Implementation of a Quantum Linear Solver

Motivation

Sources

[1] D.Dervovic, M. Herbster, P. Mountney,S. Severini, N. Usher, L. Wossnig *Quantum linear systems algorithms: a primer* (https://arxiv.org/abs/1802.08227)

[2] A. Harrow, A. Hassim, and S. Lloyd *Quantum algorithm for linear systems of equations*

[3] C. Bravo-Prieto, R. LaRose, M. Cerezo, Y. Subasi, L. Cincio, and P. Coles *Variational Quantum Linear Solver: A Hybrid Algorithm for linear systems*

[4] https://qiskit.org/textbook (https://qiskit.org/textbook)

Outline

Quantum Algorithms

Qiskit

HHL

VQLS

Quantum advantage

Reality of Quantum Computers

- NISQ
- Simulation

Quantum Algorithms

- Leveraging quantum information

Gate Model with Unitary matrices

Most common gates

- Hadamard, CNot
- Pauli X, Y, Z
- Rotations $R_x, R_y R_z$

Qiskit

In [2]:

```
import qiskit
from qiskit import QuantumCircuit, QuantumRegister, ClassicalRegister
from qiskit import IBMQ, Aer, execute, circuit
from qiskit.providers.ibmq import least_busy
from qiskit.providers.aer import noise

# HHL
from qiskit.aqua import run_algorithm
from qiskit.quantum_info import state_fidelity
from qiskit.aqua.algorithms.classical import ExactLSsolver

#Visualize
import matplotlib.pyplot as plt
from qiskit.tools.visualization import plot_histogram
from qiskit.tools.visualization import plot_bloch_vector

import math
import random
import numpy as np
from scipy.optimize import minimize

qiskit.__qiskit_version__
```

Out[2]:

```
{'qiskit-terra': '0.12.0',
 'qiskit-aer': '0.4.0',
 'qiskit-ignis': '0.2.0',
 'qiskit-ibmq-provider': '0.4.6',
 'qiskit-aqua': '0.6.4',
 'qiskit': '0.15.0'}
```

Four Elements Earth - Air - Water - Fire

- Compile
- OpenQasm
- Backend

Gateset

- U_1, U_2, and Cnot
  U_3

Where

$$U_1(\lambda) = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\lambda} \end{bmatrix}$$

$$U_2(\theta, \lambda) = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -e^{i\lambda} \\ e^{i\theta} & e^{i(\lambda+\theta)} \end{bmatrix}$$

$$U_3(\phi, \theta, \lambda) = \begin{bmatrix} cos(\frac{\theta}{2}) & -e^{i\lambda} sin(\frac{\theta}{2}) \\ e^{i\theta} sin(\frac{\theta}{2}) & e^{i(\lambda+\theta)} \end{bmatrix}$$

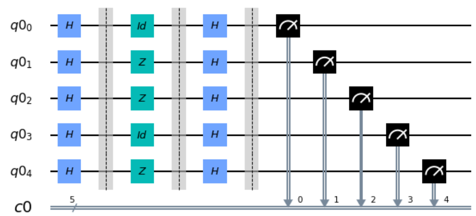
Simulations

- Perfect
- Noise

In [3]:

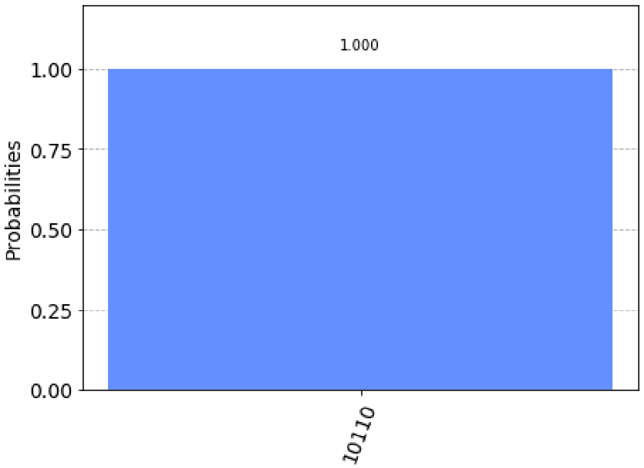
```
# Bernstein-Vazirani Algorithm
nQubits = 5  # number of qubits needed
s = 22       # Secret number


qr = QuantumRegister(nQubits)
# Classical bits
cr = ClassicalRegister(nQubits)

bvCircuit = QuantumCircuit(qr, cr)
barriers = True

# Apply Hadamard gates
for i in range(nQubits):
    bvCircuit.h(qr[i])

# Apply barrier
if barriers:
    bvCircuit.barrier()

# Oracle
for i in range(nQubits):
    if (s & (1 << i)):
        bvCircuit.z(qr[i])
    else:
        bvCircuit.iden(qr[i])

# Apply barrier
if barriers:
    bvCircuit.barrier()

#Apply Hadamard gates after querying the oracle
for i in range(nQubits):
    bvCircuit.h(qr[i])

# Apply barrier
if barriers:
    bvCircuit.barrier()

# Measurement
bvCircuit.measure(qr, cr)
bvCircuit.draw(output='mpl')
```

Out[3]:



In [4]:
```
# use local simulator
backend = Aer.get_backend('qasm_simulator')
shots = 1024
results = execute(bvCircuit, backend=backend, shots=shots).result()
answer = results.get_counts()
plot_histogram(answer)
```

Out[4]:

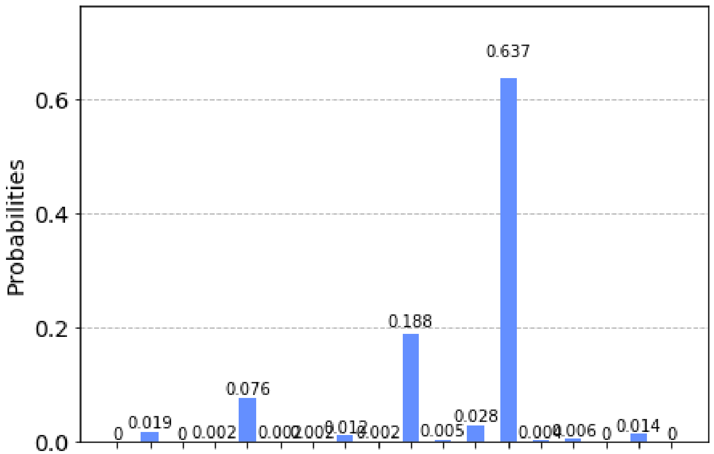


In [5]:

```python
# Use noise
IBMQ.load_account()
provider = IBMQ.get_provider(group='open')
device = provider.get_backend('ibmq_16_melbourne')
properties = device.properties()

coupling_map = device.configuration().coupling_map
noise_model = noise.device.basic_device_noise_model(properties)
basis_gates = noise_model.basis_gates

noise_results= execute(bvCircuit, backend,
                  coupling_map=coupling_map,
                  noise_model=noise_model,
                  basis_gates=basis_gates).result()

noiseanswer = noise_results.get_counts(bvCircuit)


plot_histogram(noiseanswer)
```

```
C:\Users\sigag\Anaconda3\envs\quantum\lib\site-packages\qiskit\providers\aer\noise\de
vice\basic_device_model.py:112: DeprecationWarning: This function is been deprecated
and moved to a method of the`NoiseModel` class. For equivalent functionality use `Noi
seModel.from_backend(properties, **kwargs).
  DeprecationWarning)
```

Out[5]:



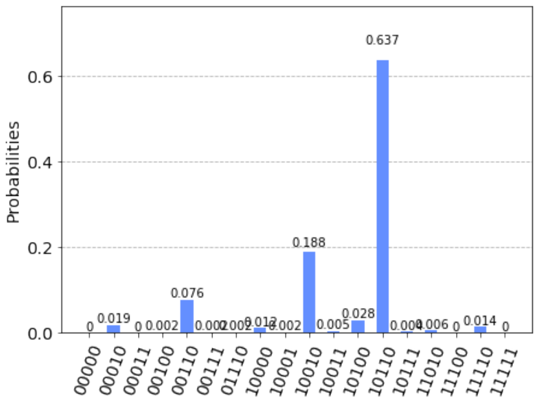
Quantum Linear Solver

In [5]:

```
# Use noise
IBMQ.load_account()
provider = IBMQ.get_provider(group='open')
device = provider.get_backend('ibmq_16_melbourne')
properties = device.properties()

coupling_map = device.configuration().coupling_map
noise_model = noise.device.basic_device_noise_model(properties)
basis_gates = noise_model.basis_gates

noise_results= execute(bvCircuit, backend,
                  coupling_map=coupling_map,
                  noise_model=noise_model,
                  basis_gates=basis_gates).result()

noiseanswer = noise_results.get_counts(bvCircuit)


plot_histogram(noiseanswer)
```

```
C:\Users\sigag\Anaconda3\envs\quantum\lib\site-packages\qiskit\providers\aer\noise\de
vice\basic_device_model.py:112: DeprecationWarning: This function is been deprecated
and moved to a method of the`NoiseModel` class. For equivalent functionality use `Noi
seModel.from_backend(properties, **kwargs).
  DeprecationWarning)
```

Out[5]:



Quantum Linear Solver

Quantum Formulation of the linear solver

Let A be an $N \times N$ *Hermitian matrix with a unit determinant. Let b and x be N-dimentional vectors such that* $x := A^{-1}b$*. Let the quantum state on* $\lceil log(N) \rceil$ *qubits |b> be given by*

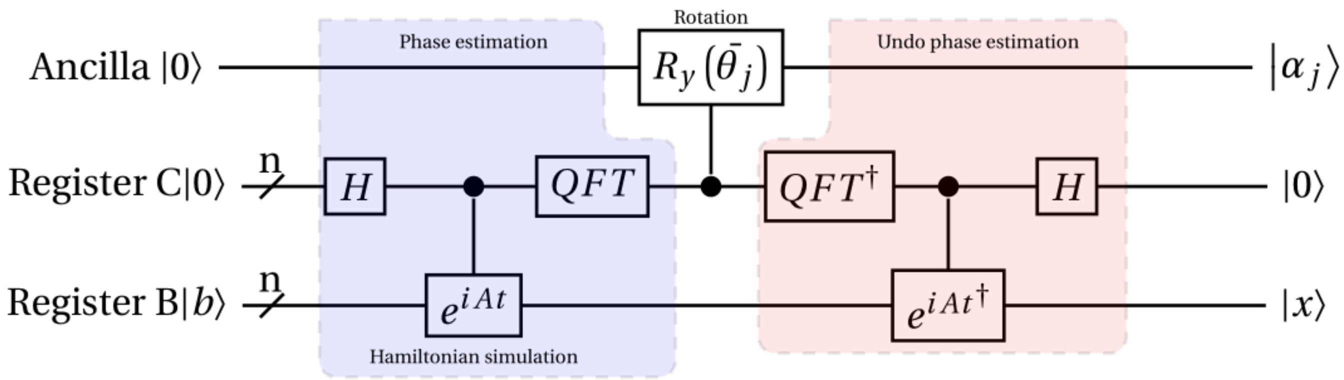
$$\frac{\sum_i b_i |i>}{||\sum_i b_i |i>||_2}$$

and for |x> by

$$\frac{\sum_i x_i |i>}{||\sum_i x_i |i>||_2}$$

where b_i*,* x_i *are the i-th components of b and x respectively. Given A and |b>, output a state* $|\tilde{x}>$ *such that* $||(|\tilde{x}> - |x>)||_2 \leq \epsilon$*, with some probability larger than 1/2.*

HHL


Rotation
Phase estimation
Undo phase estimation
Ancilla $|0\rangle$
Register C$|0\rangle$
Register B$|b\rangle$
n
n
H
QFT
$R_y(\bar{\theta}_j)$
$QFT^\dagger$
H
e^{iAt}
$e^{iAt^\dagger}$
Hamiltonian simulation
$|\alpha_j\rangle$
$|0\rangle$
$|x\rangle$

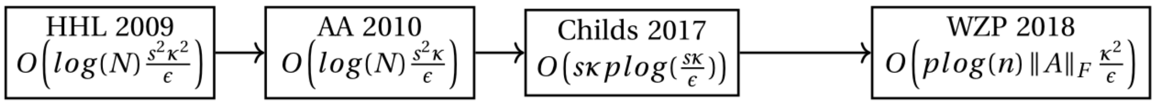
Hamiltonian Simulation

- Turning A into a Unitary operator e^{iAt}
- ~$O(log(N)s^2t)$

Phase Estimation

- Quantum Fourier transform
- Requires n extra control qubits
- Estimates the eigenvalues of A

Improvements and variations



In [6]:

```
params = {
    'problem': {
        'name': 'linear_system'
    },
    'algorithm': {
        'name': 'HHL'
    },
    'eigs': {
        'expansion_mode': 'suzuki',
        'expansion_order': 2,
        'name': 'EigsQPE',
        'num_ancillae': 3,
        'num_time_slices': 50
    },
    'reciprocal': {
        'name': 'Lookup'
    },
    'backend': {
        'provider': 'qiskit.BasicAer',
        'name': 'statevector_simulator'
    }
}

def fidelity(hhl, ref):
    solution_hhl_normed = hhl / np.linalg.norm(hhl)
    solution_ref_normed = ref / np.linalg.norm(ref)
    fidelity = state_fidelity(solution_hhl_normed, solution_ref_normed)
    print("fidelity %f" % fidelity)
```

In [7]:

```
matrix = [[1, 0], [0, 2]]
vector = [1, 4]
params['input'] = {
    'name': 'LinearSystemInput',
    'matrix': matrix,
    'vector': vector
}
```

In [8]:
```
hhl_output = run_algorithm(params)
print("solution ", np.round(hhl_output['solution'], 5))

result_ref = ExactLSsolver(matrix, vector).run()
print("classical solution ", np.round(result_ref['solution'], 5))

print("probability %f" % hhl_output['probability_result'])
fidelity(hhl_output['solution'], result_ref['solution'])
```

```
C:\Users\sigag\Anaconda3\envs\quantum\lib\site-packages\qiskit\aqua\qiskit_aqua.py:11
9: DeprecationWarning: Declarative API will be removed next Aqua release. Please cons
truct classes and call appropriate methods.
  warnings.warn(aqua_globals.CONFIG_DEPRECATION_MSG, DeprecationWarning)

solution  [1.05859-0.j 1.99245-0.j]
classical solution  [1. 2.]
probability 0.024630
fidelity 0.999389
```

Hybrid Algorithms

- Both Quantum and Classical
- Desegined wiht NISQ in mind

VQLS

In [10]:

```
def apply_fixed_ansatz(qubits, parameters):
    for iz in range(0, len(qubits)):
        circ.ry(parameters[0][iz], qubits[iz])

    circ.cz(qubits[0], qubits[1])
    circ.cz(qubits[2], qubits[0])

    for iz in range(0, len(qubits)):
        circ.ry(parameters[1][iz], qubits[iz])

    circ.cz(qubits[1], qubits[2])
    circ.cz(qubits[2], qubits[0])

    for iz in range(0, len(qubits)):
        circ.ry(parameters[2][iz], qubits[iz])
```

In [11]:

```
def had_test(gate_type, qubits, ancilla_index, parameters):
    # Creates the Hadamard test
    circ.h(ancilla_index)

    apply_fixed_ansatz(qubits, parameters)

    for ie in range(0, len(gate_type[0])):
        if (gate_type[0][ie] == 1):
            circ.cz(ancilla_index, qubits[ie])

    for ie in range(0, len(gate_type[1])):
        if (gate_type[1][ie] == 1):
            circ.cz(ancilla_index, qubits[ie])

    circ.h(ancilla_index)
```

In [12]:
```
def control_fixed_ansatz(qubits, parameters, ancilla, reg):

    for i in range(0, len(qubits)):
        circ.cry(parameters[0][i], qiskit.circuit.Qubit(reg, ancilla),
                 qiskit.circuit.Qubit(reg, qubits[i]))

    circ.ccx(ancilla, qubits[1], 4)
    circ.cz(qubits[0], 4)
    circ.ccx(ancilla, qubits[1], 4)

    circ.ccx(ancilla, qubits[0], 4)
    circ.cz(qubits[2], 4)
    circ.ccx(ancilla, qubits[0], 4)

    for i in range(0, len(qubits)):
        circ.cry(parameters[1][i], qiskit.circuit.Qubit(reg, ancilla),
                 qiskit.circuit.Qubit(reg, qubits[i]))

    circ.ccx(ancilla, qubits[2], 4)
    circ.cz(qubits[1], 4)
    circ.ccx(ancilla, qubits[2], 4)

    circ.ccx(ancilla, qubits[0], 4)
    circ.cz(qubits[2], 4)
    circ.ccx(ancilla, qubits[0], 4)

    for i in range(0, len(qubits)):
        circ.cry(parameters[2][i], qiskit.circuit.Qubit(reg, ancilla),
                 qiskit.circuit.Qubit(reg, qubits[i]))
```

In [14]:

```
def special_had_test(gate_type, qubits, ancilla_index, parameters, reg):

    circ.h(ancilla_index)

    control_fixed_ansatz(qubits, parameters, ancilla_index, reg)

    for ty in range(0, len(gate_type)):
        if (gate_type[ty] == 1):
            circ.cz(ancilla_index, qubits[ty])

    control_b(ancilla_index, qubits)

    circ.h(ancilla_index)
```

In [15]:

```
def calculate_cost_function(parameters):
    global opt

    overall_sum_1 = 0

    parameters = [parameters[0:3], parameters[3:6], parameters[6:9]]

    for i in range(0, len(gate_set)):
        for j in range(0, len(gate_set)):

            global circ

            qctl = QuantumRegister(5)
            qc = ClassicalRegister(5)
            circ = QuantumCircuit(qctl, qc)

            backend = Aer.get_backend('statevector_simulator')

            multiply = coefficient_set[i]*coefficient_set[j]

            had_test([gate_set[i], gate_set[j]], [1, 2, 3], 0, parameters)

            job = execute(circ, backend)

            result = job.result()
            outputstate = np.real(result.get_statevector(circ, decimals=100))
            o = outputstate

            m_sum = 0
            for l in range (0, len(o)):
                if (l%2 == 1):
                    n = o[l]**2
                    m_sum+=n

            overall_sum_1+=multiply*(1-(2*m_sum))

    overall_sum_2 = 0

    for i in range(0, len(gate_set)):
        for j in range(0, len(gate_set)):

            multiply = coefficient_set[i]*coefficient_set[j]
            mult = 1
```

The problem is

$$A = \alpha_1 \mathbb{Z} + \alpha_2 \mathbb{I}$$

$$b = \begin{bmatrix} \frac{1}{\sqrt{8}} \\ \cdots \\ \frac{1}{\sqrt{8}} \end{bmatrix}$$

In [16]:

```
coefficient_set = [0.55, 0.45]
gate_set = [[0, 0, 0], [0, 0, 1]]

out = minimize(calculate_cost_function, x0=[float(random.randint(0,3000))/1000 for i in
range(0, 9)], method="COBYLA", options={'maxiter':200})
print(out)

out_f = [out['x'][0:3], out['x'][3:6], out['x'][6:9]]

circ = QuantumCircuit(3, 3)
apply_fixed_ansatz([0, 1, 2], out_f)

backend = Aer.get_backend('statevector_simulator')

job = execute(circ, backend)

result = job.result()
o = result.get_statevector(circ, decimals=10)

a1 = coefficient_set[1]*np.array([[1,0,0,0,0,0,0,0], [0,1,0,0,0,0,0,0], [0,0,1,0,0,0,0,0
], [0,0,0,1,0,0,0,0], [0,0,0,0,-1,0,0,0], [0,0,0,0,0,-1,0,0], [0,0,0,0,0,0,-1,0], [0,0,0
,0,0,0,0,-1]])
a2 = coefficient_set[0]*np.array([[1,0,0,0,0,0,0,0], [0,1,0,0,0,0,0,0], [0,0,1,0,0,0,0,0
], [0,0,0,1,0,0,0,0], [0,0,0,0,1,0,0,0], [0,0,0,0,0,1,0,0], [0,0,0,0,0,0,1,0], [0,0,0,0,
0,0,0,1]])
a3 = np.add(a1, a2)

b = np.array([float(1/np.sqrt(8)),float(1/np.sqrt(8)),float(1/np.sqrt(8)),float(1/np.sqr
t(8)),float(1/np.sqrt(8)),float(1/np.sqrt(8)),float(1/np.sqrt(8)),float(1/np.sqrt(8))])

print((b.dot(a3.dot(o)/(np.linalg.norm(a3.dot(o)))))**2)
```

```
0.9889759175732641
0.9984515010031692
0.9941961555905245
0.7962256723668557
0.7683322520541489
0.7554331064406511
0.744020216776846
0.5715702194434831
0.6217323263729735
0.591788596885295
0.5764212965342479
```

Thesis questions

- What approaches are there to solve linear systems on Quantum computers
- Can you get near term supremacy with these approaches?
- Can you build on these implamentations?