MSc-project proposal
Solving Poisson's equation using dataflow computing

MAXELER Technologies
MAXIMUM PERFORMANCE COMPUTING

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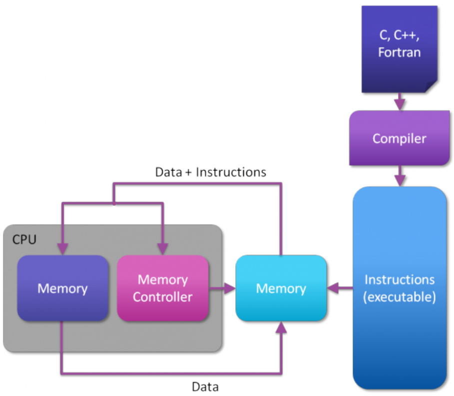
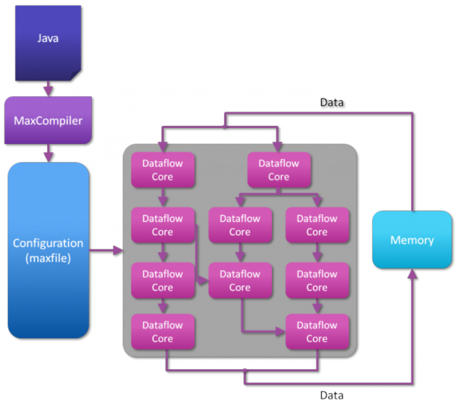
Introduction

Many practical engineering problems require the accurate and computationally efficient solution of partial differential equations (PDEs) by means of numerical discretization and solution methods. A very recent discretization approach is Isogeometric Analysis (IgA) [Hu05], which is an extension of the Finite Element Method (FEM). In essence, IgA adopts the same mathematical tools, namely, high-order B-Spline basis functions, to parameterize the problem geometry and to approximate the numerical solution. Since B-Splines and NURBS (non-uniform rational B-Splines) are the de-factor standard for describing complex geometries in Computer-Aided Design (CAD) programs, IgA is particularly useful for solving PDE-problems coming from industrial applications.

In practice, the numerical computation of approximate solutions is very time consuming due to the large number of degrees of freedom (dofs) that are necessary to achieve sufficient accuracy. It is nowadays common practice in high-performance computing is to use processors with multiple compute cores (multicore CPUs) and accelerate the most expensive parts of the simulation by using accelerator cards (e.g. programmable Graphics Processing Units (GPUs)). All these compute devices are based on the ***control-flow computing*** paradigm. As depicted in the figure on the right, the program source is transformed into a list of instructions, which is then loaded into the memory attached to the processor. Data and instructions are read from memory into the processor code, where operations are performed and the results are written back to memory. This model is inherently sequential and its performance is mainly limited by the latency of data movement in this loop.



In ***dataflow computing***, the program source is transformed into concrete representation of the numerical algorithm 'in hardware'. That is, a configuration file is created that describes the operations, layout and inter-connections of computational units. This configuration file is then used to realise the algorithm on a field-programmable gate array (FPGA). In contrast to the traditional control-flow computing approach, in dataflow computing the data is loaded once from memory and streamed through the entire chain of



operations, thereby passing intermediate results from one computational unit to the next without writing them back to off-chip memory.

This makes dataflow computing very attractive for large-scale numerical simulations, where the main bottleneck in todays computing devices are not the raw computational costs but the transfer of data between computational units and off-chip memory.

Problem description

The aim of this project is to develop an efficient solution algorithm for Poisson's equation in two and, possibly, three space dimensions on Maxeler's dataflow engines [Max]. The isogeometric analysis approach will be employed for the discretization of this elliptic PDE-problem. Bi-quadratic and -cubic B-Spline basis functions will be adopted on a multi-dimensional yet simple domain (single patch geometry). The resulting linear system of equations is very large but sparse with a regular sparsity pattern (sparse-banded matrix) if the unknowns are numbered in line-wise manner. It will be solved by the Conjugate Gradient solution methods, implemented in a matrix-free version.

Challenges

The challenges of this project are formulated in the following research questions:

- ***How to realize an efficient evaluation of matrix entries stemming from an IgA discretization within a matrix-free Conjugate Gradient solution method?*** Quadrature-based element-by-element matrix formation as in classical FEM is known to be prohibitively expensive for higher-order B-Spline basis functions. Alternative matrix assembly techniques such as sum factorization [An15], weighted quadrature rules [Ca16], low-rank tensor approximation [Ma14] and combinations thereof have been developed recently. However, their use within matrix-free iterative solution algorithms has not been investigated so far and shall be pursued in this project.
- ***How to implement the solution algorithm efficiently on dataflow engines?*** Isogeometric Analysis makes use of multi-variate B-Spline basis functions, which result from tensor-product construction of uni-variate functions. This fact shall be exploited by maximizing the reuse of partially computed data, e.g., by making use of sum-factorization strategies [An15]. Moreover, the influence of different floating-point precisions adopted in the different steps of the algorithm on the overall accuracy shall be investigated. A prototypical implementation of the matrix-free Conjugate Gradient solution method on dataflow engines for unstructured FEM is presented in [Bu15] and can be used as starting point for code development in this project.

Time schedule

The following tasks are foreseen:

- Familiarization with Maxeler's data flow computing technology and the development tools (MaxJ programming language, MaxGenFD library)
- Literature study on isogeometric analysis with special focus on efficient matrix assembly techniques with application to matrix-free iterative solution algorithms
- Development of a matrix-free Conjugate Gradient solver for IgA-based discretizations of Poisson's equation using B-Spline basis functions (reference CPU code)
- Assessment of Maxeler's MaxGenFD library as computational framework to implement B-Spline based IgA discretizations in Maxeler's dataflow engines.

- Development of a matrix-free Conjugate Gradient solver for IgA-based discretizations of Poisson's equation using B-Spline basis functions on Maxeler's dataflow engines and analysis of its performance in terms of total computing times and memory consumption (per degree of freedom) with respect to the baseline CPU code
- Thesis writing

Further information

For further information please contact Matthias Möller (m.moller@tudelft.nl) or Georgi Gaydadjiev (g.n.gaydadjiev@tudelft.nl). Access to a dedicated Maxeler server with four MAX3 dataflow engine cards will be granted. Moreover, Maxeler Technologies Ltd. offers the possibility for a short stay at its headquarter in London, UK for hands-on training. Technical support from Maxeler experts will be available during the entire project.

Literature

[An15] P. Antolin, A. Buffa, F. Calabro, M. Martinelli, and G. Sangalli, Efficient matrix computation for tensor- product isogeometric analysis: The use of sum factorization. Comput. Methods Appl. Mech. Engrg. 285 (2015) 817-828.

[Bu15] P. Burovskiy, P. Grigoras, S. Sherwin, and W. Luk, Efficient assembly for high order unstructured FEM meshes. In: Proceedings of the 25th Int. Conf. on Field Programmable Logic and Applications (2015) 1-6.

[Ca16] F. Calabro, G. Sangalli, and M. Tani, Fast formation of isogeometric Galerkin matrices by weighted quadrature. ArXiv 1605.01238, 2016.

[Hu05] T.J.R.Hughes, J.A.Cottrell, and Y.Bazilevs: Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement. Comput. Methods Appl. Mech. Engrg. 194 (2005) 4135-4195.

[Ma14] A. Mantzaflaris, B. Jüttler, B. Khoromskij, and U. Langer, Matrix generation in isogeometric analysis by low rank tensor approximation. 2014. hal-01084421.

[Max] Maxeler Technologies Inc. http://maxeler.com