Space-Time Parallel Algorithms for Boundary Element Methods

Supervisors: Carolina Urzúa-Torres and Kees Vuik.

The numerical solution of evolution partial differential equations (PDEs) is usually based on semi-discretizations, either in space (method of lines), or in time (Rothe method). As a consequence, the resulting linear systems inherit a tensor product structure of space and time variables, which typically induces that their solution method is sequential in time. In contrast, space-time discretizations treat time just as another space variable. This allows for adaptivity and parallelization in space and time simultaneously without additional effort [3, 4]. Given today's computing capabilities, this feature is of great advantage, as it paves the way to develop algorithms that can use large numbers of cores more efficiently.

Indeed, space-time methods for evolution PDEs have been gaining popularity in recent years. Although a lot of progress has been made for evolution problems on bounded domains, the case of unbounded domains has received little attention yet, in spite of its many applications in medicine, enviromental sciences, and engineering.

One should note that an effective way to treat PDEs on unbounded domains is via boundary integral equations (BIEs). However, to the best of our knowledge, the only work addressing space-time parallel algorithms for BIEs is [1, 2]. There, they tackle the heat equation and obtain an improved parallel scalability in comparison to time-stepping methods. Given that a key ingredient of their approach is the special structure of the fundamental solution of the heat equation, an interesting question is if this can also be done efficiently for other time dependent BIEs coming from applications.

Research description

The goal of this master project is to device a new space-time parallel algorithm for BEM that is designed to work with different time dependent BIEs.

Within this master project the following tasks are foreseen:

1. Literature research on space-time formulations and discretizations for time dependent BIEs.
2. Literature research on current state-of-the-art space-time solution algorithms.
3. Construction and implementation of a new space-time parallel solving technique for more general time-dependent BIEs.
4. Test the proposed space-time algorithm for BEM for the heat equation.
5. Analyse the computational performance of the algorithm and its suitability to other time dependent BIEs.
6. Master thesis.

No previous knowledge on BIE or BEM is needed, but the student is expected to have basis knowledge of numerical methods, discretization schemes, finite elements, and an interest in parallel programming.

References

[1] S. Dohr, M. Merta, G. Of, O. Steinbach, J. Zapletal: *A parallel solver for a preconditioned space-time boundary element method for the heat equation*. Technical report 2018/3, Institut für Numerische Mathematik,, TU Graz.

[2] S. Dohr, J. Zapletal, G. Of, M. Merta: *A parallel spacetime boundary element method for the heat equation*. Computers and Mathematics with Applications 78 (2019) 28522866.

[3] M.J. Gander, M. Neumüller: *Analysis of a new space-time parallel multigrid algorithm for parabolic problems*. SIAM J. Sci. Comput. 38(4), A2173A2208 (2016).

[4] C. Schwab, R. Stevenson: *Space-time adaptive wavelet methods for parabolic evolution problems*. Math. Comput. 78(267), 12931318 (2009).