An implicit dual interface capturing method for general polygonal control volumes
MSc thesis project

Introduction

Computational models for the simulation of incompressible two phase flow consist of equations for momentum and mass conservation, and an evolution equation for the interface that separates both phases.

Dual interface capturing methods use congruent *level set* and *Volume of Fluid* fields to represent the interface. This level set field is typically chosen as the signed-distance to the interface, while the Volume of Fluid is a grid function that represents the cell average of a piecewise constant marker function that changes value at the interface. While the level set field is smooth in the vicinity of the interface, the marker function (and its cell average) is discontinuous at the interface.

The synergy between the two representations leads to an algorithm that is much more efficient than standard Volume of Fluid interface models. At the same time a global level set field representation is available to accurately impose interface conditions in the discretisation of the equation for momentum conservation and for extraction of the interface curvature.

Matching level set and Volume of Fluid fields

To make the two fields congruent a mass-conserving correction is applied to the level set field, based on the local Volume of Fluid values. This mass-conserving correction is the solution of a nonlinear system of equations of dimension equal to the number of interface cells, i.e. all cells intersected by the interface.

It is straightforward to show that the interface reconstruction that is at the heart of the computation of the mass-conserving correction is not unique. In the dual formulation of Oud [3] the interface normal vector is computed from the level set field, and corrections are iteratively applied until the gradient of the corrected level-set field is in correspondence with the assumed interface orientation.

To accurately solve the transport equation for the level set field on triangular control volumes in his dual method, Raees[4] uses a 2nd order discontinuous Galerkin discretisation of the level set field. In the latter approach the definition of the level set gradient becomes local, and individual control volumes can be corrected independently.

This presents opportunities to impose additional constraints apart from mass conservation on the level set field, by formulating a constrained optimization problem for the mass conserving correction.

Volume of Fluid transport equation

Two approaches can be applied to advance the Volume of Fluid field:

- Formulating a balance equation based on donating regions, e.g. Oud[3].

- Computing a pre-image or post-image of a control volume, e.g. Raees[4].

Oud [3] uses the dimensionally split Volume of Fluid advection scheme of Weymouth and Yue. Contrary to the schemes proposed by Pucket et al. and Sussman, this scheme is mass conserving up to machine precision and ensures the Volume of Fluid only attains physical values under a (severe) constraint on the size of the nondimensional time step. Although attempts have been made to extend the donating region approach to unstructured grids and more general control volumes, the resulting algorithms get extremely complicated and/or lose the property of exact mass conservation.

Raees[4] uses a fully Lagrangian approach and computes a post image of each control volume based on a flowmap of the vertices of the control volume. However, because for general velocity fields the post-image of a straight-sided control volume will not remain straight-sided, an adhoc mass-correction and redistribution step has to be included in the algorithm. This approach is relatively straightforwardly extended to more general control volumes.

Imposing a mass conserving constraint

To extend dual methods to more general control volumes and/or relax the constraint on the size of the nondimensional time step the explicit solution of the transport equation for the Volume of Fluid method should be eliminated from the algorithm. To enforce exact mass conservation an *implicit* constraint should be imposed on the level set field, that does not rely on a separate Volume of Fluid solution. This type of approach has been proposed by Kuzmin[] in the context of finite element discretisation schemes for the level set field.

The aim of the current project is to incorporate an implicit constraint of conservation of mass in a Cartesian dual interface capturing method, explore different optimization goals and strategies and investigate the extension of this approach to general polygonal control volumes.

Preliminary research questions

- Can dual interface methods with an implicit constraint of conservation of mass be more efficient than dual interface methods where the mass conservation constraint is based on an explicitly avanced Volume of Fluid field?
- Is it possible to improve the properties of the interface model by enforcing additional constraints on the solution next to exact mass conservation?
- Can the implicit mass conservation constraint also be imposed on general polyhedral control volumes?

Literature study

In the literature study the student should focus on the following subjects:

- Implicitly constrained level set methods [1,2].
- Dual interface capturing methods [3,4]
- Discontinuous Galerkin discretisation methods for the linear convection equation on Cartesian and general polygonal control volumes.
- Linear an nonlinear optimization algorithms.

Project outline

The project consists of two parts. In the first phase an implicit Dual interface capturing method will be formulated for discretisation on two-dimensional Cartesian control volumes in the following steps:

a Implement the baseline *explicit* MCLS dual interface capturing method for a Cartesian two-dimensional computational grid, with a discontinuous Galerkin discretisation of the LS field and the dimensionally split mass-consering advection scheme of Weymouth and Yue.

b Formulate an implicit mass-conserving correction of the LS solution as a nonlinear optimization problem.

c Formulate a set of possible goal functions for optimization, for different applications, e.g. extraction of curvature. Analyse accuracy of the solution and the complexity of solving the nonlinear optimization problem.

d Implement the implicit mass-conserving correction and evaluate its performance in comparison to the baseline explicit algorithm for classical testcases, e.g. corner flow deformation of a circular region, the *Zalesak disk* and the *vortex-shear testcase*.

In the second phase the focus will be on extending the developed method to general polyhedral control volumes.

a Formulate a discontinuous Galerkin discretisation of the level set transport equation on general polyhedral control volumes.

b Extend the Cartesian formulation of the implicit mass-conserving correction of the LS solution to the case of general polyhedral control volumes.

c Apply the general polyhedral formulation of the algorithm for the same testcases as were used to verify the Cartesian algorithm.

References

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[5] Weymouth, G.D. and Dick K.-P. Yue, Conservative Volume-of-Fluid method for free-surface simulations on Cartesian-grids, *JCP* 229:8(2010) 2853-2865.