Towards arbitrage-free implied volatility surfaces with diffusion probabilistic models

Shuaiqiang Liu, Cornelis Vuik

January 2024

1 Introduction

Generating realistic implied volatility surfaces with generative AI models is an evolving area of research [1, 5, 4]. However, even with the powerful deep neural networks, it is non-trivial to process high dimensional complex patterns in finance, such as volatility smile, term structure. A major challenge is to ensure the generated samples meet financial constraints, e.g., arbitrage-free conditions.

This research project aims to extend the previous work [3] by integrating arbitrage-free conditions into the generation of implied volatility surfaces through a new state-of-the-art family of deep generative models i.e., diffusion probabilistic models [2, 6].

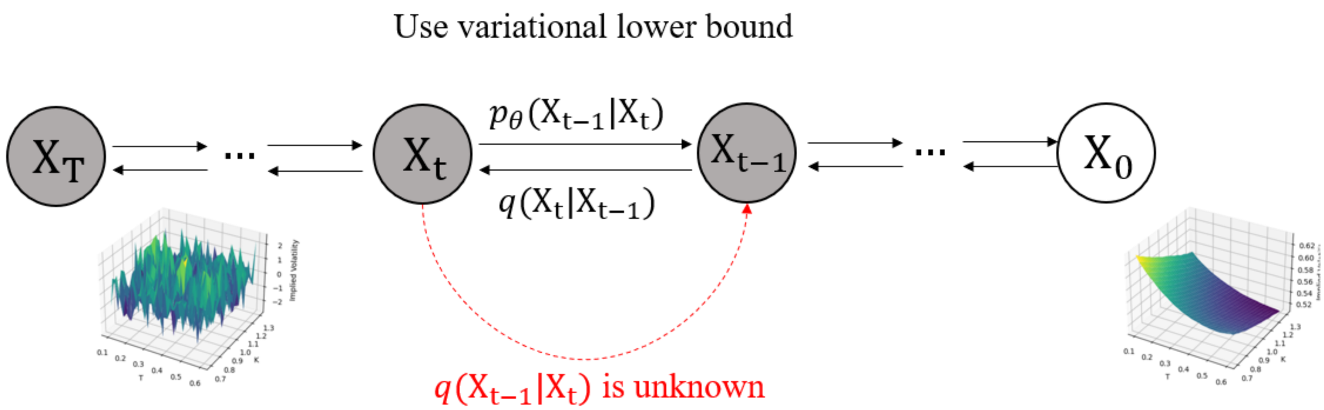

Figure 1: The forward and reverse process in diffusion models to generate volatility surfaces [3].

The diffusion models are characterized by a two-stage process. Initially, they undergo a forward phase where increasing levels of random noise are progressively added to the original data, a stage called "diffusion." This is followed by a reverse phase, successively eliminating the noise to generate new data samples. From a mathematical perspective, diffusion models exhibit a strong connection with stochastic differential equations [6]. Thus, this project is also expected to shed light on the relationship between stochastic volatility models and deep generative models, potentially enhancing the understanding of these distinct methods when dealing with implied volatility surfaces.

2 Objectives

1. Conduct a literature review about arbitrage-free implied volatility surfaces and diffusion models.
2. Develop diffusion probabilistic models, tuning hyper-parameters, providing best practices, etc.
3. Introduce appropriate hard or soft constraints (neural network architectures or loss functions) to guarantee arbitrage-free constraints into implied volatility surfaces generation.
4. Explore the interpret-ability of the developed diffusion models in the context of option pricing.
5. Applications to financial option pricing and risk management (including computer codes).

References

[1] Bergeron, M., Fung, N., Hull, J., Poulos, Z., and Veneris, A. (2022). Variational autoencoders: A hands-off approach to volatility. *The Journal of Financial Data Science*, 4(2):125–138.

[2] Ho, J., Jain, A., and Abbeel, P. (2020). Denoising diffusion probabilistic models. *arXiv*, 2006.11239.

[3] Ma, X. (2023). Diffusion Probabilistic Model for Implied Volatility Surface Generation and Completion. *TU Delft Thesis*.

[4] Na, A., Zhang, M., and Wan, J. (2023). Learning volatility surfaces using generative adversarial networks. *arXiv preprint arXiv:2304.13128*.

[5] Ning, B., Jaimungal, S., Zhang, X., and Bergeron, M. (2021). Arbitrage-Free Implied Volatility Surface Generation with Variational Autoencoders. *arXiv*, 2108.04941.

[6] Song, Y., Sohl-Dickstein, J., Kingma, D. P., Kumar, A., Ermon, S., and Poole, B. (2021). Score-based generative modeling through stochastic differential equations.