**American Option Pricing (AC/CL/JK/NB/LS)**

**Description**

American-style derivatives can be exercised at any date before expiry, as opposed to European-style derivatives, which can only be exercised at expiry. These derivatives are found in all major financial markets, including equity, commodity, foreign exchange, and many others. The valuation of these products is a challenging problem in derivatives finance, due to its mathematical complication. The mathematical theory links this problem to the definition of optimal stopping times (see for instance [1]). The most popular methods that are used are finite difference methods [2], binomial trees [3] and Monte Carlo methods [4]. Quadrature methods have also been considered [8]. Convergence results for these kinds of methods are rather technical and can be found, for instance, in [5]. More recent methods focus on solving equations for the optimal exercise boundary. That is, the minimal exercise value which makes it convenient to exercise the option before expiry, for example [6].

There are already literature reviews, such as [7], but a complete trade-off analysis between all the different approaches does not exist, as far as our knowledge goes. There are already some experimental analyses such as [9], but they are by no means complete. As such, there is space to contribute to the field.

To achieve such a goal, it is necessary to start with a restricted scope. We propose to start analysing the pricing problem under Black-Scholes dynamics with no dividends in a choice of three different methods. These methods should be carefully implemented, with attention to their robustness and performance.

The candidate should start with the theoretical study on the problem, then get an overview of the methods existing in literature, for example by studying existing literature reviews (such as [7]). Then specific models should be selected for an in-depth analysis, based on their known effectiveness, theoretical novelty, and practical interest.

Of particular interest is the novel approach presented in [10], which proposes an alternative to Longstaff-Schwartz based on *stochastic exercises rates* rather than on exercises boundaries. A critical component of both algorithms is the choice of *basis functions*, which can drastically affect the convergence rate towards the correct price of the option. Therefore, a study of alternative basis functions and the convergence rate they provide is also of interest.

Another interesting approach is the one from [11], based on the concept of signatures. The signature of a stochastic process is an encoding based on iterative integrals of the stochastic process, and therefore it encapsulates all the information contained in the original process. In some applications, it can be beneficial to work with the space of signatures, instead of the original space where the process lives (see [11]).

A drawback of the signature approach is that it relies on the continuity of the underlying process, while in some scenarios (e.g. path-dependent options) it may be desirable to allow for sudden jumps in the underlying dynamics. Therefore, a study about whether the signatures of simple jump processes such as the Poisson process can be computed would be beneficial for many applications.

The candidate is asked to perform an experimental and theoretical analysis of the advantages and drawbacks of the chosen models, in the context of commodity derivative pricing. Creativity will be required in finding corner cases to challenge a model choice, together with mathematical rigor to analyse the problem and coding skills for the implementations.

Depending on time, the candidate can explore increasingly complicated asset dynamics, e.g., starting with the classical Black-Scholes, or stochastic volatilities or even jumps. Another possible area of expansion is the evaluation of Greeks and the hedging techniques.

Coding will also be a large part of this topic. As such, the student should be confident in his/her programming skills.

**Objectives**

1. Review the existing literature on American option pricing, starting from the theoretical framework and simple dynamics cases (e.g., Black and Scholes).
2. Comparison of the classical Longstaff-Schwartz method with the approach from [10] based on exercise rates. A study of alternative basis functions to increase the convergence rate of these methods would be interesting. A theoretical study of the convergence rate would be ideal, but possibly very challenging.
3. Become acquainted with the concept of "signature" as a tool to encode a stochastic process and apply this concept for American option pricing as in [11].
4. The theoretical results about signatures assume that paths are continuous. Can this be extended to simple jump processes like the Poisson process?
5. Alternatively---and only if time permits---if the student is specially interested in programming, applying parallelization schemes to existing methodologies could also be a way to increase the performance of certain models.

**References**

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