MSc-project: Advancement of a 1D Spline-based higher-order Material Point Method to 2D

jointly by Deltares (Enabling Delta Life) and TU Delft (Delft University of Technology)

Motivation

The Material Point Method (MPM) is a relatively new numerical method that is particularly designed to study dynamic geotechnical problems involving large soil deformations such as submarine land-slides and pile driving. A 3D MPM code is developed at Deltares, a Dutch research and consulting company situated in Delft, in the frame of a research community consisting further of University of Cambridge, UPC Barcelona, TU Hamburg-Harburg, University of Padova and TU Delft. The MPM makes use of a fixed background mesh on which the equilibrium equations are solved by the Finite Element Method (FEM) and a cloud of material points representing a deforming solid that moves through the mesh [Su94, Su95, AL13].

Within standard MPM, piecewise linear Finite Element basis functions are commonly used for the mapping of data between material points and the background mesh, whereas high-order interpolation functions would be desirable for several reasons. Firstly, high-order functions enable a more accurate reproduction of physical quantities such as stresses. Secondly, interpolation of higher order allows utilizing coarser FE meshes, thus reducing the computational costs while preserving the same spatial accuracy or even improving it. Introducing high-order basis functions in MPM is, however, not trivial [Be12]. In the frame of a precursor master study, a solution approach using B-Spline interpolation functions as employed in the Isogeometric Analysis framework [Hu05] has been pursued for 1D [Ti16]. It rendered a high convergence rate and accurate results with studied benchmarks. Deltares and TU Delft therefore intend to further pursue this approach with the aim of eventually obtaining a 3D solution.

Problem description and challenges

The task of this master thesis is to develop a high-order 2D MPM that makes use of quadratic B-Spline basis functions for the accurate interpolation of quantities on FE background meshes consisting of unstructured triangulations. Based on the existing work in 1D, the extension of this approach to multiple dimensions requires the combination of recent ideas from Isogeometric Analysis with the MPM. The main challenge is the design of quadratic B-Spline basis functions that preserve higher-order continuity across element boundaries even on unstructured triangulations. In order to ensure relevance to geotechnical analyses, devised solutions should be thoroughly validated using benchmarks.

Time schedule

The following tasks are foreseen:

The student will first familiarize with the FEM and MPM based on existing work. This involves the study of the master thesis of Tielen [Ti16] including a 1D FEM/MPM code implemented in Matlab and benchmarks computed with it. Furthermore, this stage involves familiarization with 2D FE analyses by means of a small code to be developed and validated with simple benchmarks.

Afterwards, the developed 2D code will be extended to quadratic interpolation functions, first for FE analyses then for MPM analyses. Here, the approach developed in [Ti16] will be pursued but it may well require adaptation.

The study of 2D B-Spline interpolation will require special attention in this work. The extension of B-Spline interpolation functions to triangles with higher-order inter-element continuity represents a challenging task [Ja14, Sp12]. The student is expected to decide on the basis of a literature study on existing approaches on one of them that will be implemented into the 2D FEM/MPM code. Special attention should be paid to the treatment of discontinuities within a solid, i.e. soil layer boundaries, as well as of traction boundary conditions.

The numerical properties of the novel 2D B-Spline MPM will be studied numerically for well-established benchmark problems, i.e. rigid strip footing on elastic soil [Gi72].

Contact

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Literature

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