Msc thesis proposal:

Replacing the Monte Carlo Simulation by the COS Method for PFE (Potential Future Exposure) Calculations

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**Background**

The quantification of the counterparty credit risk (CCR) is the subject of this thesis. CCR is the risk that a bank suffers from economic losses in the OTC (Over-the-Counter) derivatives and Repos due to the default of the counterparty of those trading positions.

A common measure for the quantification of the CCR is the so-called Potential Future Exposure (PFE), which is defined as the 97.5% quantile of the future exposure distribution of a netting set or of a higher-level portfolio at a future time point, as illustrated by Figure 1.



Figure Illustration of the future exposure distribution

The netting set is a portfolio of trades, whereby the MtM prices of the trades are allowed to be netted based on a bilateral netting agreement between the bank and its counterparty.

The future exposure, i.e. the total positive MtM price, of a netting set is a random variable, since the driving risk factors of the trades (such as interest rates, FX rates, credit spreads, etc) at a future time point are not yet known today.

PFE is usually adopted by banks to define the trading limit to control CCR. The standard numerical method, if not the only method, in industry for PFE calculations is the Monte Carlo simulation.

In this MC framework, the risk factors are first simulated according to pre-selected stochastic models, then the simulated scenarios are fed to the pricing functions of the trades to yield their MtM prices at a future time point, and at last we aggregate them up to yield the future exposure distribution of the whole portfolio and return the 97.5%-th quantile as the PFE value.

**Challenge**

The PFE calculation is time consuming since the MC simulation method has a low convergence rate and therefore a large number of simulation is needed to achieve an acceptable level of accuracy.

This is not very handy in practice, especially when we are only interested with the marginal impact of one new trade to the PFE value.

Hence, a faster and more accurate alternative method is highly desired.

**The goal and content of this thesis**

The goal of this thesis project is to replace the MC method by a much faster numerical method – the COS method for PFE calculations.

The COS method was first introduced in 2008, initially for the purpose of option pricing. However, the essence of the method is to recover the unknown density function from its Fourier-cosine series expansion, with the key insight that the series coefficients are almost readily available from the characteristic function which is very often easier to derive than the density function itself.

Hence, the COS method is applicable whenever the probability density function of a random variable is not known but the characteristic function can be obtained either analytically or semi-analytically.

Thus in this Msc thesis, we aim to solve the PFE from the following angle:

* First to derive the characteristic function of the future exposure for the total netting set, either analytically or semi-analytically,
* then to derive a variation of the COS method to recover the cumulative distribution function (CDF) from the characteristic function, and at last
* the PFE value can be found as VaR of the CDF that corresponds to the 97.5% quantile.

The thesis project will begin with a literature review, from which the student will grasp the basic concept about PFE, the latest efforts in literature that try to improve the PFE calculations, and the measurement of PFE using MC method.

As the 2nd step, the student needs to build the MC simulation framework as the benchmark. in which the commonly used stochastic models for interest rate and FX are employed. The simulation engine is then linked with the available pricer of two basic product types – IR swap and cross currency swap. Aggregation of the scenarios leads to the future exposure distribution.

As the 3rd step, the student needs to derive the characteristic function of the future exposure, either analytically or semi-analytically. Then by a variation of the COS method, one can recover the CDF from the characteristic function.

At last, the student compares the performance of the two methods, for a few trades and for one or two typical netting sets.

In the end, we conclude and summarize the situations in which the COS method can replace the MC method, as well as future research works.

**Contact**

If you are interested to enter the field of quantitative risk analysis, this is a very good starting point. Please feel free to contact me directly if this topic is of your interest, or if you would like to learn more details: fang.fang@ffquant.nl or f.fang@tudelft.nl

**About FF Quant Advisory B.V.**

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1. Part-time Assistant professor at the Applied Mathematics Department of TU Delft; Director of FF Quant Advisory B.V. https://fsquaredquant.nl/ [↑](#footnote-ref-2)