Msc thesis proposal:

Using Machine Learning Methods to Fit the Characteristic Function of Combined Stochastic Processes

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In the past a few years, there has been increasing interests in applying machine-learning techniques for fast pricing of derivatives and fast calibration of model parameters.

This thesis project is dedicated to explore a different way of employing the machine learning techniques than seen in literature. That is, we try to let the machine learn the characteristic function (ch.f.) of combined stochastic processes, which drive the underlying dynamics of exotic options such as barrier options.

From a theoretical point of view, fitting the ch.f. is more efficient than fitting the option pricing function itself, since the parameter data sets for the machine to “learn” from contain only the model parameters, not including the parameters defining the option contracts. Hence, either the training time can be less, or the accuracy of the end approximation could be higher than fitting the option pricing function directly.

Once the ch.f. is approximated, the COS method [8] can be used for valuation of options, which is efficient enough for real-time calibration.

**Background**

The tremendous increase in computing power and data storage capacity during the last decade has resulted in the rapid development of machine learning and data mining with diverse applications in economics, finance, science, engineering, and technology. In the finance area, machine learning models have elicited considerable attention from many researchers because of their predictive power.

A few recent articles from literature have attempted to solve the derivative pricing problem and/or related implied volatility calibration problem using various types of machine learning methods.

In [3], a generative Bayesian learning model is proposed, which incorporates a prior reflecting a risk-neutral pricing structure to provide fair prices for the deep ITM and the deep OTM options that are rarely traded.

In [1], the authors illustrate that, for many classical problems, speed-ups of several orders of magnitude by deploying machine learning techniques based on Gaussian process regression (GPR). The price one has to pay for this extra speed is some loss of accuracy. To be more precise, they start with showing the strengths of the method with fitting a non-trivial Gamma profile. Next, they illustrate the fitting ability by letting the machine learn the implied volatility surface of a given underlier on a given day. As a second line of applications, they apply the techniques in the setting of the pricing of exotic derivatives under advanced models.

Authors in [2] use convolutional neural networks (CNN) to find the Hölder exponent of simulated sample paths of the rBergomi model, a recently proposed stock price model used in mathematical finance.

A deep learning method is employed in [6] to approximate expected exposures and potential future exposures of Bermudan options.

A data-driven approach called CaNN (Calibration Neural Network) is proposed in [5] to calibrate financial asset price models using an Artificial Neural Network (ANN). Determining optimal values of the model parameters is formulated as training hidden neurons within a machine learning framework, based on available financial option prices.

Instead of using CNN or DNN (Deep Neural Nets), authors of [7] built Chebyshev tenors, either directly or with the help of the Tensor Extension Algorithms, to tackle the computational bottleneck associated with the calibration of the rough Bergomi volatility model. Results are encouraging as the accuracy of model calibration via Chebyshev Tensors is similar to that when using Deep Neural Nets, but with building efforts that range between 5 and 100 times more efficient in the experiments run.

**Challenge**

The Bayesian learning methods exhibit a lower level of accuracy than other two types of methods.

CNN and DNN methods are subject to poor mathematical tractability, as the networks are within a black-box based on off-the-shelf machine learning techniques. Further, mathematically there is no proof of error convergence to ensure the stability and accuracy of the approximations. At last, the activation function is still chosen based on “experiences” without any mathematical justification or by accounting for the problem-specific information.

Tenor-based methods are much more tractable mathematically, but face the curse of dimensionality and are seen to be too complicated to be embraced by the industry.

**The goal and content of this thesis**

We notice that the key point in derivative pricing actually lies in the recovery of the distribution function of the (combined) underlying stochastic process. Hence, all we need is actually to approximate this distribution accurately. The next step of option pricing can be done semi-analytically using the COS method, which is sufficiently efficient to support real-time calibration in practice.

Hence, this thesis project is designed to explore the feasibility of using machine-learning methods to fit the ch.f., which is the one-to-one equivalence of the concerned distribution function and is the input needed by the COS method.

Aiming to provide a mathematically tractable yet simple-enough method for practical usage, we propose to

* start the research by replicating [1] and then adjust the methodology to cope with our aim of fitting the (analytically-known) ch.f. of some Levy processes (such as GBM and CGMY), linking to the underlying dynamic of an European option;
* try to improve the performance of the method by replacing the basis functions used in [1] by orthogonal functions in the functional space, such as cosine functions, preparing for a mathematical proof of the existence of the solution and the error convergence;
* Increase the complexity by fitting the ch.f. of the barrier-bridging process of a Levy variable, linking to the underlying dynamic of a barrier option;
* Feed the approximated ch.f. to the COS method to price barrier options and benchmark the results those of existing option pricing methods, such as COS-based methods and/or MC-based methods.

**Reference**

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**Contact**

If you are interested to enter the field of quantitative risk analysis, this is a very good starting point. Please feel free to contact me directly if this topic is of your interest, or if you would like to learn more details: fang.fang@ffquant.nl or f.fang@tudelft.nl

**About FF Quant Advisory B.V.**

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1. Part-time Assistant professor at the Applied Mathematics Department of TU Delft; Director of FF Quant Advisory B.V. https://fsquaredquant.nl/ [↑](#footnote-ref-2)