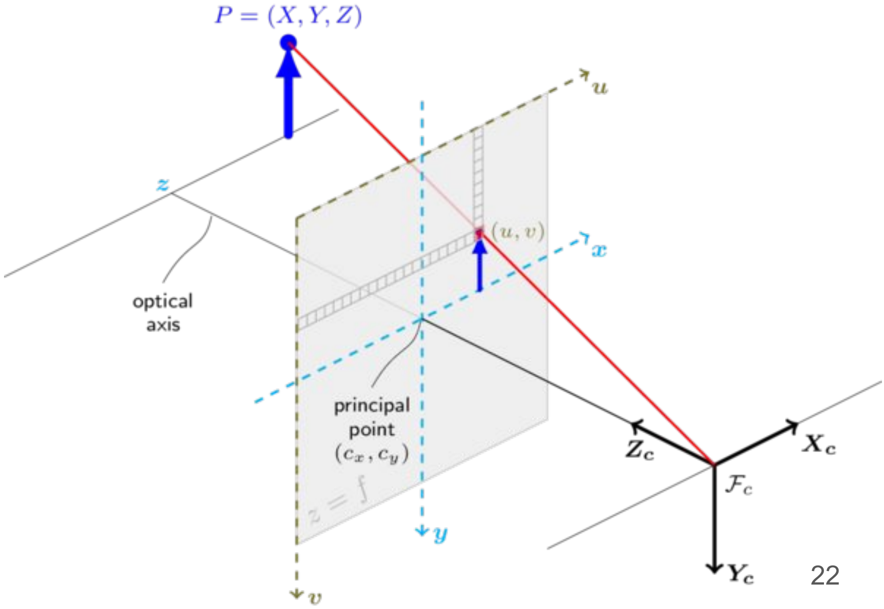
Calibration

- A. Dermanis - Mathematical Foundations of Analytical Photogrammetry
- Estimate camera's internal and external parameters
- Collinearity condition
- Collinearity equations:

$$u = u_0 - f\,\frac{R_{11}\,(x - x_0) + R_{12}\,(y - y_0) + R_{13}\,(z - z_0)}{R_{31}\,(x - x_0) + R_{32}\,(y - y_0) + R_{33}\,(z - z_0)}$$

$$v = v_0 - f\,\frac{R_{21}\,(x - x_0) + R_{22}\,(y - y_0) + R_{23}\,(z - z_0)}{R_{31}\,(x - x_0) + R_{32}\,(y - y_0) + R_{33}\,(z - z_0)}$$

- Set ground control points



Calibration

- Direct Linear Transformation method

$$u = \frac{L_1 x + L_2 y + L_3 z + L_4}{L_9 x + L_{10} y + L_{11} z + 1}$$

$$v = \frac{L_5 x + L_6 y + L_7 z + L_8}{L_9 x L_{10} y + L_{11} z + 1}$$

- 11 unknowns, 6 ground control points

Calibration

- $A\mathbf{l} = \mathbf{0}$

$$A = \begin{bmatrix} x_1 & y_1 & z_1 & 1 & 0 & 0 & 0 & 0 & -u_1 x_1 & -u_1 y_1 & -u_1 z_1 & -u_1 \\ 0 & 0 & 0 & 0 & x_1 & y_1 & z_1 & 1 & -v_1 x_1 & -v_1 y_1 & -v_1 z_1 & -v_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_n & y_n & z_n & 1 & 0 & 0 & 0 & 0 & -u_n x_n & -u_n y_n & -u_n z_n & -u_n \\ 0 & 0 & 0 & 0 & x_n & y_n & z_n & 1 & -v_n x_n & -v_n y_n & -v_n z_n & -v_n \end{bmatrix}$$

- find null space of A using SVD
- edges of container don’t contain info about depth,
  resulting in 3 zero columns of A and a non-unique solution $\mathbf{l}$

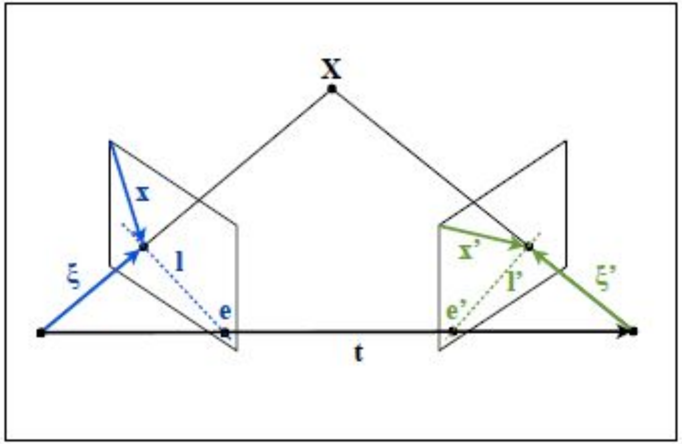
Coplanarity condition

- Schindler, Konrad. "Mathematical foundations of photogrammetry." *Handbook of Geomathematics*. Springer, 2015. 3087-3103.
- $\xi \cdot (\mathbf{t} \times \xi') = 0$
- this relates to the fundamental matrix

$$\mathbf{x}^T F \mathbf{x}' = 0$$

- camera matrices can be obtained

$$[I \mid \mathbf{0}]\,,\quad [[\mathbf{e}']_\times F \mid \mathbf{e}']$$



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Fundamental matrix

- F 3x3 matrix with rank 2
- $B\mathbf{f} = \mathbf{0}$

$$B = \begin{bmatrix} x'_1 x_1 & x'_1 y_1 & y'_1 x_1 & y'_1 y_1 & x'_1 & y'_1 & x_1 & y_1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \\ x'_n x_n & x'_n y_n & y'_n x_n & y'_n y_n & x'_n & y'_n & x_n & y_n & 1 \end{bmatrix}$$

- find null space of B using SVD
- solve with 8 point correspondences

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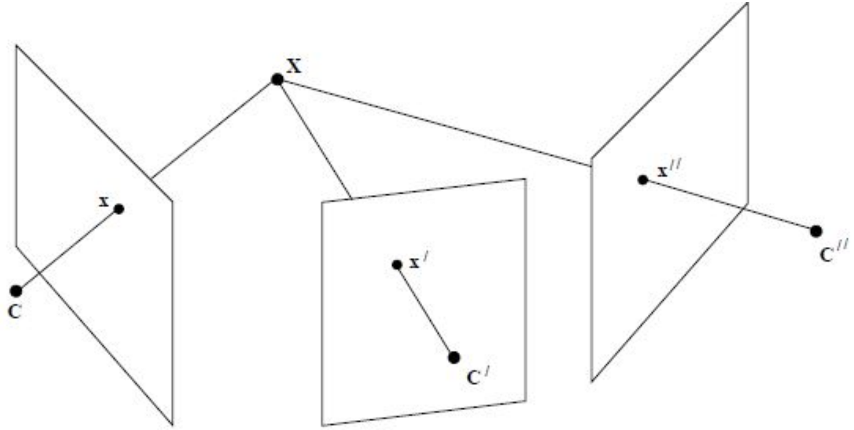
Trifocal tensor

- Hartley, Richard, and Andrew Zisserman. *Multiple view geometry in computer vision.* Cambridge university press, 2003.
- Extension to three camera views
- Three 3x3 matrices T_i

$$[\mathbf{x}']_\times \left(\sum_i x^i \mathrm{T}_i\right)[\mathbf{x}'']_\times = 0_{3\times 3}.$$



Next steps

- Explore trifocal tensor
- From fundamental matrix/trifocal tensor to more accurate 3D reconstruction in the reference system of object
- Matching points