Numerieke methoden voor de stationaire ondiepwatervergelijkingen

Femke van Wageningen-Kessels

afstudeercommissie
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TU Delft
Delft University of Technology

Achtergrond: rivierbeheer

Achtergrond: software

WAQUA

- sinds circa 1970 -

- tijdsafhankelijke oplossing
- nauwkeurig
- veel mogelijkheden

QuickFlow

- sinds 2006 -

- alleen stationaire oplossing
- WAQUA nodig voor beginoplossing
- plaatsdiscretisatie WAQUA

Inhoud

- De ondiepwatervergelijkingen
- Correct oplossen
  - stabiliteit
- Snel oplossen
  - methoden
  - testresultaten
- Conclusie

2D ondiepwatervergelijkingen

$$
\begin{aligned}
\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} - fV + g\frac{\partial \zeta}{\partial x} - \frac{\tau_{\mathrm{bottom},x}}{\rho_0 H} - \nu_{\mathrm{H}}\left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2}\right) &= 0 \\
\frac{\partial V}{\partial t} + U\frac{\partial V}{\partial x} + V\frac{\partial V}{\partial y} + fU + g\frac{\partial \zeta}{\partial y} - \frac{\tau_{\mathrm{bottom},y}}{\rho_0 H} - \nu_{\mathrm{H}}\left(\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2}\right) &= 0 \\
\frac{\partial \zeta}{\partial t} + \frac{\partial HU}{\partial x} + \frac{\partial HV}{\partial y} &= 0
\end{aligned}
$$

Discretisatie & randvoorwaarden $\Rightarrow \quad \mathrm{M}\dfrac{\partial \mathbf{x}}{\partial t} = \mathbf{F}(\mathbf{x})$

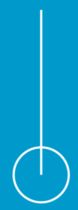
Stationair $\Rightarrow \quad \mathbf{F}(\mathbf{x}) = \mathbf{0}$

Correcte oplossing

- Residuenanalyse
- Continuering
- Stabiliteitsanalyse

Stabiliteit

Stationair



Oplossing verandert niet meer als tijd verstrijkt

Stabiel

Als er een kleine verstoring optreedt blijft de oplossing ongeveer gelijk

Eigenwaardenanalyse: theorie

Lineair probleem $\frac{dx}{dt} = Ax$

QuickFlow: $M\frac{dx}{dt} = F'(x)x$

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Oplossing $x(t) = x_0 e^{At} = x_0 S e^{Jt} S^{-1}$

J Jordan vorm van A: eigenwaarden op hoofddiagonaal

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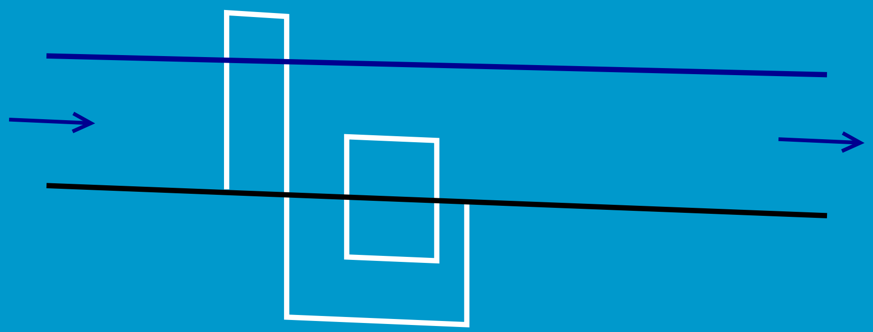
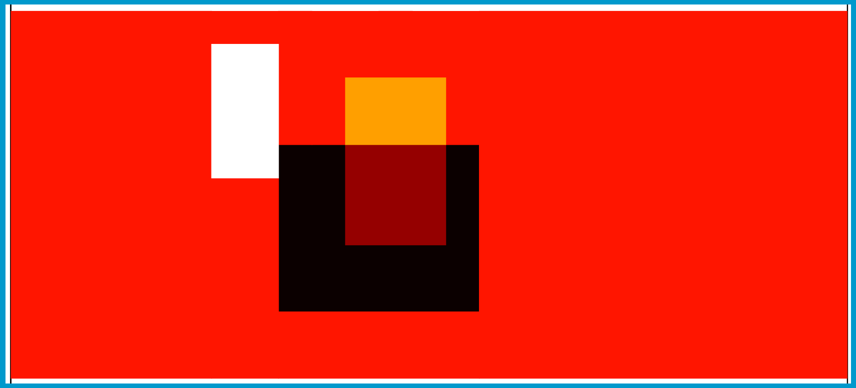
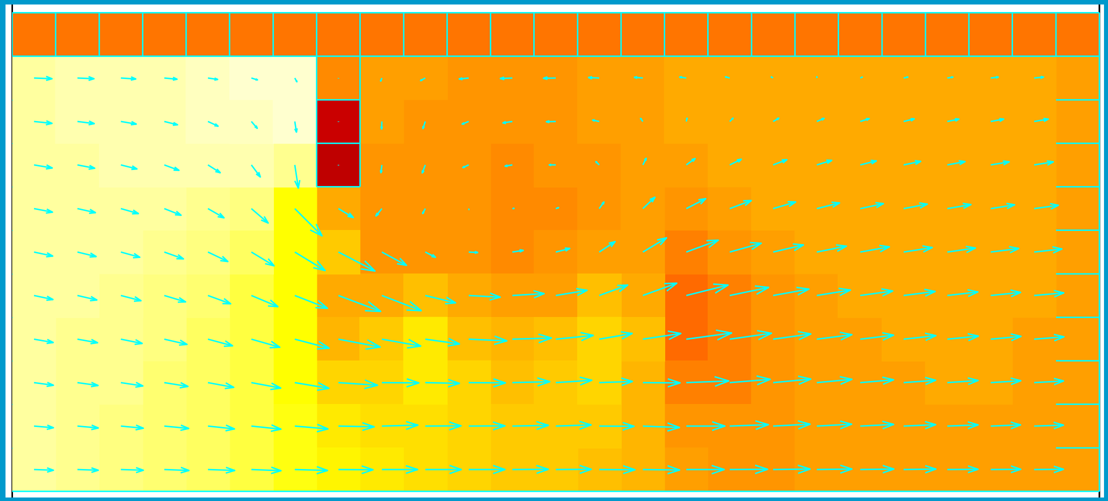
Als $Re(\lambda) < 0$ dan $e^{Jt} \rightarrow 0$ als $t \rightarrow \infty$

Dus $x(t) = 0 \Rightarrow x = 0$ is stabiele oplossing

Eigenwaardenanalyse: QuickFlow







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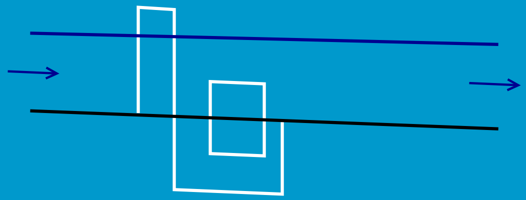
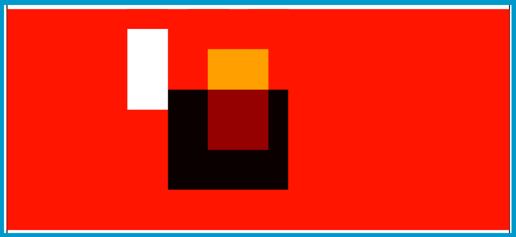
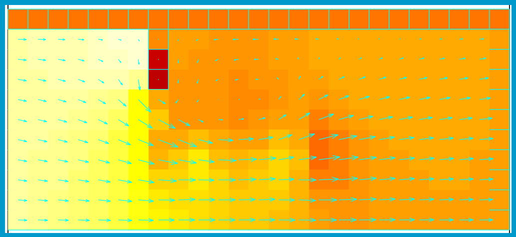
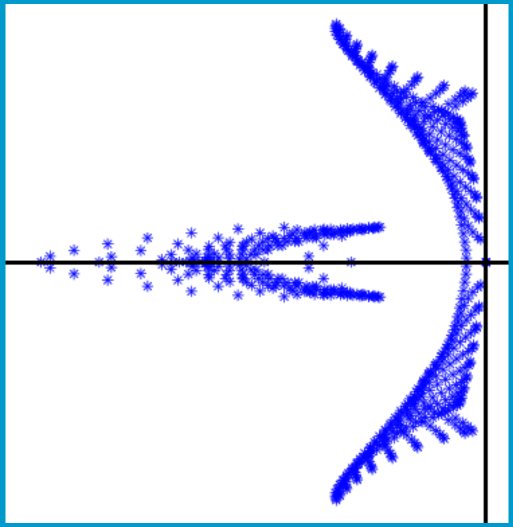


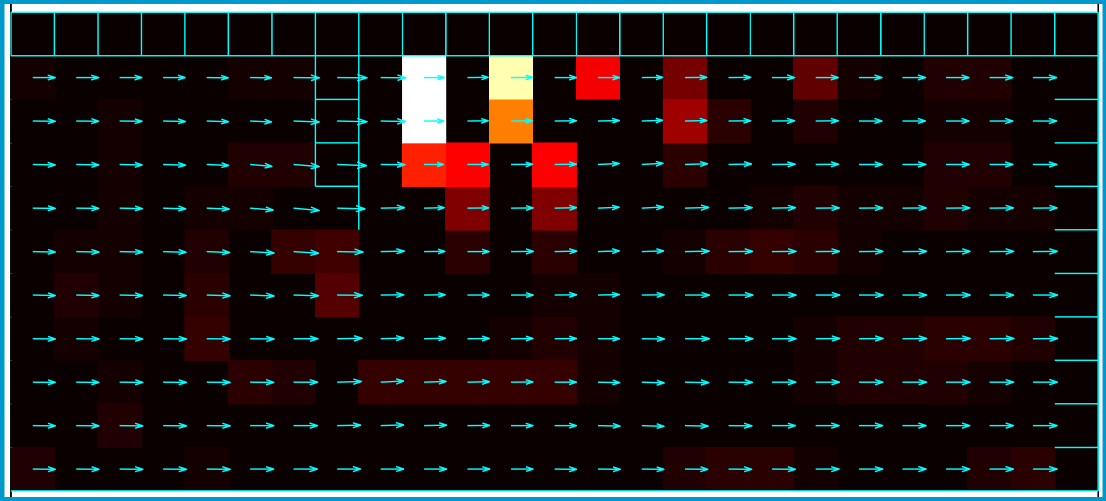




Eigenwaarden

Eigenvector





Stabiliteit: Crank-Nicolson

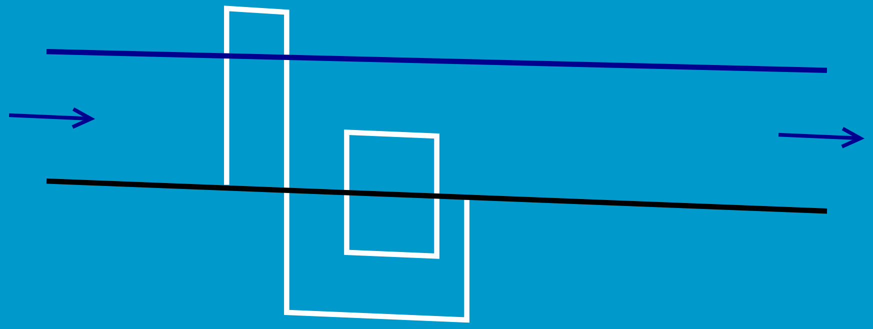
$$\begin{aligned} M\frac{dx}{dt} &= F(x) \\ M\frac{x_{\text{nieuw}} - x_{\text{oud}}}{\Delta t} &= \frac{F(x_{\text{nieuw}}) + F(x_{\text{oud}})}{2} \end{aligned}$$

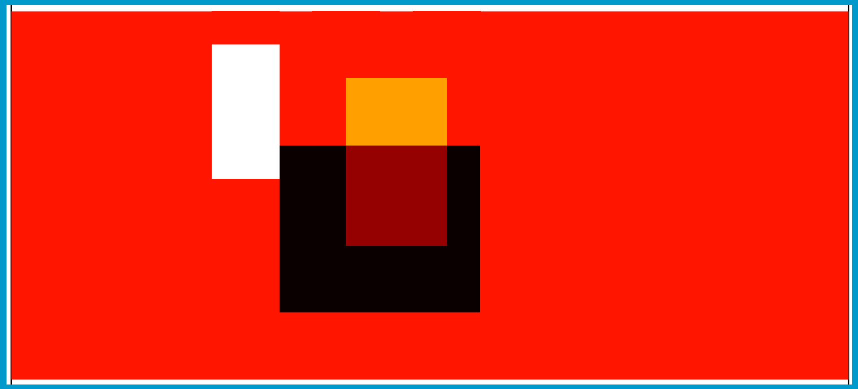
Beginoplossing stabiel $\Rightarrow$

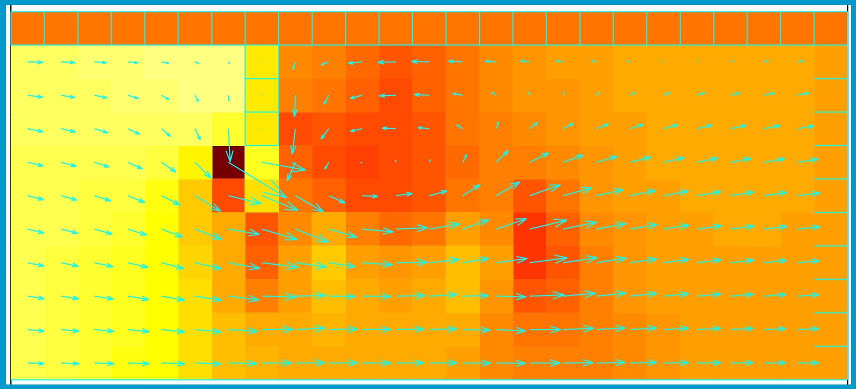
Crank-Nicolson onvoorwaardelijk stabiel $\Rightarrow$

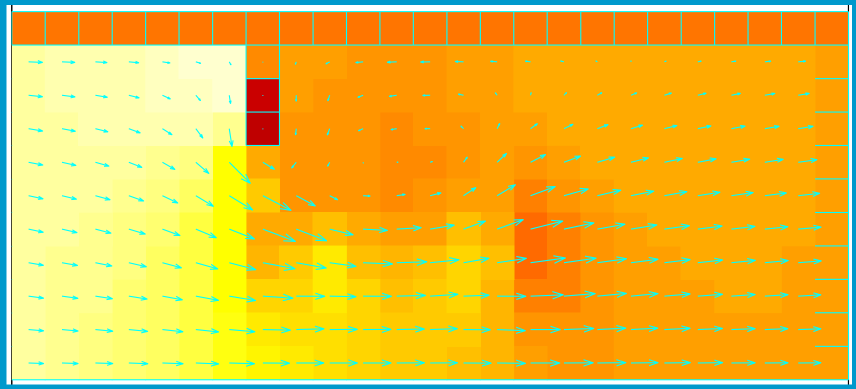
oplossing wijkt nooit ver af van beginoplossing

Toepassing Crank-Nicolson

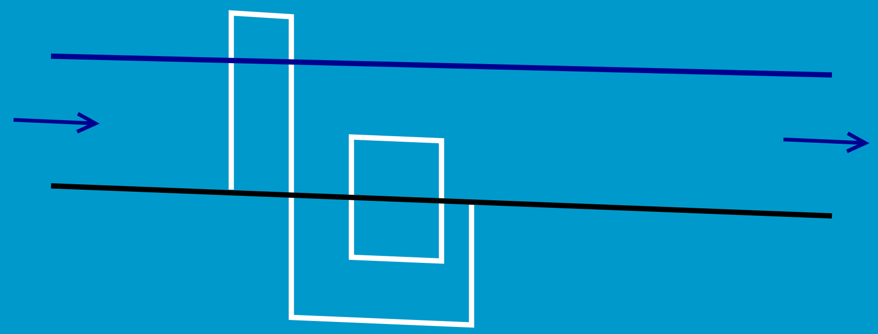


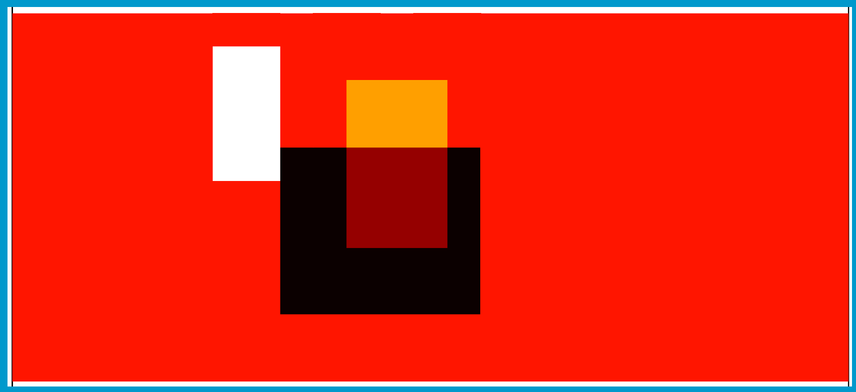


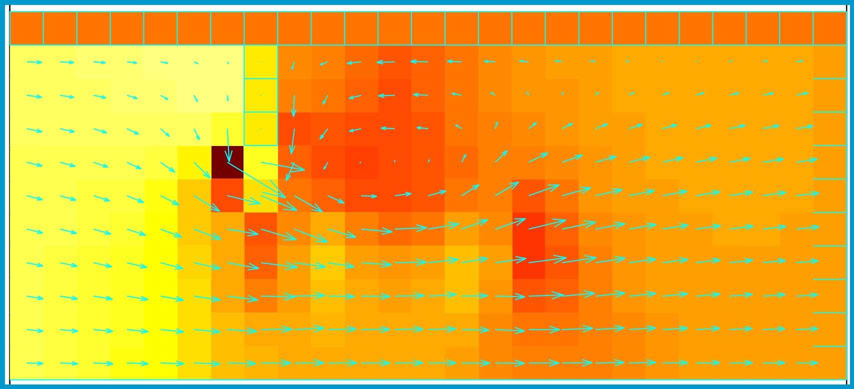


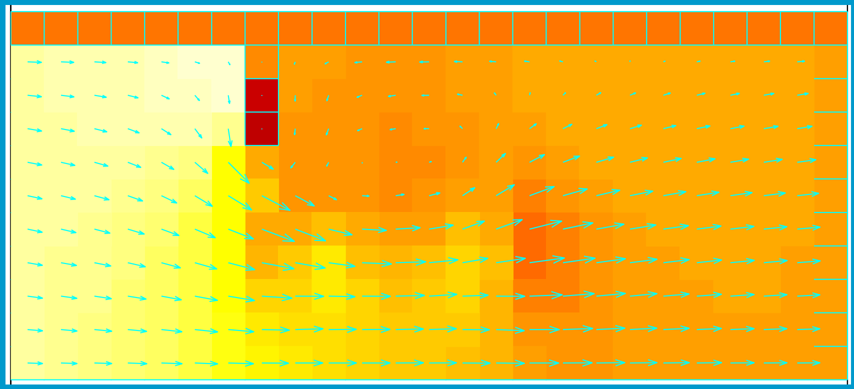


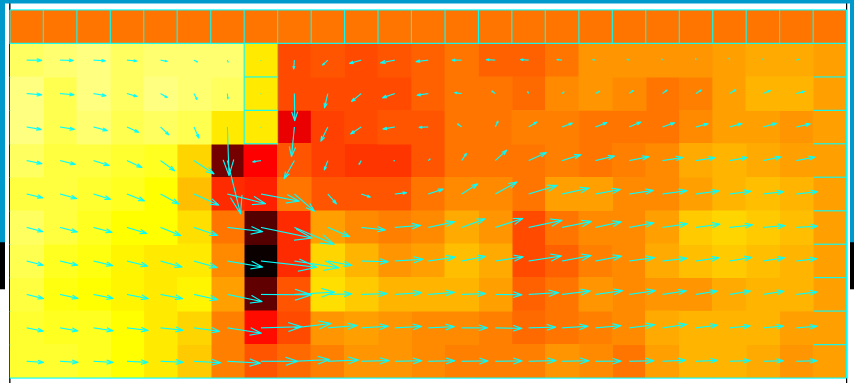
Toepassing Crank-Nicolson

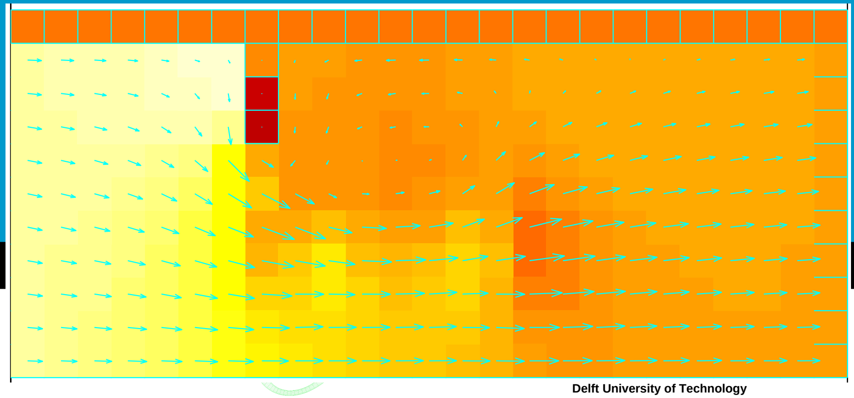




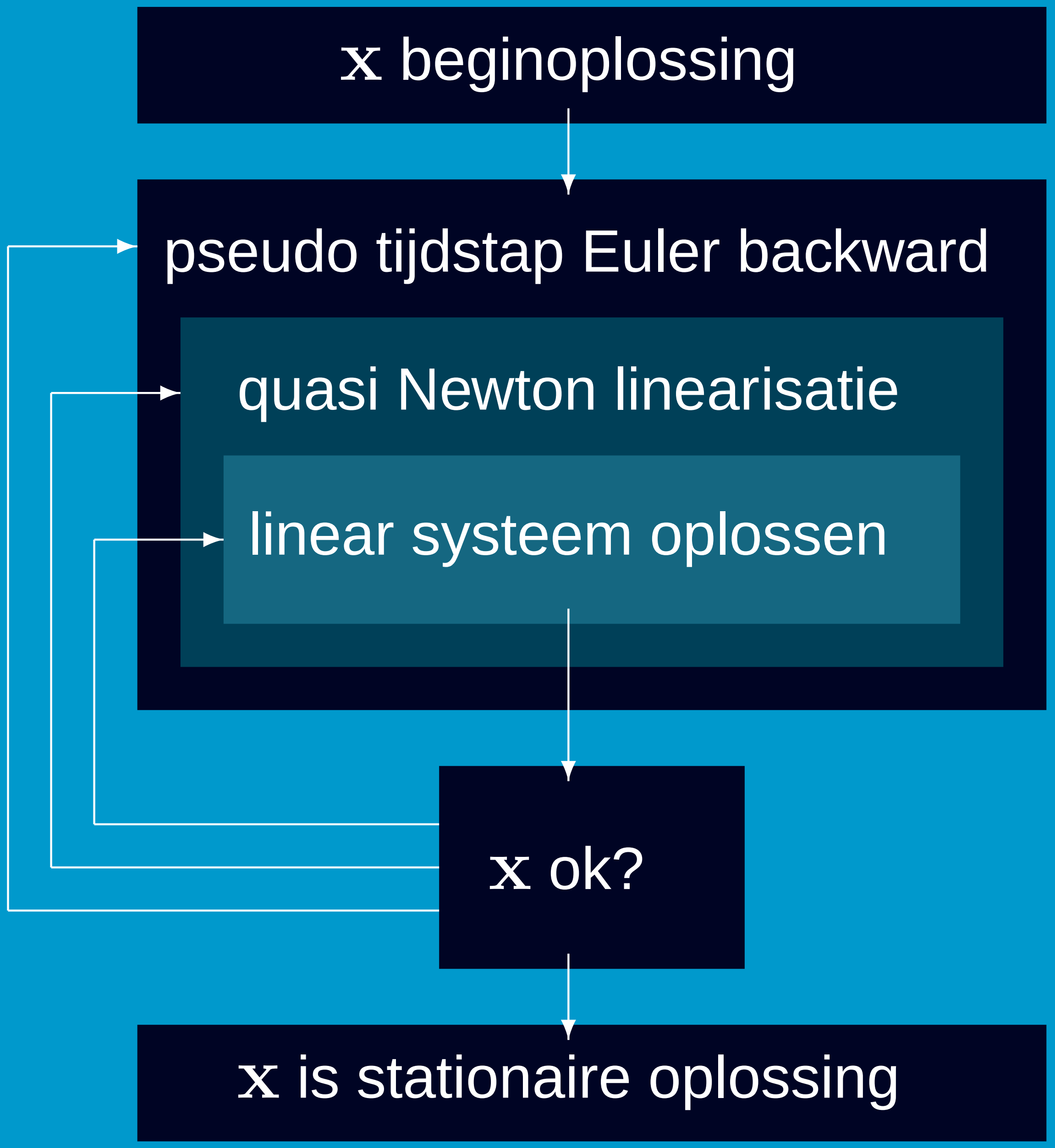






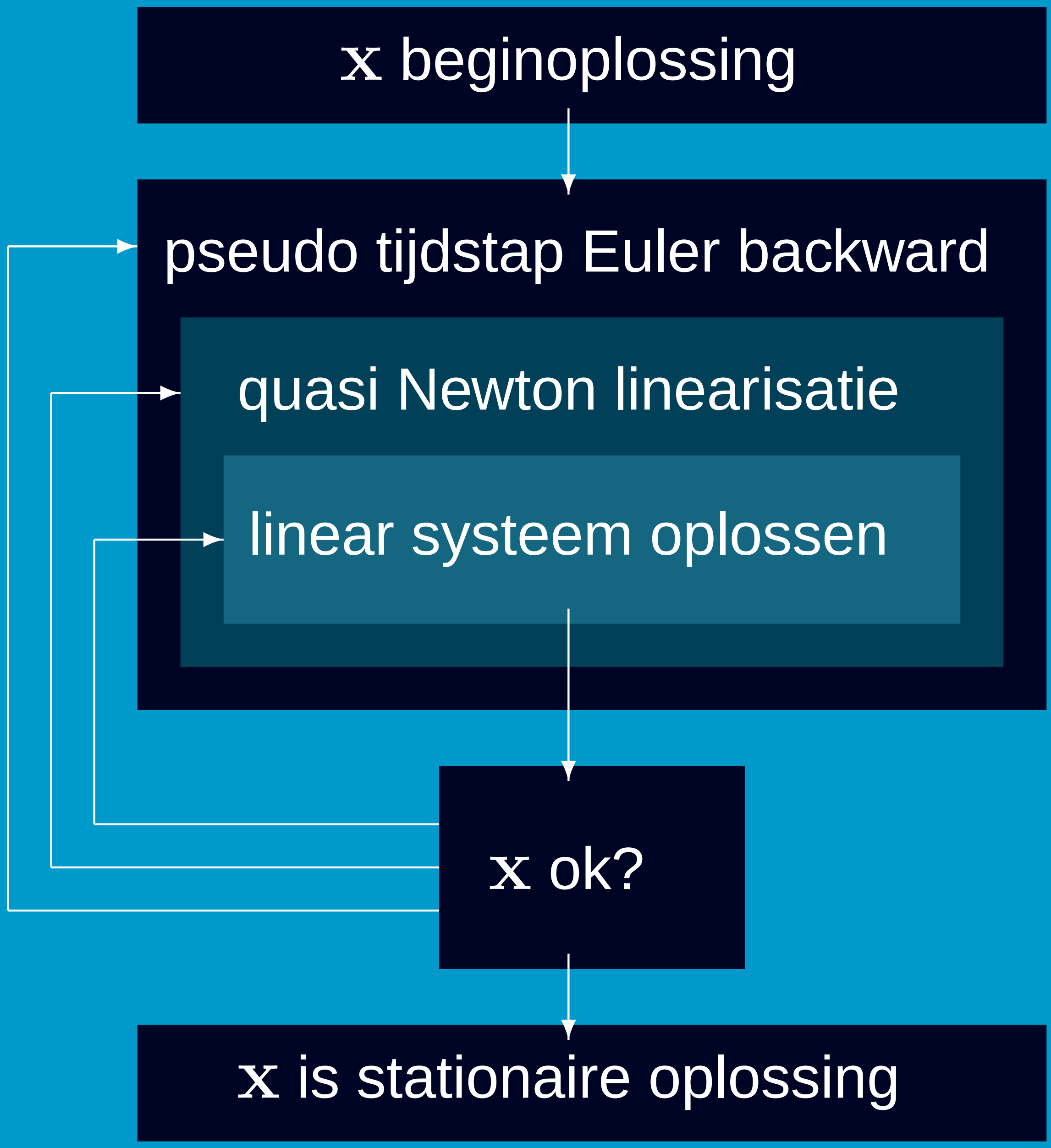


Snel oplossen



Snel oplossen

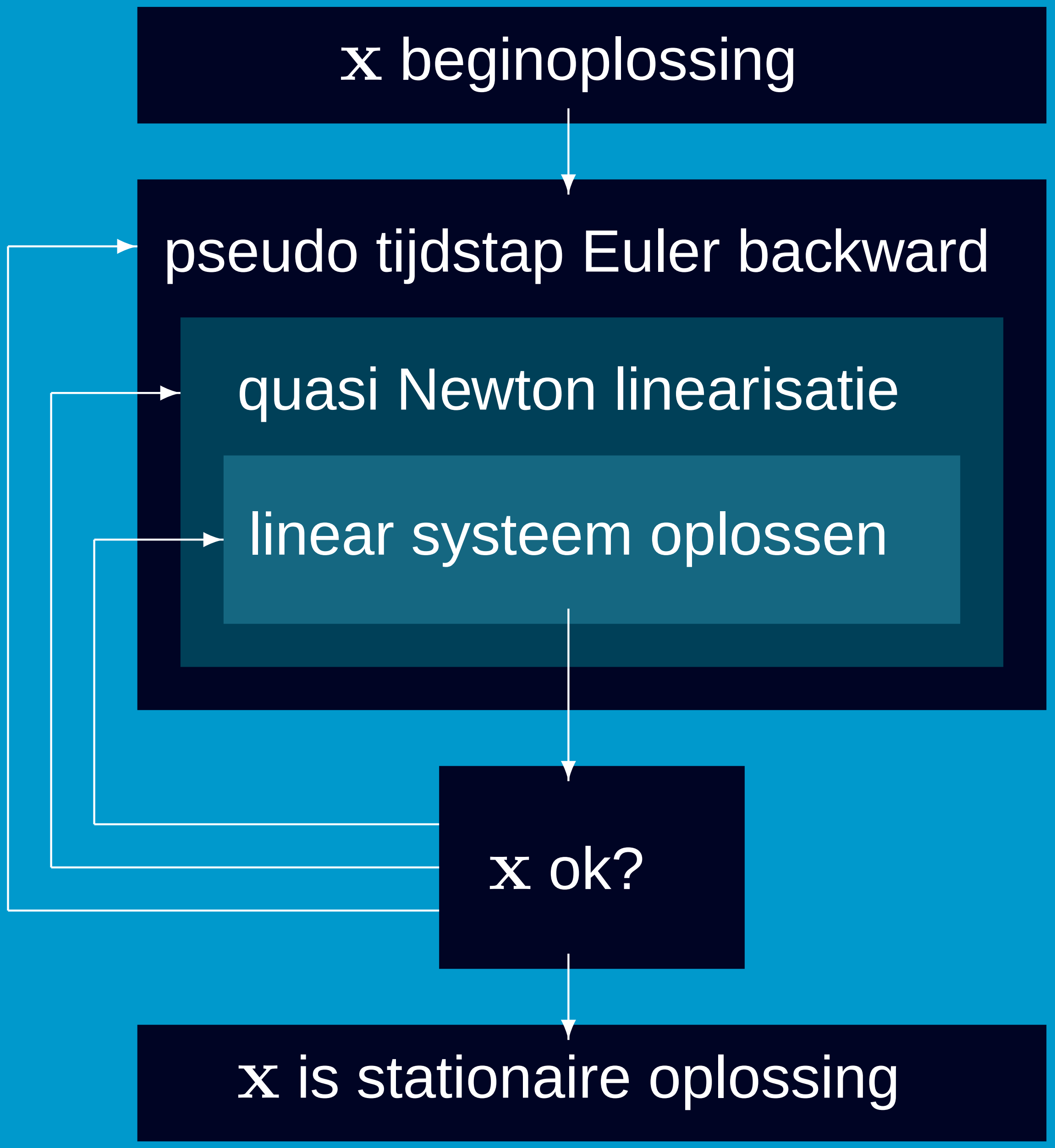
grote tijdstappen



Snel oplossen

grote tijdstappen

line search



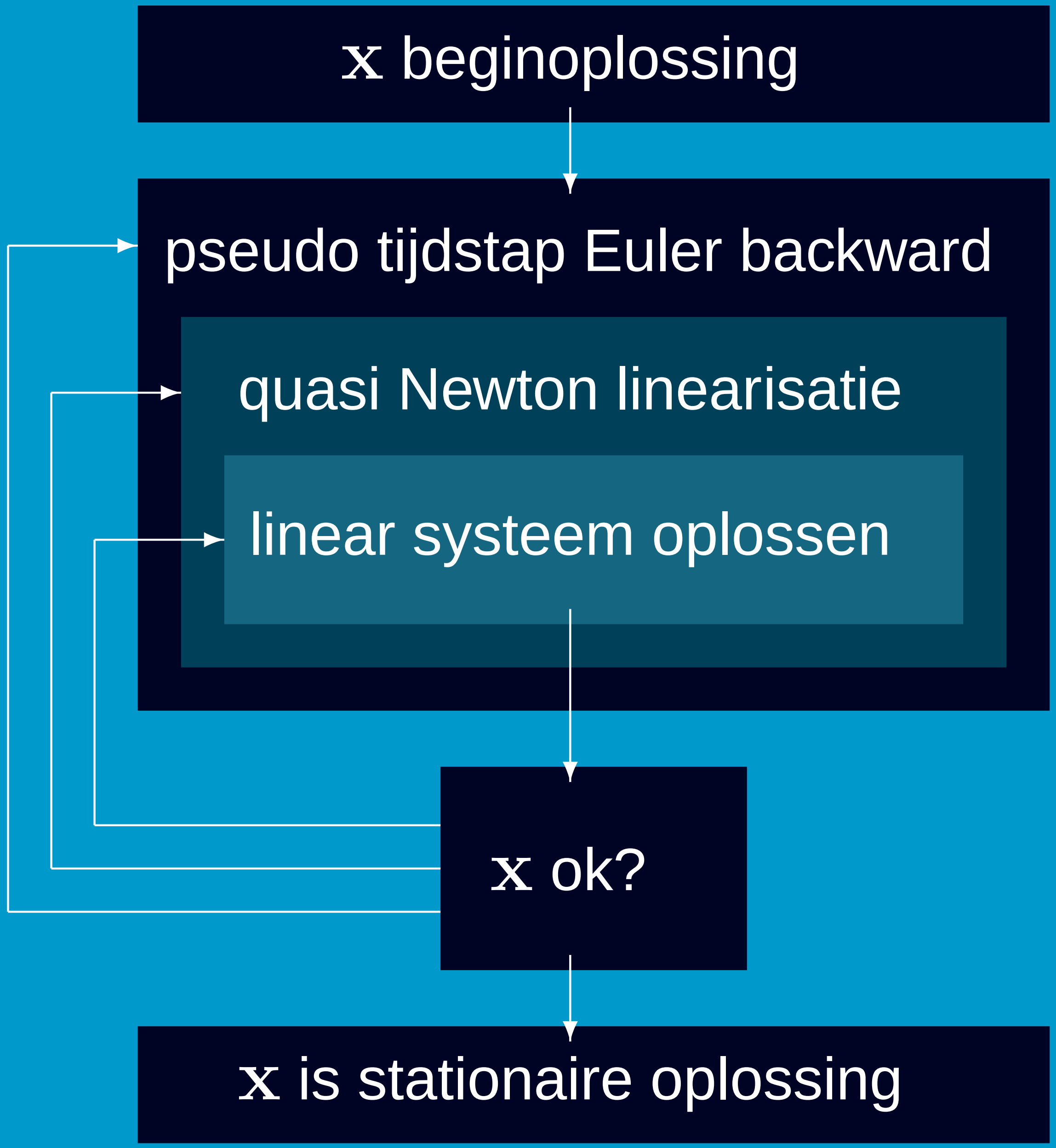
Snel oplossen

grote tijdstappen

line search

LU preconditioner

BiCGSTAB



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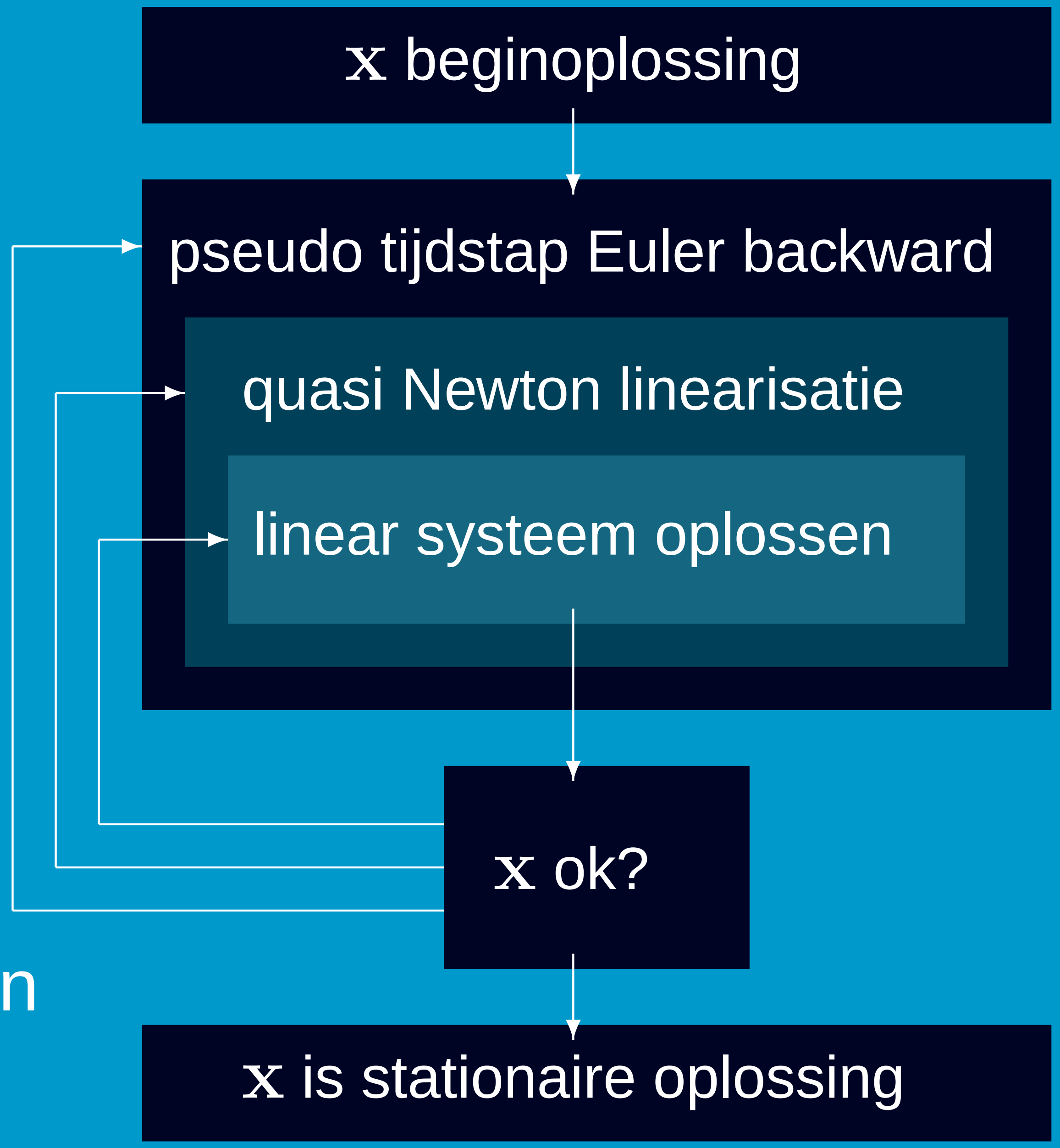
grote tijdstappen

line search

LU preconditioner

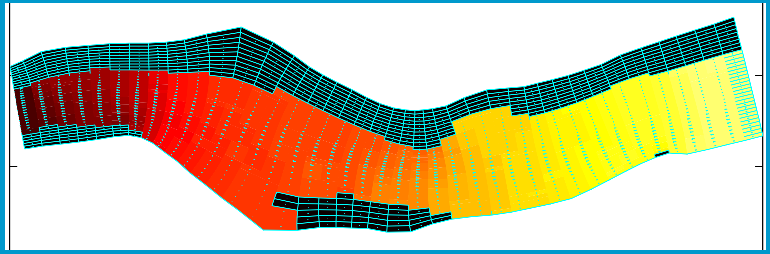
BiCGSTAB

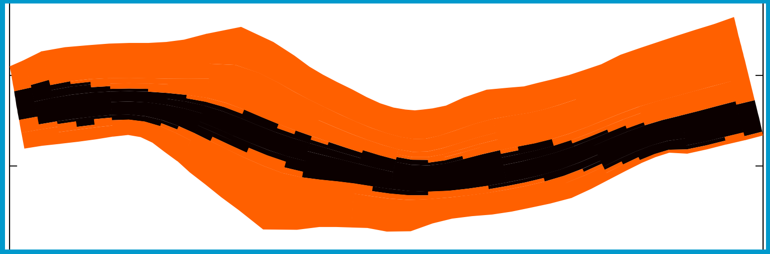
Jacobiaan en LU
weinig herberekenen



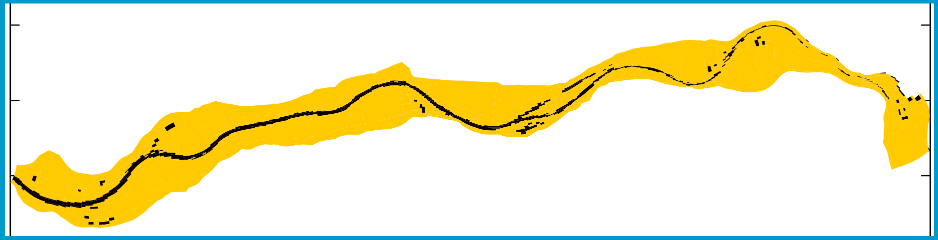
Tests

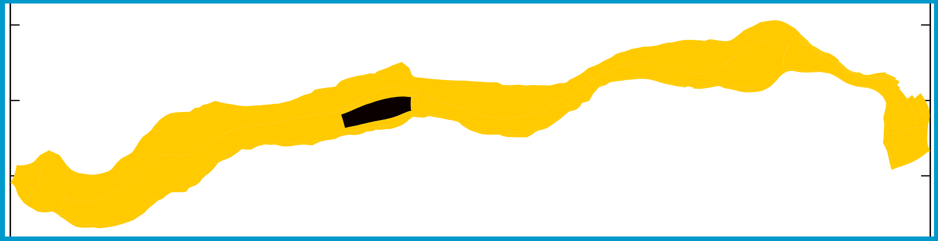
Lek



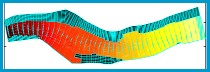
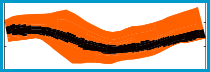
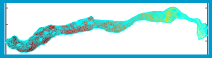
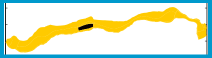


Randwijk





Testresultaten

	Lek		Randwijk		
					

Testresultaten

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	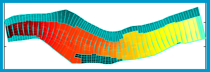	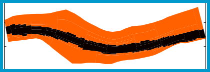	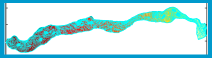		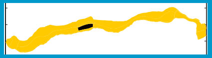
oud	21	37	476	1435	2092

Testresultaten

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	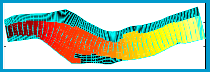	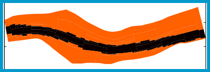	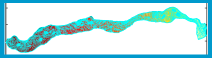		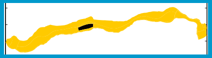
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pseudotijdstap	7	12	55	74	361

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	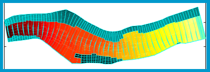	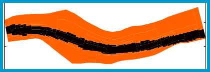	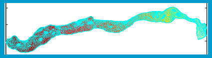		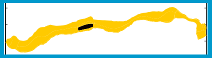
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quasi Newton	6	18	50	82	?

Testresultaten

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	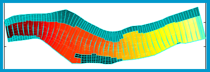	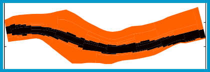	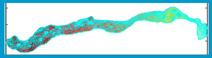		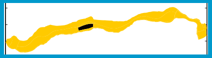
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Testresultaten

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	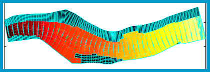	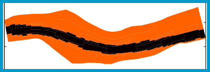	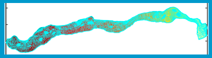		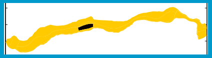
oud	21	37	476	1435	2092
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quasi Newton	6	18	50	82	?
BiCGSTAB	4	12	25	36	68
totale verbetering	5 ×	3 ×	19 ×	40 ×	30 ×

Conclusie

- Correct oplossen $\longrightarrow$ ook moeilijke problemen
- Stabiliteitsanalyse:
  - Eigenwaarden
  - Crank-Nicolson
- Snel oplossen $\longrightarrow$ 3 tot 40 keer sneller

Vragen of opmerkingen?

VORTECH BV
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TU Delft
Delft University of Technology

Vragen of opmerkingen?

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TUDelft
Delft University of Technology

Vanavond: borrel!

20:00 – ’t Boterhuis – Markt 15-17a