Msc thesis proposal:

Pricing Energy Quanto Options using Dimension-reduced Cosine Expansion

Griselda Deelstra[1] and Fang Fang[2]

2024-09-26

Background

Energy quanto options are risk management tools that have a payoff similar to the product of the payoffs of two options, each written on an energy-related underlying.

The payoff an energy quanto option reads

$$\Lambda(T) = \max\left(\tilde{S}_1(T) - K_1, 0\right) \max\left(\tilde{S}_2(T) - K_2, 0\right),$$

where

- The underlying assets $\tilde{S}_1$ and $\tilde{S}_2$ are correlated. Typically, $\tilde{S}_1$ and $\tilde{S}_2$ can be average of or futures on energy spot prices, or the average of or futures on weather indices;
- K_1 and K_2 are strike prices;
- T is maturity.

For renewable electricity, the weather conditions have an important impact on the price of electricity. The demand for the electricity can be lower in a warm winter and the decrease in demand can yield a decline in the electricity price. These options take this correlation into account.

Hence, as opposed to standardized contracts that only account for price risk, these options are designed to manage both volumetric and price risk in energy markets. That is why they are becoming very popular.

Let us consider energy quanto options written on two future contracts with maturity $\tau \in \mathbb{R}$, with prices at time t denoted by $F_1(t,\tau)$ and $F_2(t,\tau)$. The time-0 price $V(F_1, F_2, T)$ of an energy quanto option with maturity T on these 2 futures prices $F_1(T,\tau)$ and $F_2(T,\tau)$ reads, with $T \leq \tau$:

$$V(F_1, F_2, T) = \mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^T r_s ds}\, (F_1(T,\tau) - K_1)^+ (F_2(T,\tau) - K_2)^+\right],$$

where

- r_s is the short rate.

---

[1] Professor in the Service Sciences Actuarielles at Université Libre de Bruxelles (ULB)
[2] Assistant professor at the Applied Mathematics Department of TU Delft (TUD); Director of FF Quant Advisory B.V. https://fsquaredquant.nl/

Challenge

The valuation of these options is more complex than that of equity options, due to characteristic features of energy markets such as seasonality, mean reversion, and price spikes.

Markov-Modulated Additive processes, among others, are considered suitable candidates for capturing these dynamics. [4] considers the valuation of such an option on futures when the underlying futures prices are governed by Markov-modulated additive processes, which have independent but non-stationary increments within each regime. A valuation formula is derived using the Fast Fourier Transform (FFT) technique under the assumption that the joint characteristic function of the log-futures prices is known analytically. Several numerical case studies illustrate that this FFT-based valuation has a high precision and is much faster than Monte Carlo estimates.

In practice, however, it is not realistic to assume a constant interest rate. Introducing a stochastic interest rate into the valuation formula results in a three-dimensional pricing problem. This would considerably increase the computational complexity of the FFT-based approach due to the "curse of dimensionality".

In literature, the COS method, introduced in [1], has been proven to be even more superior to the FFT-based method in many applications. For example, [6] extended the regime-switching model to the rich class of time-changed Lévy processes and use the COS method to price several options under the resulting models. It was reported that the COS method converges exponentially in that framework and thus outperforms all other numerical methods that have been proposed so far.

This thesis is designed to answer the following question: Can the COS method be applied to price energy quanto options under Markov-modulated framework with regime-switching stochastic interest rate assumptions?

The goal and content of this thesis

A straightforward extension of the COS method from one to higher dimensions would also suffer from the "curse of dimensionality".

To address this challenge, a dimension-reduced COS method has been developed through a series of MSc theses, such as [2] and [3], funded by and conducted at FF Quant Advisory[3]. This method utilizes Canonical Polyadic Decomposition (CPD) to decompose the Fourier-cosine coefficient tensor, which is then integrated with the COS method, hence the name "COS-CPD" method.

The aim of this MSc thesis is to apply the COS-CPD method in pricing energy quanto options with regime-switching models.

[3] https://fsquaredquant.nl/

We will reach this goal via a few steps:

- Replicate the COS-CPD method in [3] for pricing a basket option;
- Replicate the pricing method in [4] using FFT;
- Apply the COS-CPD method to energy quanto options in the same set-up as in [4], namely assuming the underlying futures prices are governed by Markov-modulated additive processes with a constant interest rate.
- Relax the constant interest rate assumption: price energy quanto options using Markov-Modulated additive processes with stochastic interest rate.

Contact

This thesis will be a collaborative effort between ULB and TUD.

The research will be carried out by an MSc student from TUD, under the daily supervision of Assistant Professor Dr. Fang Fang from TUD. Regular discussions will also be held with Professor Griselda Deelstra from ULB.

If you are interested in this topic or would like more information, please feel free to contact Assistant Professor Dr. Fang at f.fang@tudelft.nl.

Reference

[1] Fang, F. and Oosterlee, C.W., *A novel pricicng method for European options based on Fourier-cosine series expansions.* SIAM J. Sci. Comput., 2008, 31, 826–848.

[2] Dimension Reduction Techniques for Multi-dimensional Numerical Integrations Based on Fourier-cosine Series Expansion. Zhimin Cheng. TU Delft, Msc thesis, 2022.

[3] *Solving Multivariate Expectations Using Dimension-reduced Fourier-cosine Series Expansion and Its Applications in Finance*. Marnix Brands. TU Delft, Msc thesis 2023.

[4] Benth F., Deelstra G., Kozpınar S., 2023, Pricing Energy Quanto Options in the Framework of Markov-Modulated Additive Processes, IMA Journal of Management Mathematics, Volume 34, Issue 1, January 2023, Pages 187{220}.

[5] Benth F., Deelstra G., Kozpınar S., 2023, Pricing Energy Quanto Options: A Regime-switching Framework with Stochastic Interest Rates, work in progress.

[6] Tour, G., Thakoor, N., Khaliq, A. Q. M., & Tangman, D. Y. (2018). *COS method for option pricing under a regime-switching model with time-changed Lévy processes.* Quantitative Finance, 18(4), 673–692. https://doi.org/10.1080/14697688.2017.1412494