Iterative solution methods for the simulation of flow in industrial glass furnaces

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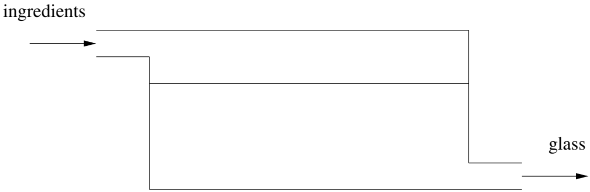
Outline

Industrial glass furnace



Mathematical simulation of flows

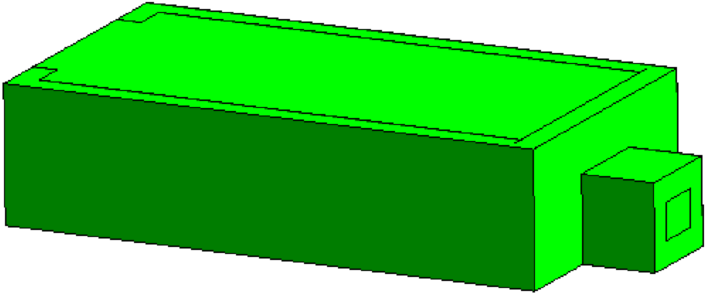
Why simulate?

- Physical experiments costly and time-consuming
- Certain physical quantities hard to measure

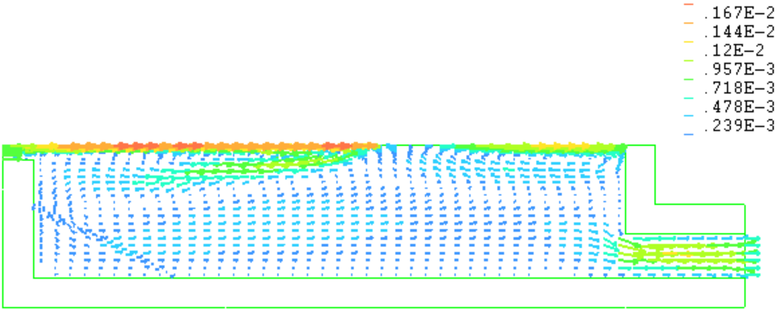
X-stream:

- CFD simulation package for glass industry
- Developed at TNO Science and Industry
- Lots of models available: combustion, turbulence, radiation, stirring, etc.

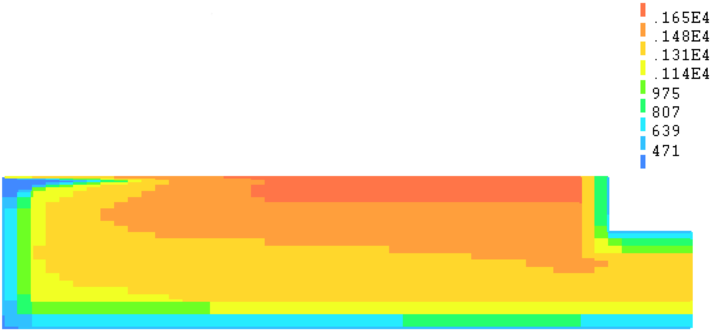
Geometry of a glass furnace



Simulation of velocities in a glass furnace

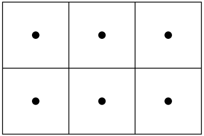


Simulation of temperature in a glass furnace

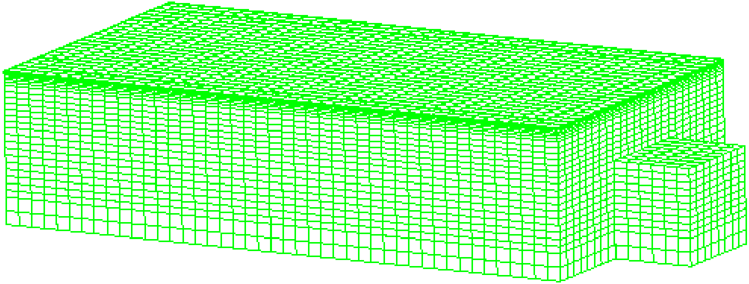


Solving mathematical flow model

- Partial differential equations arise from model
- Impossible to solve directly
- Transformation to finite number of difference equations
- Solve system of equations with iterative solution method



Grid of a glass furnace



Goal of the Master's project

Goal: improve X-stream algorithms

Focus: iterative solution methods combined with deflation

Purpose of deflation

System to be solved: $A\mathbf{x} = \mathbf{b}$

Condition number $\kappa(A) = \frac{\lambda_{max}(A)}{\lambda_{min}(A)}$ (A SPD)

Smaller condition number $\longrightarrow$ faster convergence.

Deflation removes smallest eigenvalues A.

Basic idea of deflation

System to be solved: $A\mathbf{x} = \mathbf{b}$

$$P = I - AZ(Z^TAZ)^{-1}Z^T$$
$$Q = I - Z(Z^TAZ)^{-1}Z^TA \quad (PA = AQ)$$

$$\mathbf{x} = (I - Q)\mathbf{x} + Q\mathbf{x}$$

Calculation of $\mathbf{x}$:

- $(I - Q)\mathbf{x} = Z(Z^TAZ)^{-1}Z^TA\mathbf{x} = Z(Z^TAZ)^{-1}Z^T\mathbf{b}$
- Solve $PA\tilde{\mathbf{x}} = P\mathbf{b}$ for $\tilde{\mathbf{x}}$
- Premultiply result with Q

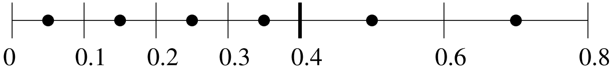
Choice of Z

Choice of Z important for convergence.

Possibilities:

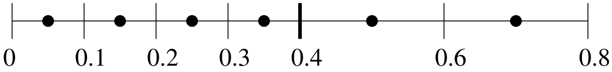
- Constant deflation (CD)
- Constant linear deflation based on grid numbering (CLD-ijk)
- Constant linear deflation based on grid coordinates (CLD-cartesian)

Constant deflation (CD)



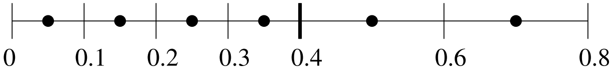
$$Z_{CD} = \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{pmatrix}$$

Constant linear deflation based on grid numbering (CLD-ijk)



$$Z_{CLD-ijk} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 1 & 3 & 0 & 0 \\ 1 & 4 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 2 \end{pmatrix}$$

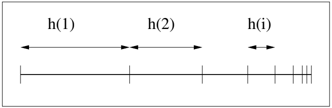
Constant linear deflation based on grid coordinates (CLD-cartesian)



$$Z_{CLD-cartesian} = \begin{pmatrix} 1 & 0.05 & 0 & 0 \\ 1 & 0.15 & 0 & 0 \\ 1 & 0.25 & 0 & 0 \\ 1 & 0.35 & 0 & 0 \\ 0 & 0 & 1 & 0.5 \\ 0 & 0 & 1 & 0.7 \end{pmatrix}$$

Experiments MATLAB

$$\frac{d^2\varphi}{dx^2} = x \sin x, \quad \varphi(0) = \varphi(\pi) = 0$$



Deflated GCR with 3 subdomains:

n	CD	CLD-ijk	CLD-cartesian
51	46	32	13
72	63	50	20

Condition number: $\kappa(A) = \frac{\lambda_{max}(A)}{\lambda_{min}(A)}$

$\kappa(PA_{CLD-ijk}) = 1.4 \cdot 10^5$

$\kappa(PA_{CLD-cart}) = 8.9 \cdot 10^4$

Experiments X-stream

Input:

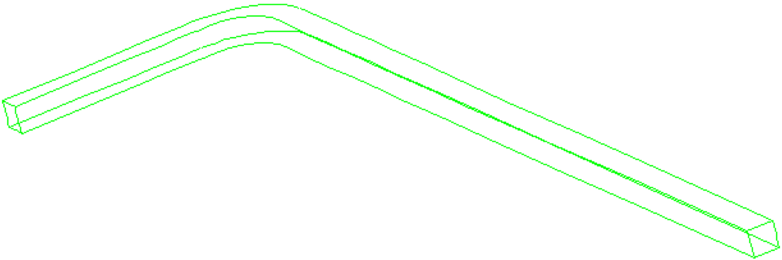
- number of inner iterations
- iterative method: SIP or GCR with CD, CLD-ijk or CLD-cartesian

Output:

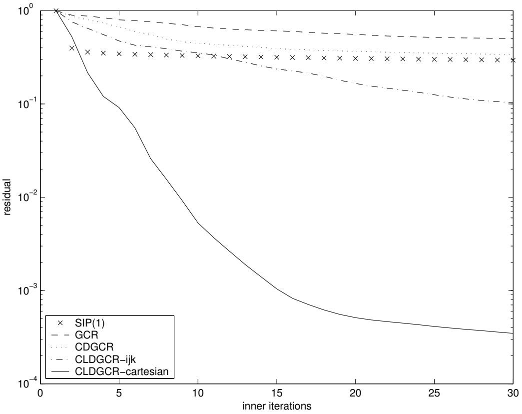
- residuals
- number of outer iterations
- wall-clock time

Note that the number of outer iterations and the wall-clock time depend on the residuals.

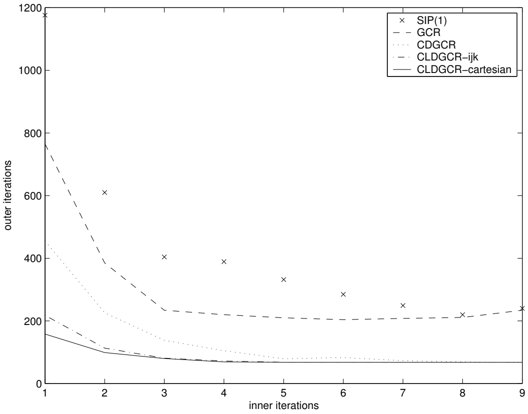
Test case channel



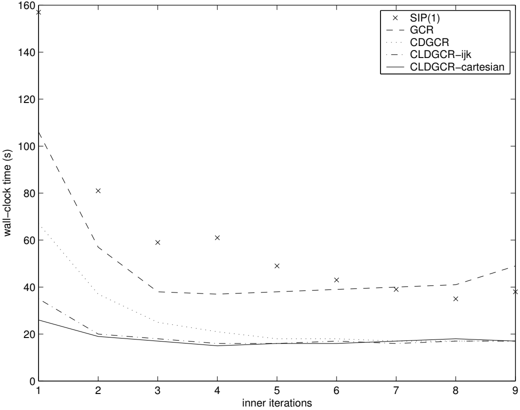
Residuals channel



Number of outer iterations channel



Wall-clock times channel



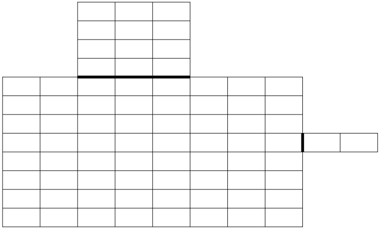
Results wall-clock times channel test case

inner it.	SIP(1) / GCR	SIP(1) / CDGCR	SIP(1) / CLDGCR-ijk	SIP(1) / CLDGCR-cart
1	1.5	2.3	4.5	6.0
2	1.4	2.2	4.1	4.3
6	1.1	2.4	2.5	2.7
9	0.8	2.2	2.2	2.2

Using optimal number of inner iterations for each method:

SIP(1) / GCR	SIP(1) / CDGCR	SIP(1) / CLDGCR-ijk	SIP(1) / CLDGCR-cart
0.9	2	2.1	2.3

Subdomain partitioning



$E = Z^T A Z$

600 subdomains:

E_{CD} 600×600-matrix

E_{CLD} 2400×2400-matrix

Conclusions

- GCR performs better with deflation
- Constant linear deflation performs better than constant deflation
- CLD-cartesian performs better than CLD-ijk
- More subdomains means less iterations
- Drawback of CLD: not suitable for large number of subdomains

Recommendations

Further research subdomain partitioning:

- How to decrease number of subdomains
- Implement a suitable sparse solver for unstructured matrices

Questions?