Frequency Transfer Operators for Increased Helmholtz Solving Efficiency

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Background

Efficiently solving the high-frequency Helmholtz equation remains a major challenge in computational wave analysis. Classical multigrid and Krylov solvers fail to scale at high wavenumbers due to the oscillatory and indefinite nature of the operator. Stabilized preconditioners such as the Complex Shifted Laplacian (CSL) and Multilevel Deflation (MLD) improve convergence but tend to degrade as frequency increases.

A new idea consists of *frequency-transfer operators*; mappings that relate Helmholtz solutions at nearby frequencies via rephasing transformations. These operators promise a new class of solvers that combine the robustness of classical preconditioners with the efficiency of coarse-frequency solves, opening new possibilities for hierarchical and data-driven multilevel methods.

Mathematical background. The problem focuses on solving the time-harmonic Helmholtz equation, represented as

$$L_\omega u = f, \tag{1}$$

where the operator is defined as

$$L_\omega = -\Delta - \omega^2 m(x), \tag{2}$$

with $m(x) = 1/c^2(x)$ the squared slowness of the medium and $x \in R^2$. The solution $u(x)$ describes the steady-state acoustic pressure field generated by a source term $f(x)$ at angular frequency ω.

Relevance to seismic applications. In seismic imaging and subsurface characterization, the Helmholtz equation governs the propagation of time-harmonic acoustic or elastic waves through the Earth's heterogeneous medium. Accurately solving this equation for high frequencies is crucial for methods such as Full Waveform Inversion (FWI) and Reverse Time Migration (RTM), where fine spatial resolution depends on the ability to capture short-wavelength phenomena.

However, seismic models often contain strong velocity contrasts and complex geometries, leading to highly indefinite and non-normal Helmholtz operators. These properties make iterative solvers unstable or slow to converge, especially when ω is large. As a result, scalable preconditioners and frequency-transfer techniques are essential to make large-scale, high-fidelity seismic simulations computationally feasible.

Research Objective

This project aims to design, analyze, and learn frequency-transfer operators that map Helmholtz solutions between frequencies, enabling preconditioning in the frequency domain rather than the spatial domain.

Central question. *How can learned frequency-transfer operators reduce the computational cost of high-frequency Helmholtz solvers while preserving stability and accuracy?*