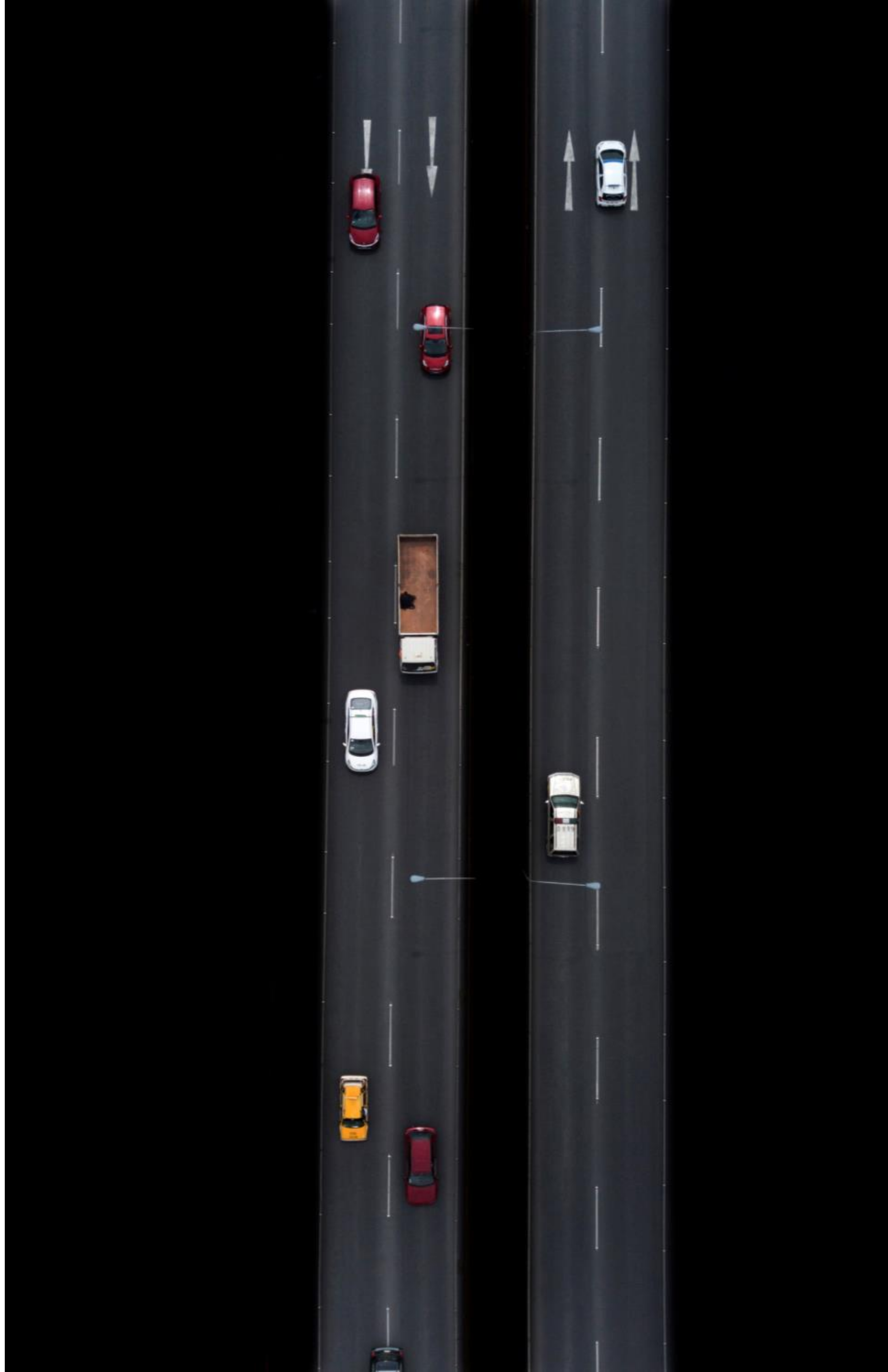
Estimating congestion and traffic patterns when planning road work

Literature study

Subjects

1. Theoretical background
    1. Different traffic models
    2. LWR model
    3. Fundamental relations
    4. The Godunov scheme
2. Combining ML and FR
3. Planning
4. Results

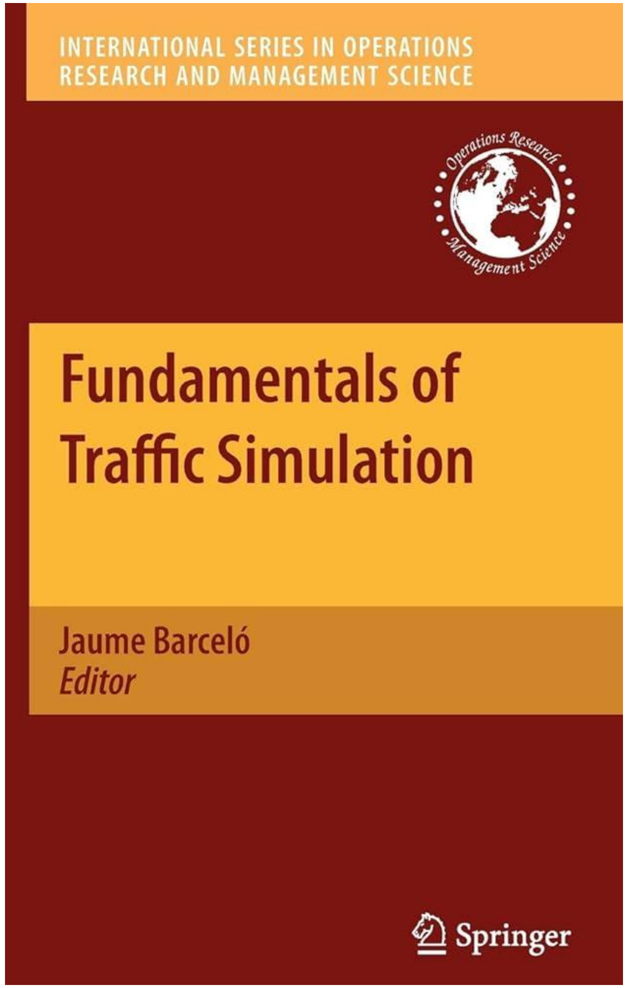


1. Theoretical background

- Internship at CGI (november 2022 – march 2023)
  - iAMLAB
  - SPIN
    - Systeem Planning en Informatie Nederland

How can mathematical models, specifically the combination of traffic models and machine learning algorithms, be used to improve estimates of the effect of road work on traffic?

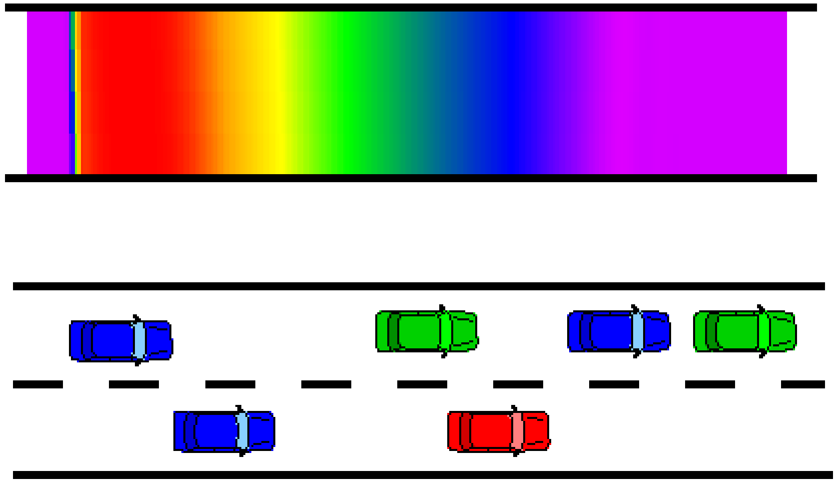
1.1 Different traffic models



1.1 Different traffic models

Macroscopic model

Microscopic model



Kesting, Arne et al. "Agents for Traffic Simulation." *Multi-Agent Systems* (2008).

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1.2 LWR model

M. J. Lighthill and G. B. Whitham, *On kinematic waves. ii. a theory of traffic flow on long crowded roads*, Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences, 229 (1955), pp. 317-345

P. I. Richards, *Shock waves on the highway*, Operations research, 4 (1956), pp. 42-51

1.2 LWR model

$$\frac{\partial q}{\partial t} + \frac{\partial \phi}{\partial x} = 0$$

Conservation law

1.2 LWR model

$$\frac{\partial q}{\partial t} + \frac{\partial \phi}{\partial x} = 0$$

Conservation law

$$\phi(q)$$

Fundamental relation

1.2 LWR model

$$\frac{\partial q}{\partial t} + \frac{\partial \phi}{\partial x} = 0$$

Conservation law

$$\phi(q)$$

Fundamental relation

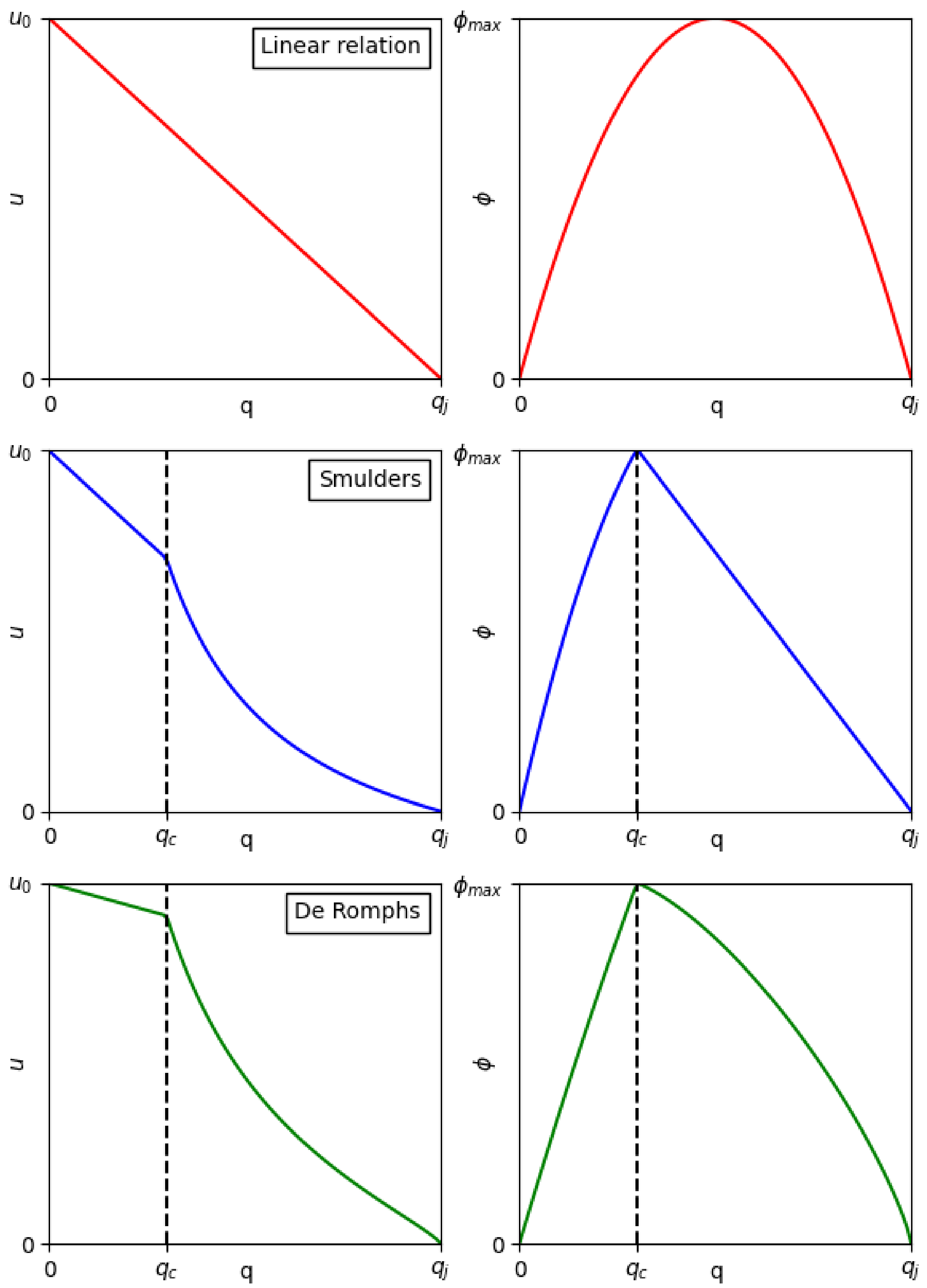
$$u(q) \qquad \phi = qu$$

1.3 Fundamental relations

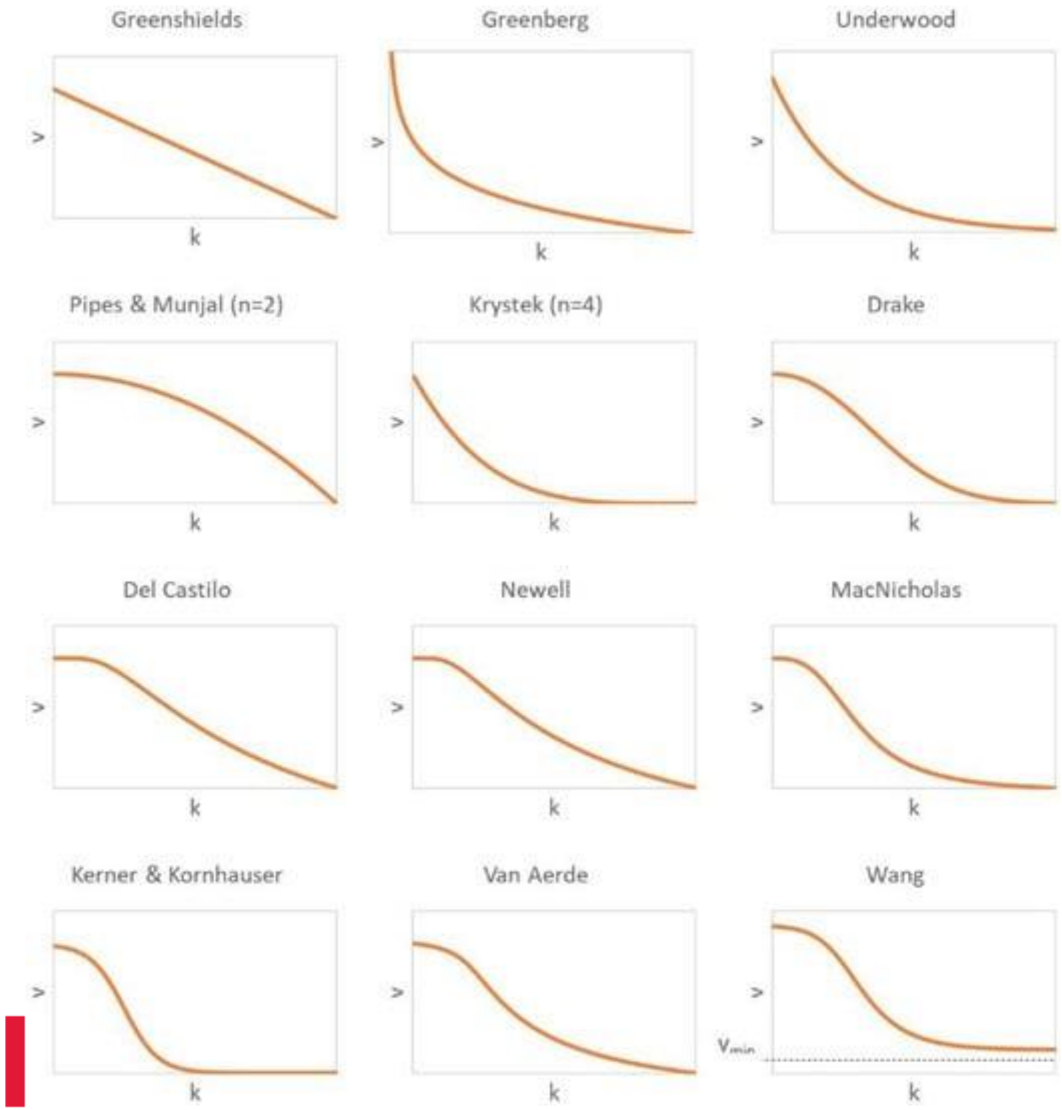
$$u_{\text{linear}}(q) = u_0\left(1 - \frac{q}{q_j}\right)$$

$$u_{\text{Sm}}(q) = \begin{cases} u_0\left(1 - \frac{q}{q_j}\right), & \text{for } q < q_c \\ \gamma\left(\frac{1}{q} - \frac{1}{q_j}\right), & \text{for } q > q_c \end{cases}$$

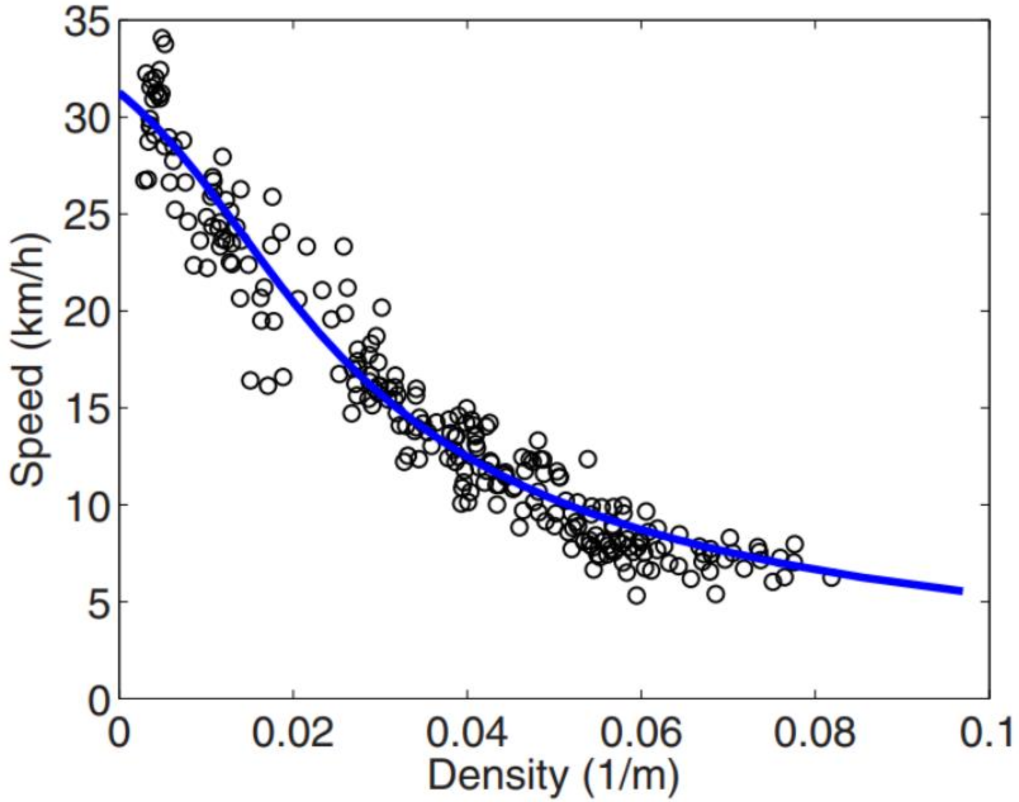
$$u_{\text{DR}}(q) = \begin{cases} u_0(1 - \alpha q), & \text{for } q < q_c \\ \gamma\left(\frac{1}{q} - \frac{1}{q_j}\right)^{\beta}, & \text{for } q > q_c \end{cases}$$



1.3 Fundamental relations



1.3 Fundamental relations



Dirk Helbing. “Derivation of a fundamental diagram for urban traffic flow”. In: *The European Physical Journal B* 70 (2009), pp. 229–241.

1.4 The Godunov scheme

1.4 The Godunov scheme

Definition 6 (Cauchy problem) *The problem*

$$q_t + \phi(q)_x = 0, \quad x \in \mathbb{R}, \quad t > 0,$$
$$q(x,0) = q_0(x), \quad x \in \mathbb{R},$$

for some function $\phi : \mathbb{R} \to \mathbb{R}$ is called a Cauchy problem [4]. In this context, ***Cauchy data*** *represents the initial conditions $q_0(x)$ from which a unique solution can be found.*

1.4 The Godunov scheme

Definition 6 (Cauchy problem) *The problem*

$$
\begin{aligned}
q_t + \phi(q)_x &= 0, \quad x \in \mathbb{R}, \quad t > 0, \\
q(x, 0) &= q_0(x), \quad x \in \mathbb{R},
\end{aligned}
$$

for some function $\phi : \mathbb{R} \to \mathbb{R}$ is called a Cauchy problem [4]. In this context, ***Cauchy data*** *represents the initial conditions $q_0(x)$ from which a unique solution can be found.*

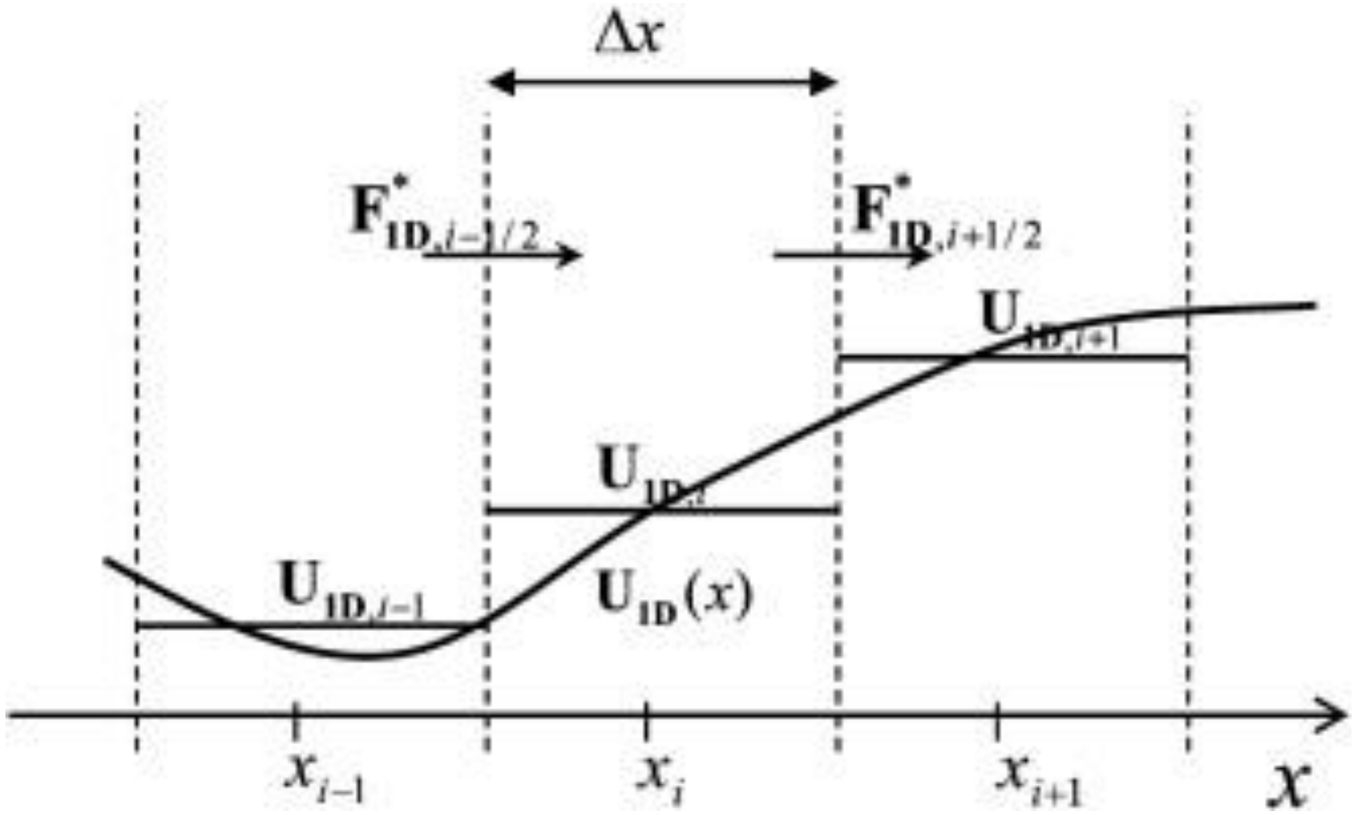
Definition 7 (Riemann problem) *A Cauchy problem with initial values*

$$
q_0(x) = \begin{cases} q_l & \text{for } x < 0 \\ q_r & \text{for } x \geq 0 \end{cases} \tag{2.11}
$$

where $q_l, q_r \in \mathbb{R}$ is called a Riemann problem. [8]

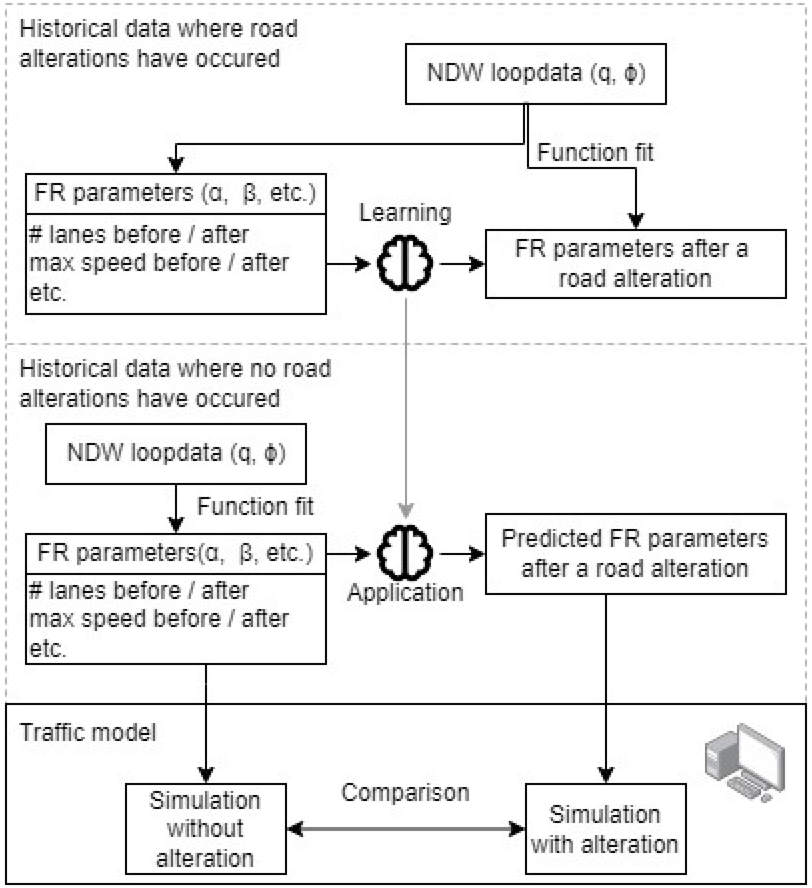
1.4 The Godunov scheme

- Finite volume method
- Conserved implicitly
- Shockwaves stay intact



https://www.sciencedirect.com/science/article/abs/pii/S0309170812000760

2. Combining ML and FR



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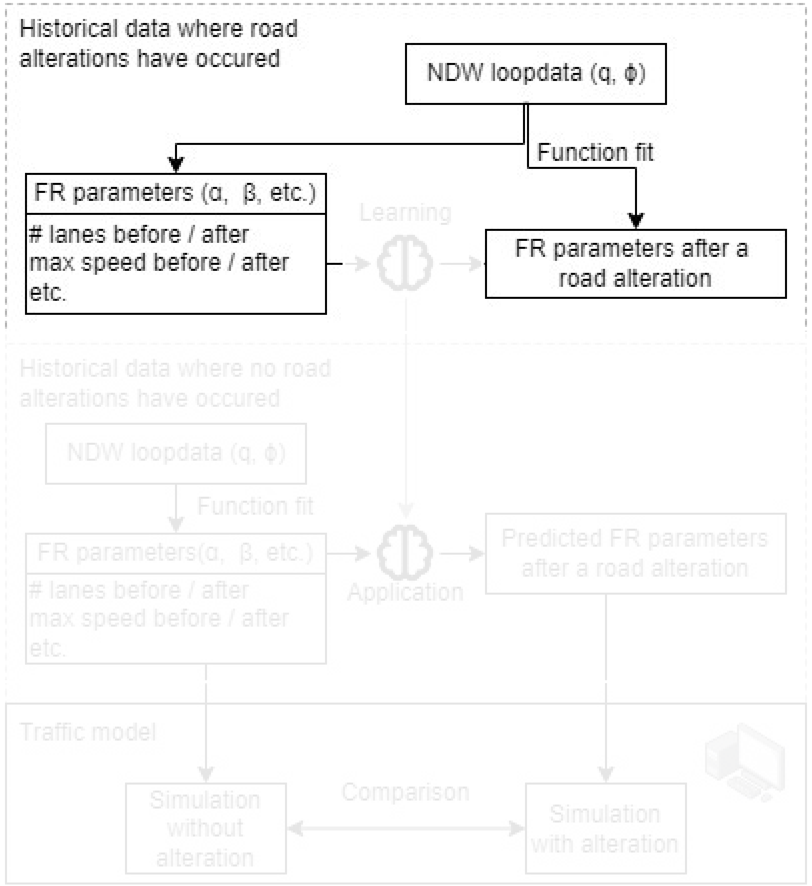

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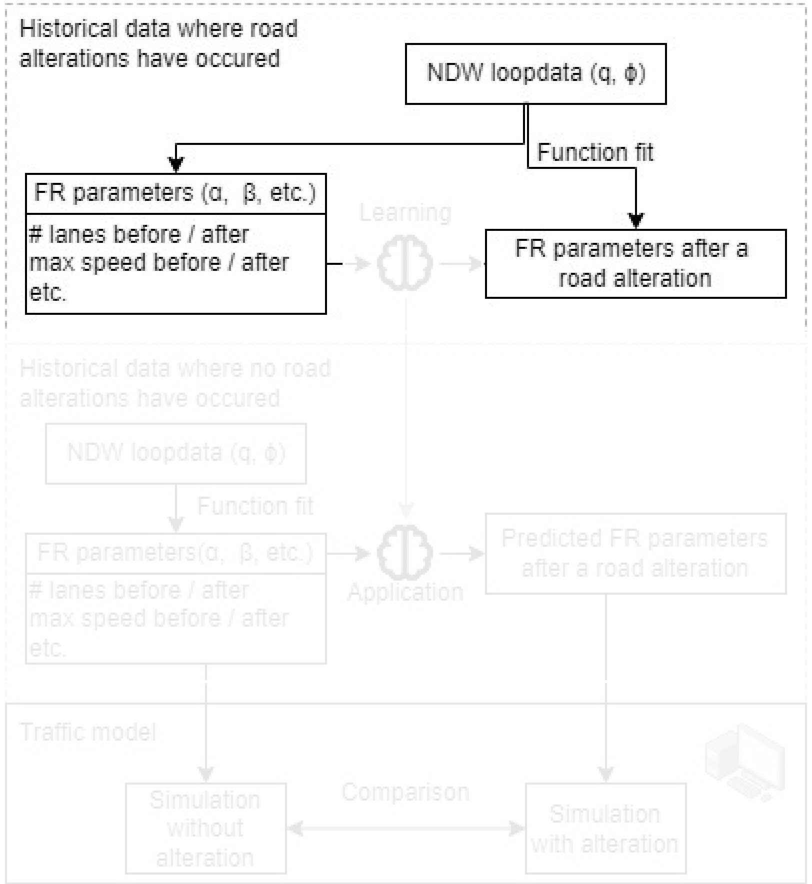

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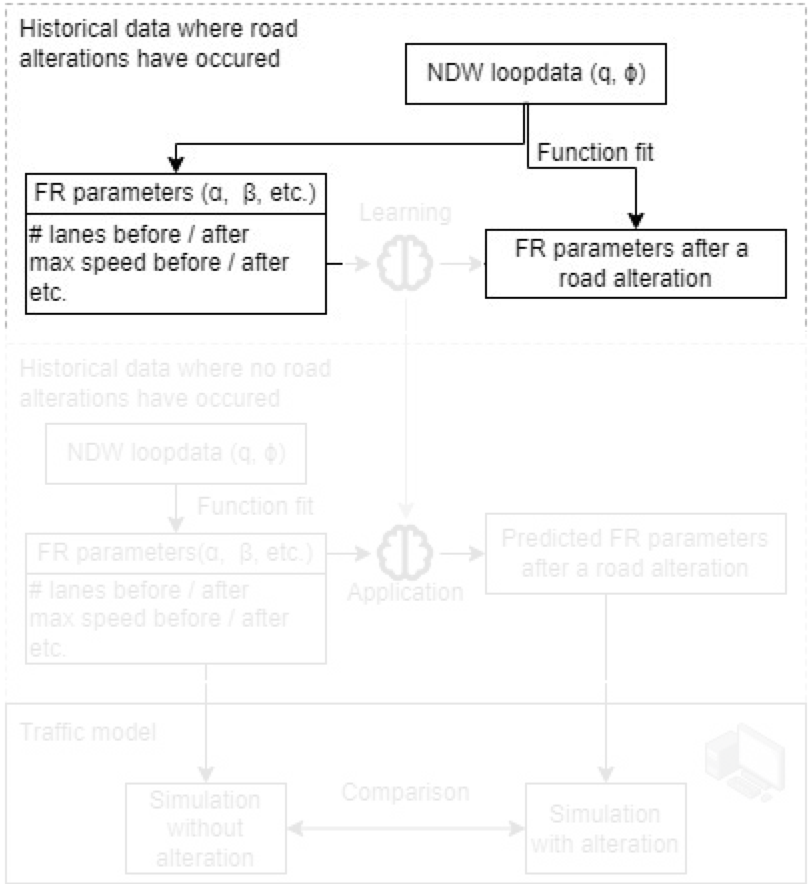
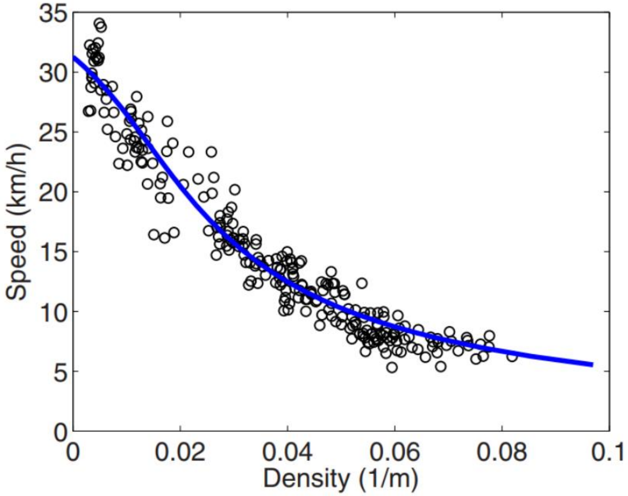
2. Combining ML and FR





Dirk Helbing. “Derivation of a fundamental diagram for urban traffic flow”. In: *The European Physical Journal B* 70 (2009), pp. 229–241.

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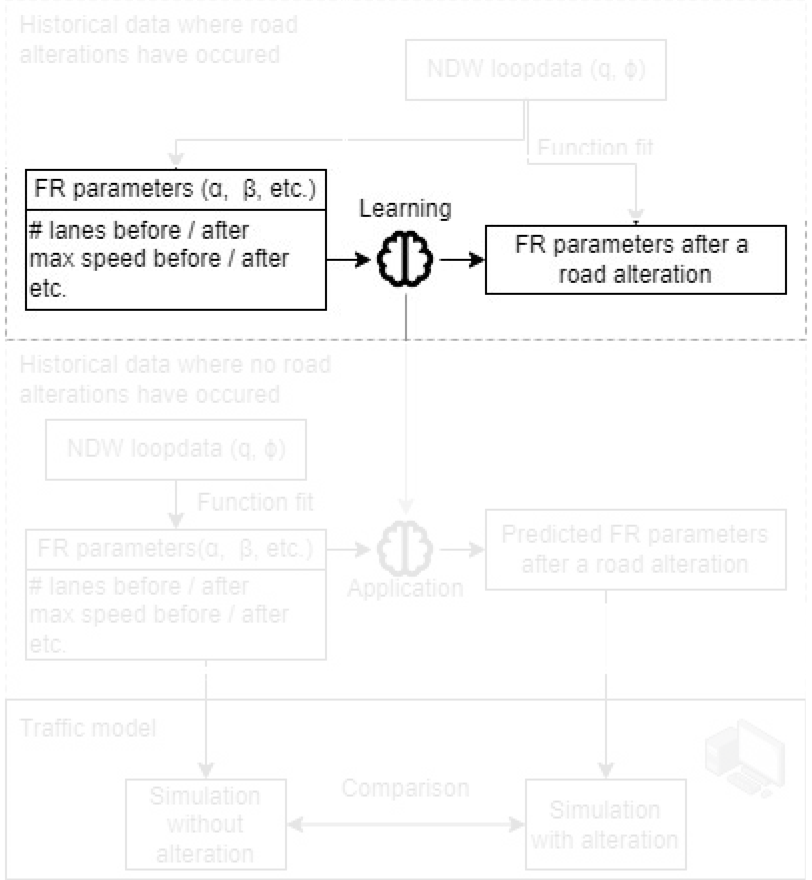

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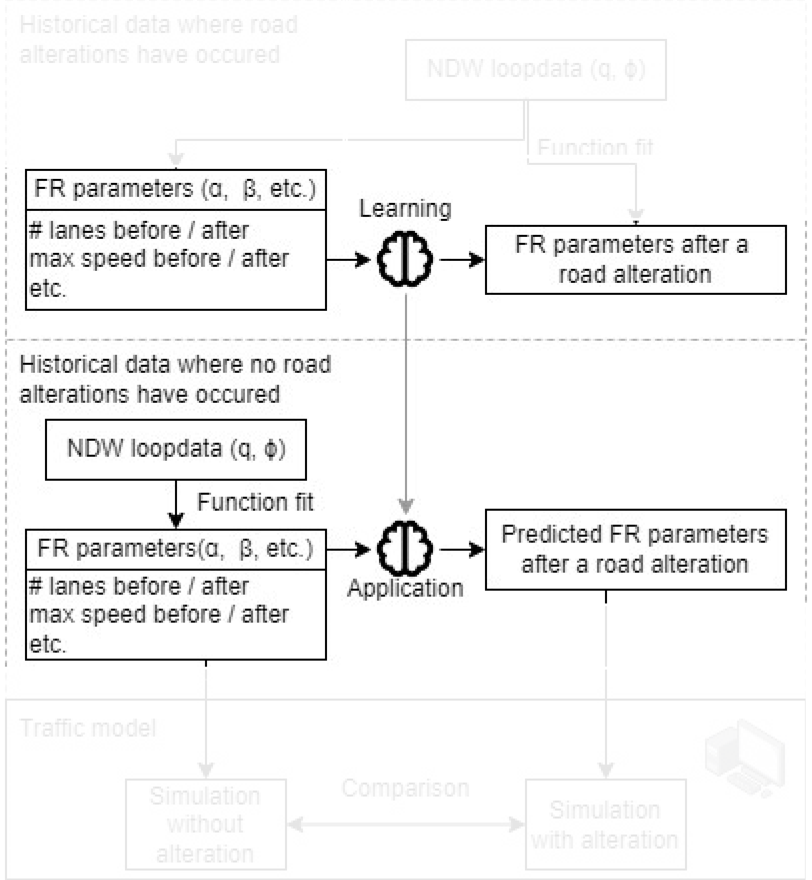

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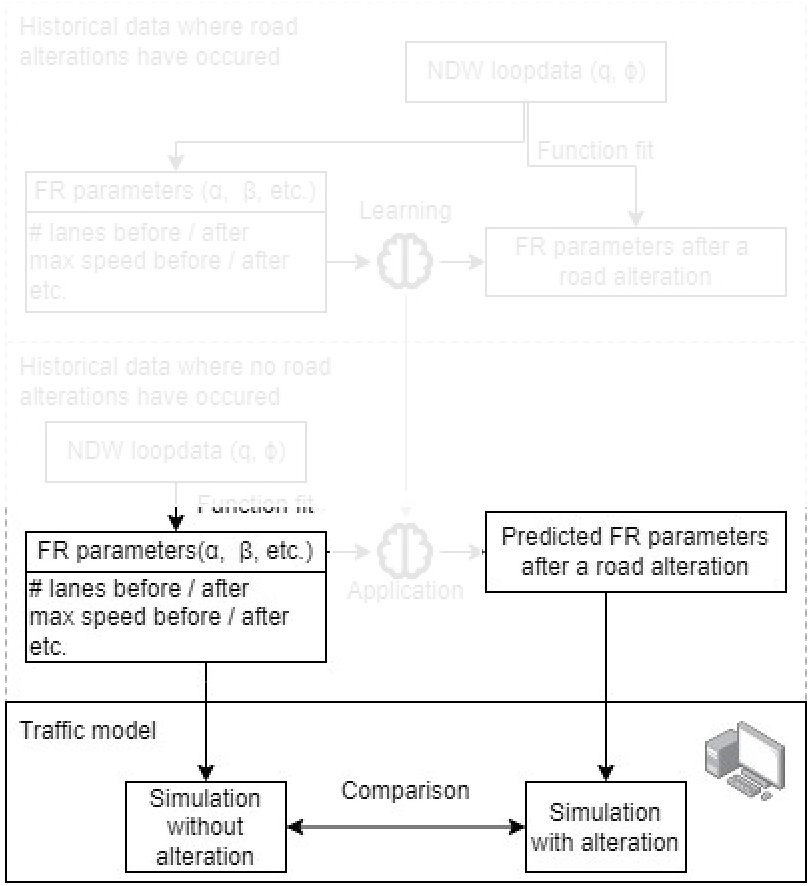

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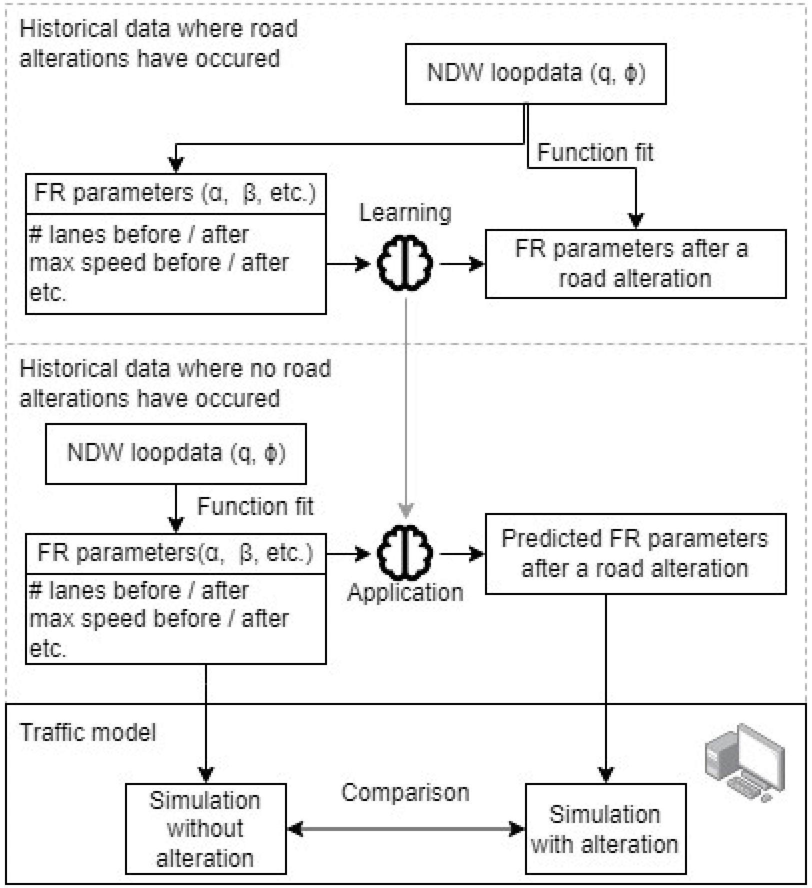

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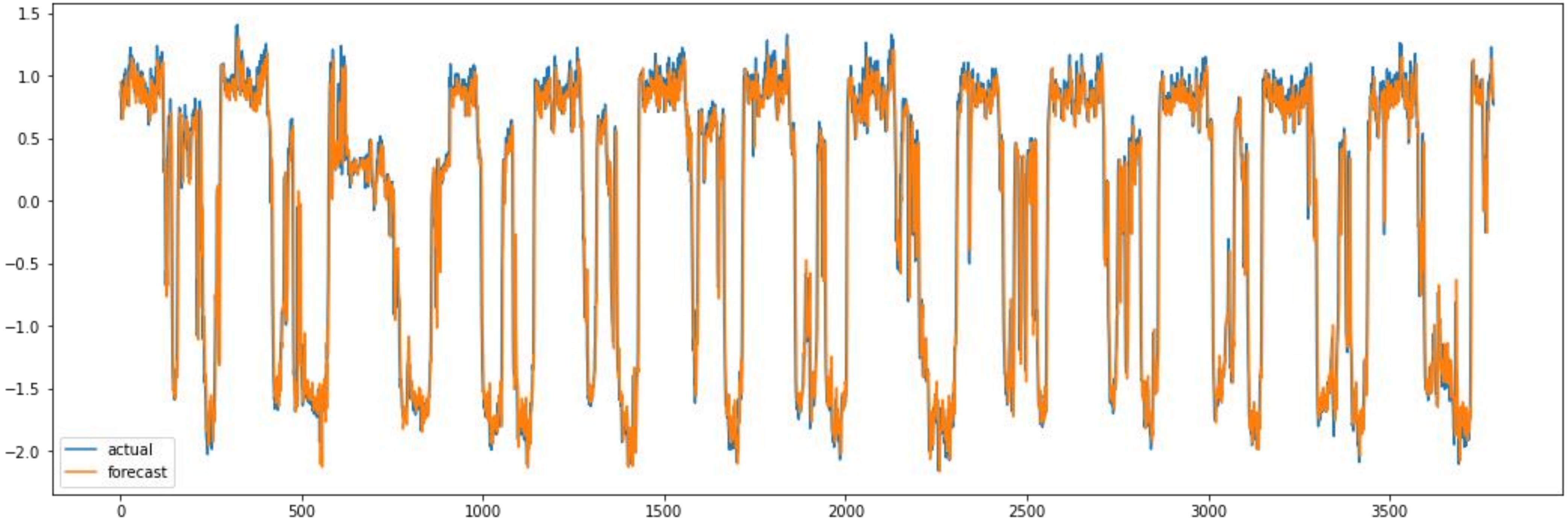

[Traffic forecasting using graph neural networks and LSTM (keras.io)]

2. Combining ML and FR

Table 2. ML – based traffic analysis summary

	Technique used	*Objectives*
Sommer and Paxson .[13]	Machine learning technique	ML methods have been applied to spam detection more effectively than intrusion detection because the detection of anomalies is best for finding different forms of known attacks.
Zhang et al.[24]	Artificial neural network using Voting Experts (VE) algorithm	Extract out the protocol features from feature words that are extracted by VE algorithm.
Wang et al [25]	Machine learning (SVM)	Classification the energy that is used in data flows.
Furno et al. [26]	Machine learning using Exploratory factor analysis (EFA)	Spatial structure analysis and bridge the temporal to mobile traffic data by using EFA technique
Mirsky et al.[27]	Artificial neural network using Kitsune	Detection of malicious traffic entering and leaving the network
Suthaharan et al.[28]	Machine learning Using supervised learning technique	Classification of network intrusion traffic by learning the network characteristics
Blowers et al.[29]	Machine learning use a DBSCAN clustering	Anomaly detection based on clustering
Laskov et al[21].	Machine learning (SVM),(kNN) γ-algorithm, k-means	Compare both supervised and unsupervised learning for detecting malicious activities
Mukkamala et al[9]	Machine learning (SVM)	To discover patterns or features that describe user behaviours to build classifiers for recognizing anomalies
Zamani et al [16]	artificial immune algorithm	Intrusion detection in distributed systems.
Bujlow et al.[30]	Machine learning using C5.0 algorithm	Classification of traffic in network
Amuna and Vinoth [31]	Machine learning using Decision Tree and Naïve Bayes ML algorithms	Classification of traffic in network
Bartos et al. [32]	Machine learning Using supervised learning technique	Detect both known and previously unseen security threats

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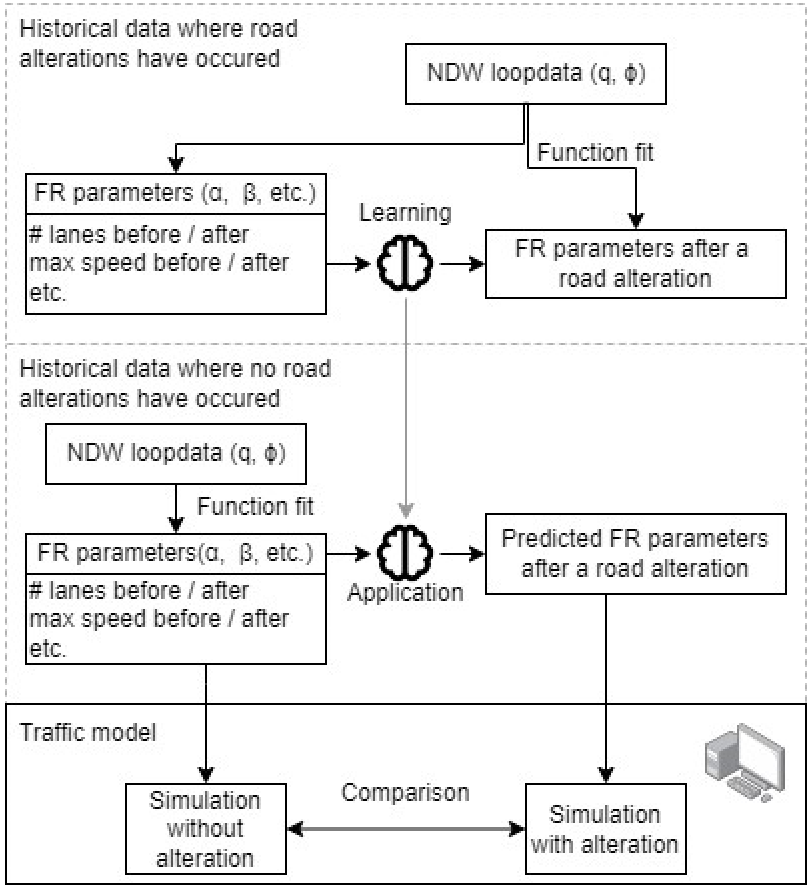
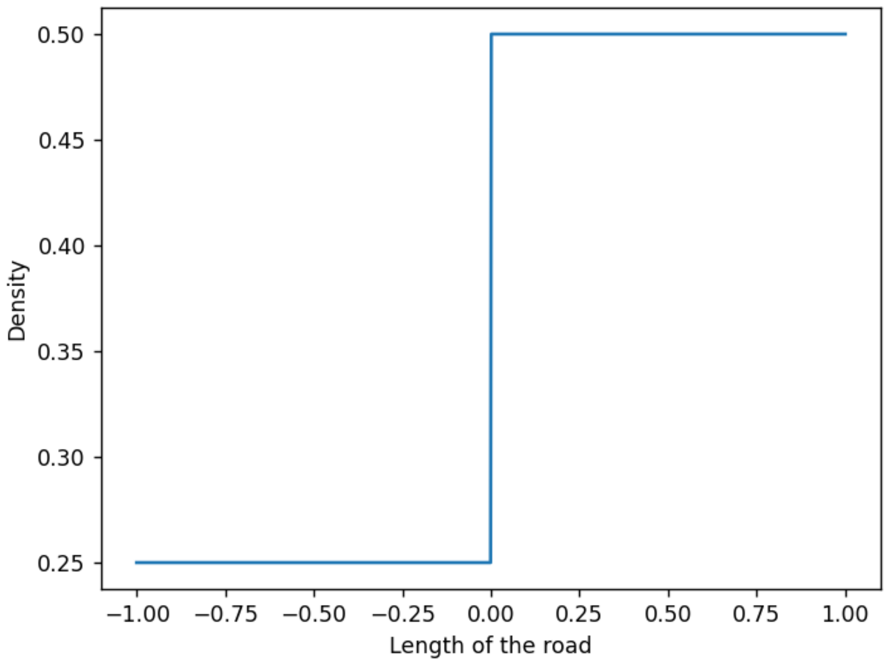
Alqudah, Nour, and Qussai Yaseen. "Machine learning for traffic analysis: a review." *Procedia Computer Science* 170 (2020): 911-916.

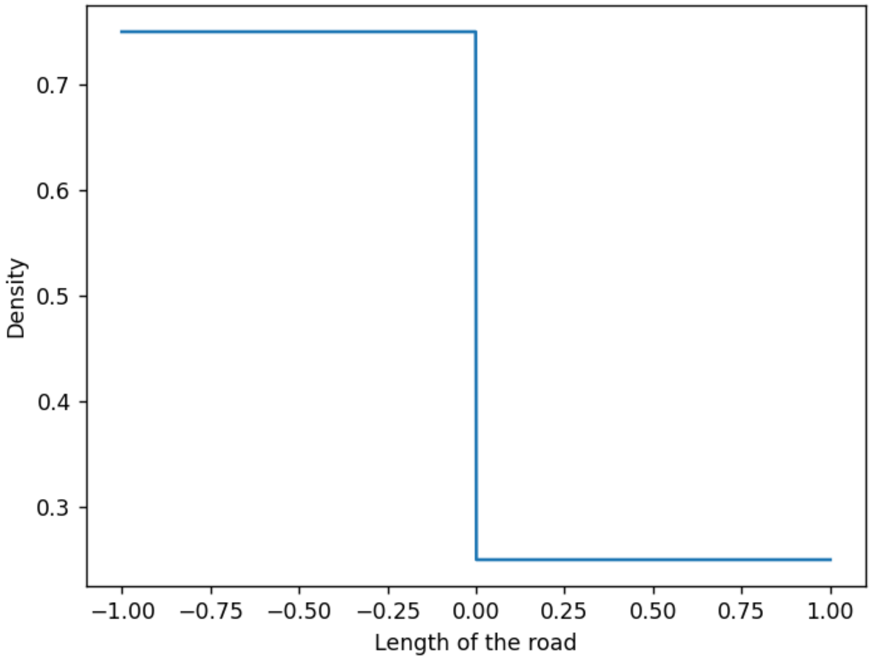
3. Planning


Historical data where road alterations have occured
NDW loopdata (q, ϕ)
Function fit
FR parameters (α, β, etc.)
lanes before / after
max speed before / after
etc.
Learning
FR parameters after a road alteration
Historical data where no road alterations have occured
NDW loopdata (q, ϕ)
Function fit
FR parameters(α, β, etc.)
lanes before / after
max speed before / after
etc.
Application
Predicted FR parameters after a road alteration
Traffic model
Simulation without alteration
Comparison
Simulation with alteration

4. Results

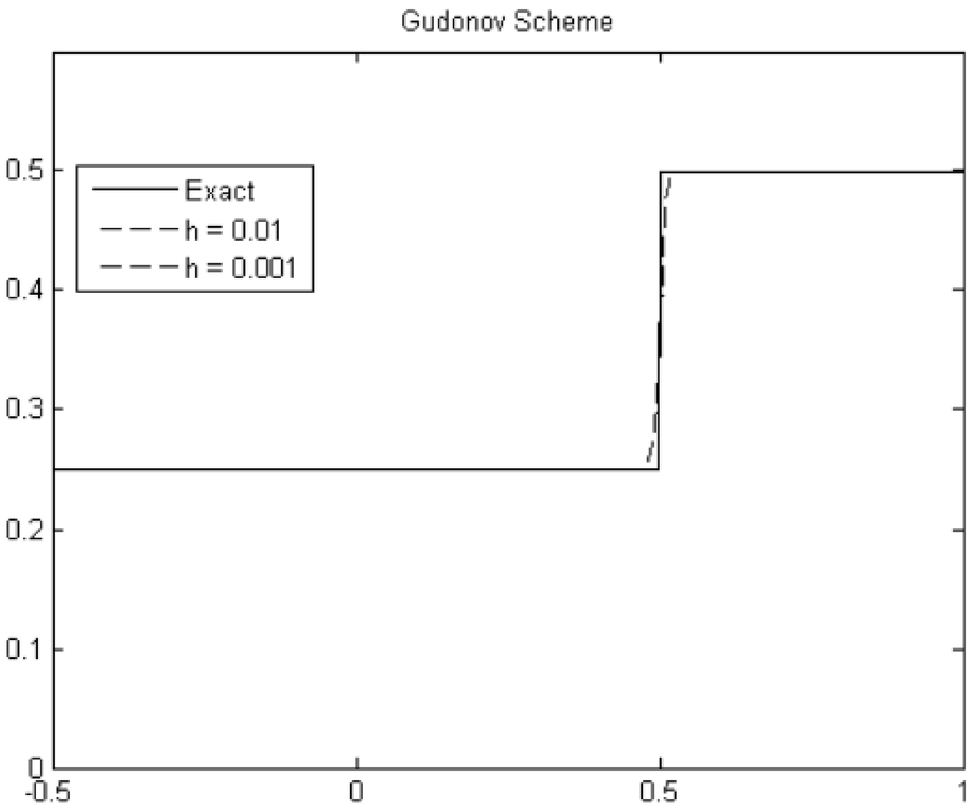


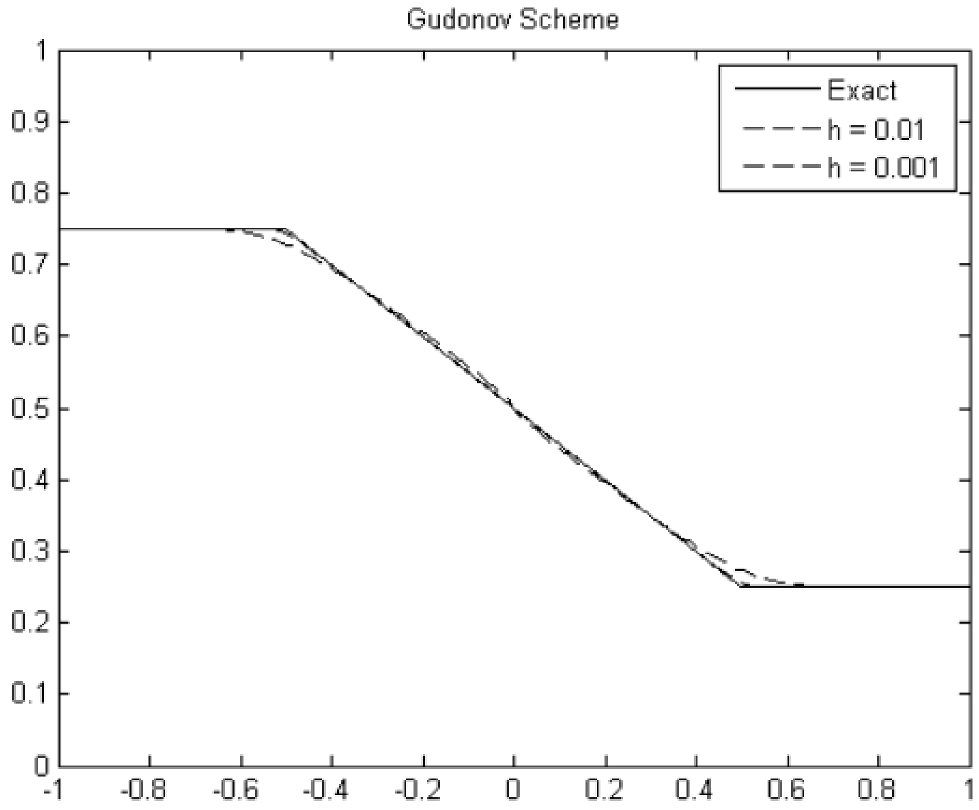
0.50
0.45
0.40
0.35
0.30
0.25
Density
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00
Length of the road



0.7
0.6
0.5
0.4
0.3
Density
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00
Length of the road

4. Results

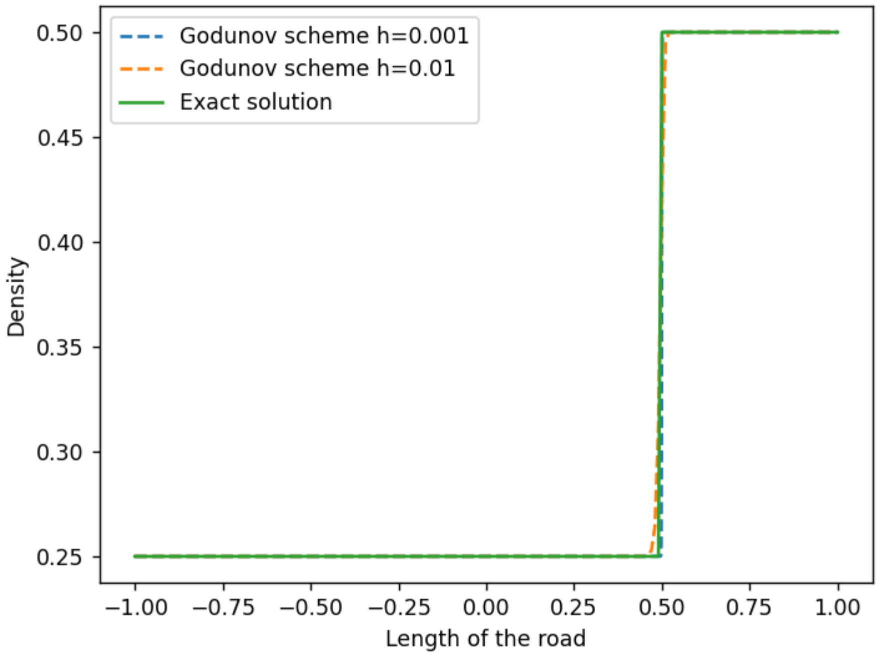


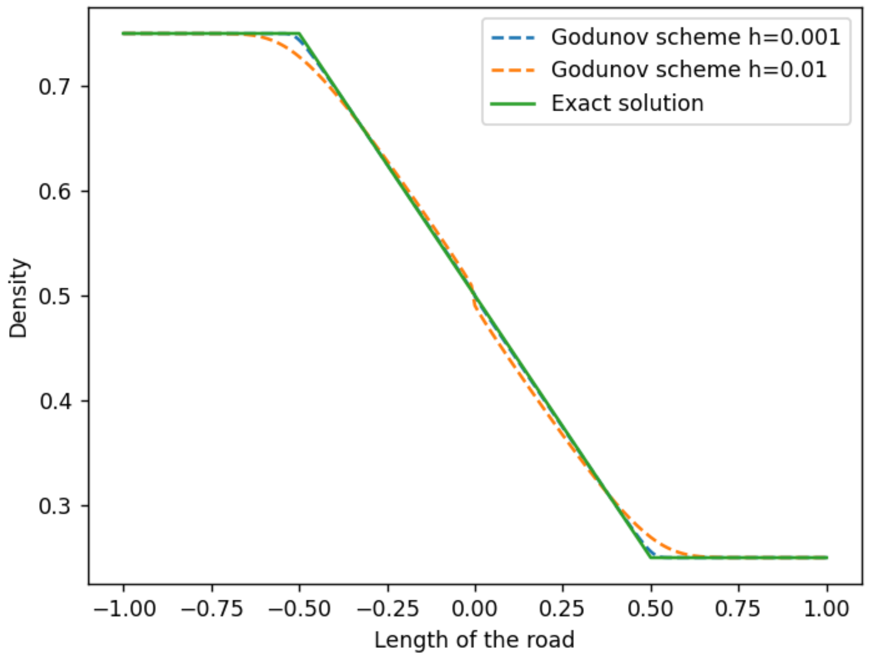


Godunov Scheme for a shock solution at T = 2 (left) and Rarefraction at T = 1 (right). k = 0.001.

[Numerical Solutions of Traffic Flow on Networks (ntnu.no)]

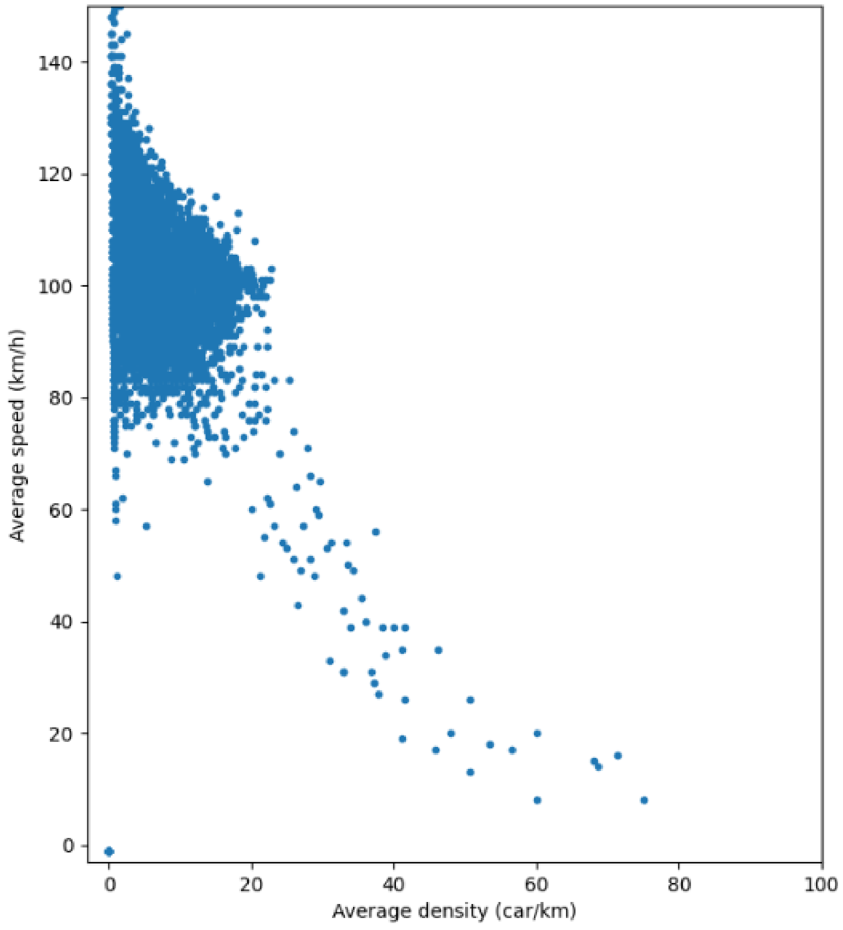
4. Results



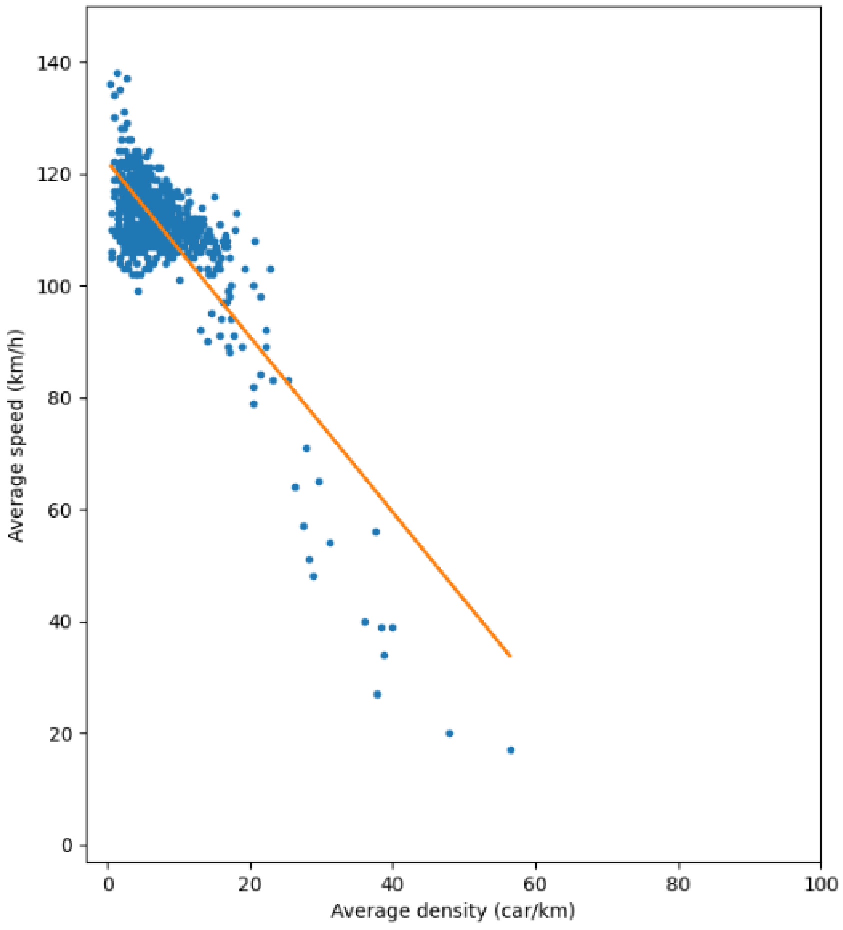


Godunov Scheme for a shock solution at T = 2 (left) and Rarefraction at T = 1 (right). k = 0.001.

4. Results



Average speed (km/h)
0
20
40
60
80
100
120
140
0
20
40
60
80
100
Average density (car/km)



Average speed (km/h)
0
20
40
60
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120
140
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80
100
Average density (car/km)

Estimating congestion and traffic patterns when planning road work

Literature study

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Subquestions

1. What is a **measure of performance** of different traffic models and ML algorithms in terms of their ability to accurately estimate the effects of road work on traffic?

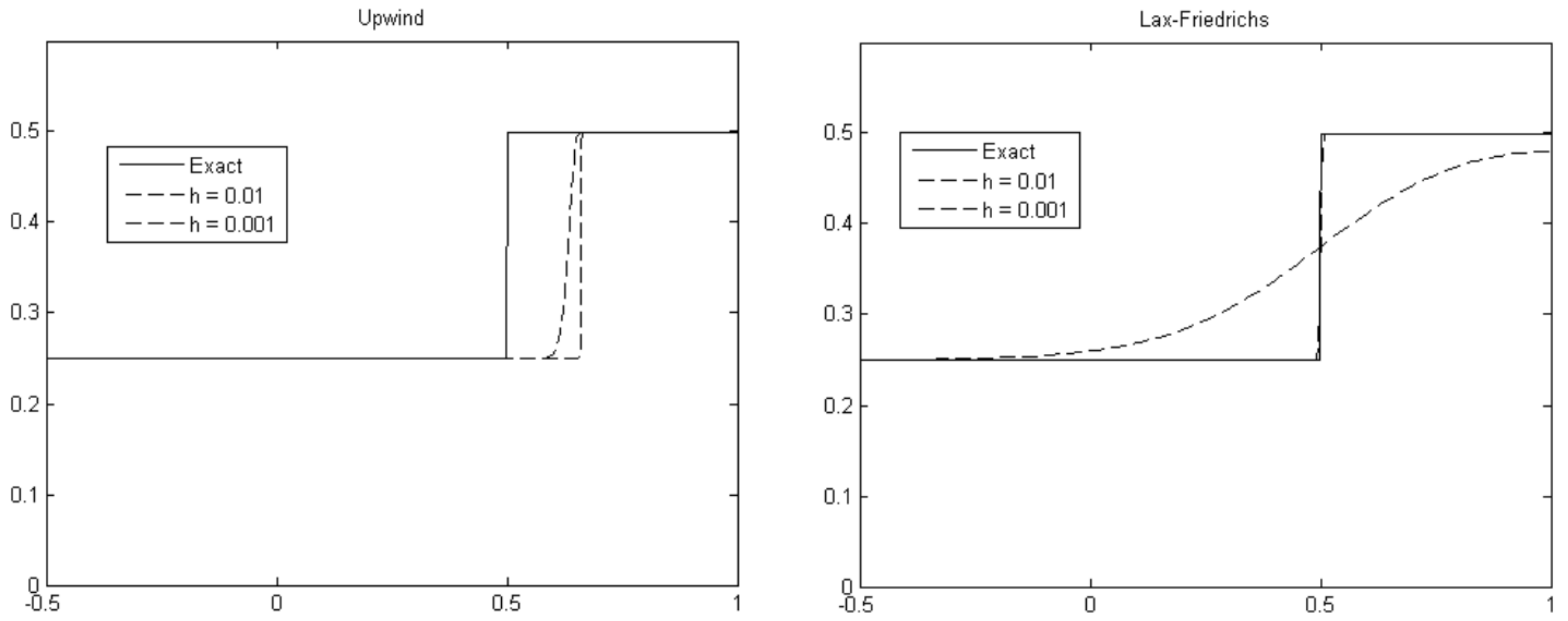
2. How can **macroscopic traffic** models be used as a **framework** to estimate the effect of road work on traffic using ML?

3. How can we use **ML algorithms** to identify and predict the **impact** of road work based on historical data?

4. How can the **insights** gained from the traffic model be used to **improve the efficiency** of road work planning processes?

FEM around shock waves

Figure 6.2.1: Exact and numerical solutions for the inviscid Burger equation using the upwind scheme(6.2.2)(left) and the Lax-Friedrichs scheme(6.2.3)(right). k = 0.001.



https://ntnuopen.ntnu.no/ntnu-xmlui/bitstream/handle/11250/259317/730608_FULLTEXT01.pdf?sequence=3&isAllowed=y

FEM around shock waves

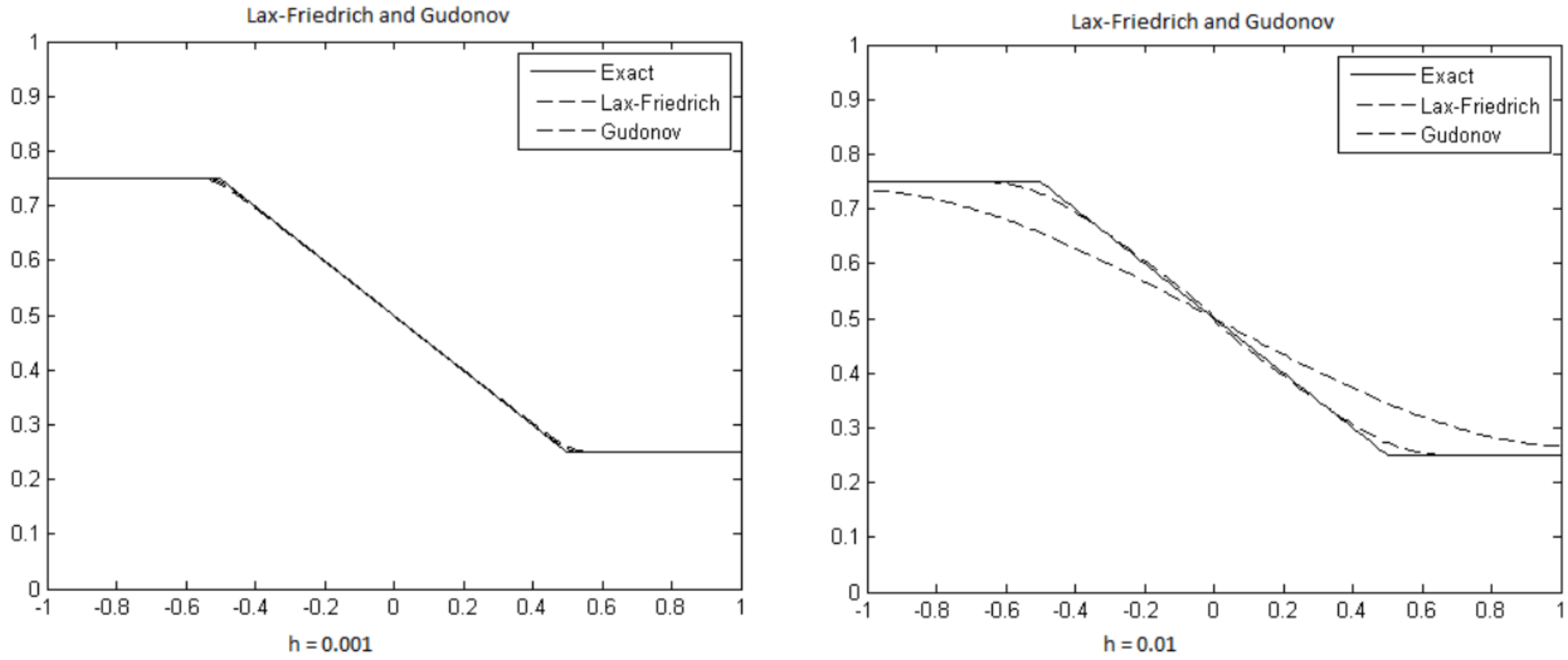


Figure 7.1.2: Godunov and Lax-friedrich for a Rarefraction solution $h = 0.001$(left) and h = 0.01(right). $k = 0.001$ and $T = 1$.

https://ntnuopen.ntnu.no/ntnu-xmlui/bitstream/handle/11250/259317/730608_FULLTEXT01.pdf?sequence=3&isAllowed=y

Pseudocode for the Godunov scheme

1. $q_l = q_r$ gives the constant solution $q(x,t) = q_0(x)$ and $\phi(q(0)) = 0$.

2. $q_l < q_r$ means that there is a higher density of traffic on the right than on the left. This higher density on the right leads to lower speeds. As traffic moves from left to right, it follows that the shockwave will stay a discontinuity. It is concluded that the solution has the form:

$$q(x,t) = \begin{cases} q_l & \text{for } x < st \\ q_r & \text{for } x \geq st \end{cases} \tag{2.12}$$

where the shock speed s is found to be

$$s = \frac{\phi(q_l) - \phi(q_r)}{q_l - q_r}. \tag{2.13}$$

This choice for s is called the Rankine-Hugoniot condition [4]. This means the value of $\phi(q(0))$ depends on the value of s; if $s > 0$, then the shock moves to the right and $q(0) = q_l$, while $s < 0$ yields $q(0) = q_r$. $s = 0$ is impossible in this situation, as that implies $\phi(q_l) = \phi(q_r)$ which is only possible if $q_l = q_r$.

3. $q_l > q_r$ has multiple weak solutions, but only one physically meaningful solution; the rarefraction wave. This means the shock will not stay a discontinuity, but it will spread out. This type of rarefraction wave is the correct solution in this situation as it satisfies the entropy condition as defined in [4] and [8]. Mathematically, this looks like this:

$$q(x,t) = \begin{cases} q_l & \text{for } x < \phi'(q_l)t \\ (\phi')^{-1}(\frac{x}{t}) & \text{for } \phi'(q_l)t \leq x \leq \phi'(q_r)t \\ q_r & \text{for } x > \phi'(q_r)t \end{cases} \tag{2.14}$$

For the Godunov method, we will need $\phi(q(0))$. This can be found from this equation:

$$q(0) = \begin{cases} q_l & \text{if } \phi'(q_l) > 0 \\ (\phi')^{-1}(0) & \text{if } \phi'(q_l) \leq 0 \leq \phi'(q_r) < 0 \\ q_r & \text{if } \phi'(q_r) < 0 \end{cases} \tag{2.15}$$

In the case of traffic models, $\phi(q)$ is a concave function and $(\phi')^{-1}(0)$ is the unique solution to $\phi'(q) = 0$ which represents the point of maximum flux. [4, 8]

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Pseudocode for the Godunov scheme

Data: Some initial $q_0(x)$, a fundamental relation $\phi(q)$ with maximum flux $\phi(q_{max})$, a domain x and boundary values $Q_{\text{boundary point}}(t)$.
Result: An approximation of the traffic flow over time.

```
begin
  Discretize t as t_n size k, and x as x_i size h
  Discretize q_0(x) as q_i^0 = (1/h) ∫_{x-h/2}^{x+h/2} q_0(x) dx
  for n in timerange do
    for all i do
      We will find q_i^* at the interface between x_i and x_{i+1}
      if φ'(q_i^n) ≥ 0 and φ'(q_{i+1}^n) ≥ 0 then  q_i^* ⟵ q_i^n ;
      if φ'(q_i^n) < 0 and φ'(q_{i+1}^n) < 0 then  q_i^* ⟵ q_{i+1}^n ;
      if φ'(q_i^n) ≥ 0 and φ'(q_{i+1}^n) < 0 then
        s ⟵ (φ(q_i^n) − φ(q_{i+1}^n)) / (q_i^n − q_{i+1}^n)
        if s ≥ 0 then q_i^* ⟵ q_i^n;
        if s < 0 then q_i^* ⟵ q_{i+1}^n;
      if φ'(q_i^n) < 0 and φ'(q_{i+1}^n) ≥ 0 then  q_i^* ⟵ q_max ;
    for all interior i do
      q_i^{n+1} = q_i^n − (k/h)(φ(q_i^*) − φ(q_{i-1}^*))
    for all boundary points i do
      q_i^{n+1} = Q_i(t_{n+1})
```

Discretization of the initial condition:

$$q_i^0 = \frac{1}{h}\int_{x-\frac{h}{2}}^{x+\frac{h}{2}} q_0(x)\,dx$$

Shock speed:

$$s \longleftarrow \frac{\phi(q_i^n)-\phi(q_{i+1}^n)}{q_i^n-q_{i+1}^n}$$

Update for interior points:

$$q_i^{n+1} = q_i^n - \frac{k}{h}\left(\phi(q_i^*) - \phi(q_{i-1}^*)\right)$$

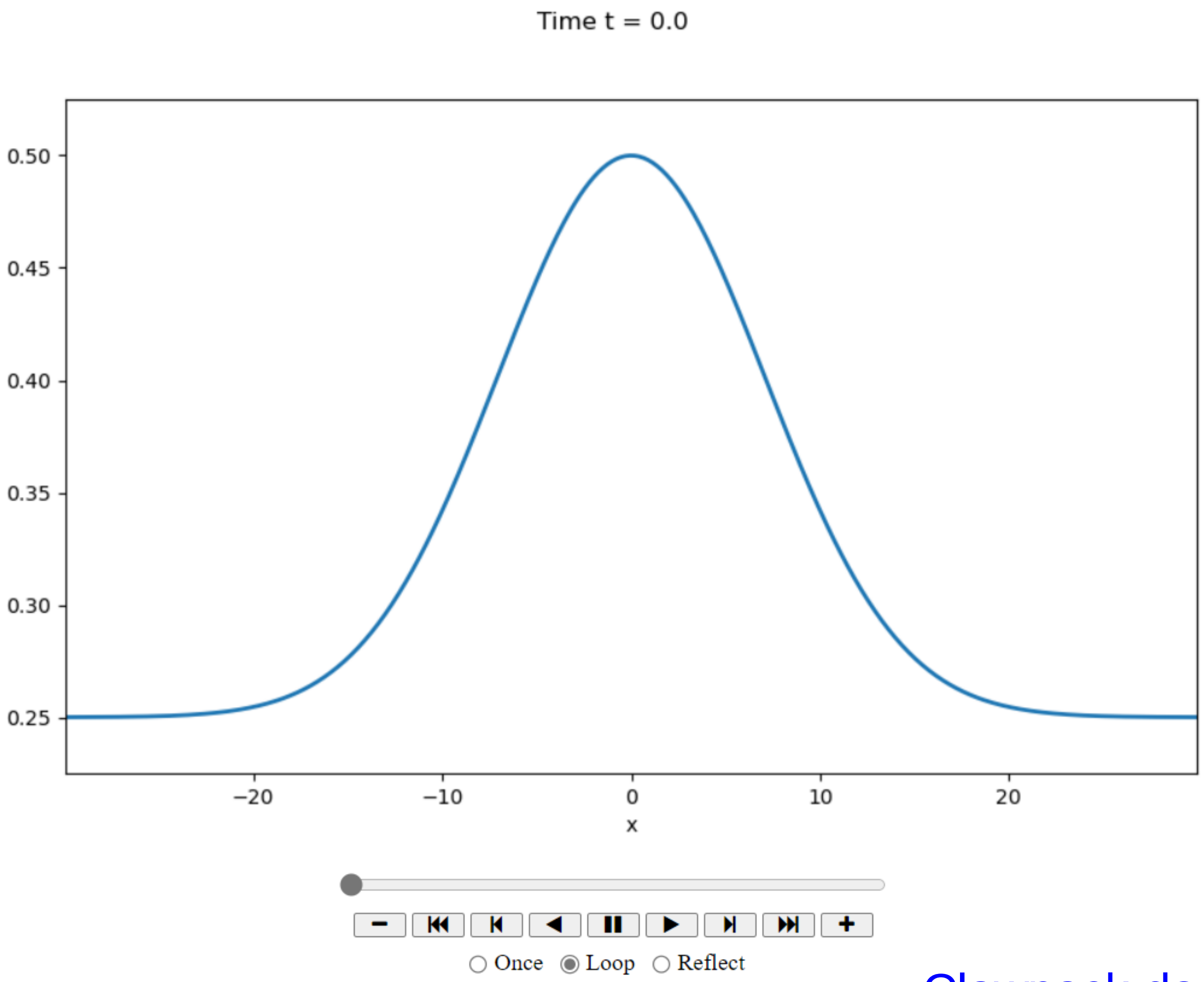
Update for boundary points:

$$q_i^{n+1} = Q_i(t_{n+1})$$

2.1 NDW data

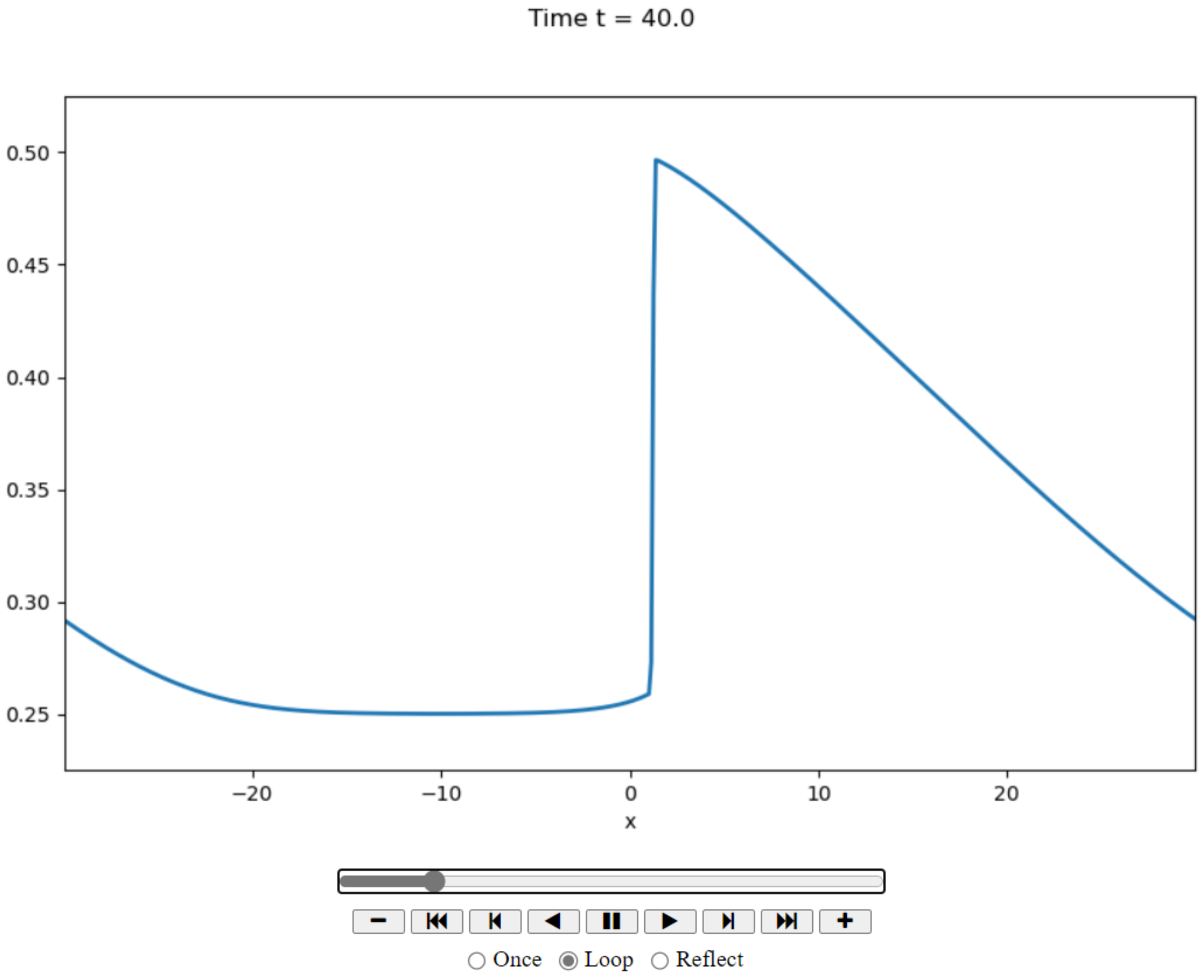
- Loopdata
- Incidents instead of (planned) road work

1.4 The Godunov scheme



[Clawpack documentation — Clawpack 5.9.x documentation]

1.4 The Godunov scheme



1.4 The Godunov scheme