Robust Algorithms for Discrete Tomography

Literature Study

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Layout

Layout

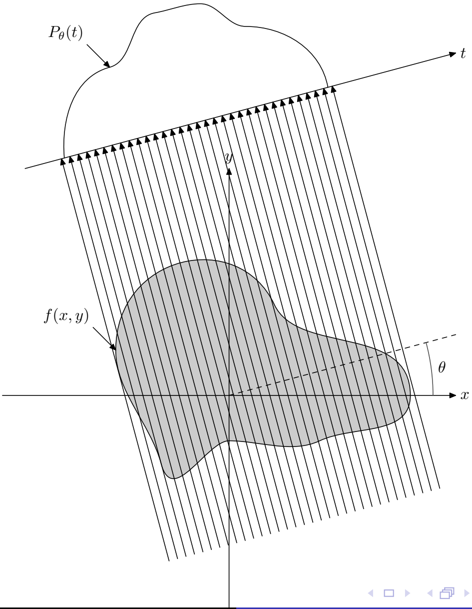
Introduction

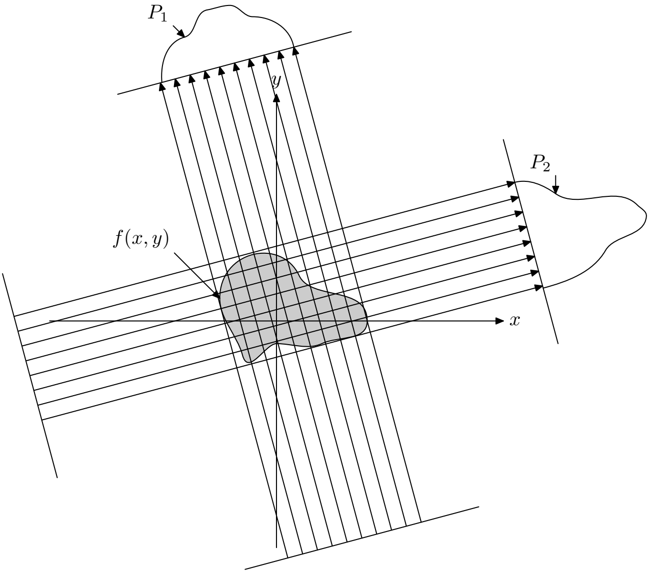
$\tau\acute{o}\mu o\sigma$ (tomos) + $\gamma\rho\acute{\alpha}\phi\epsilon\iota\nu$ (graphein) = Tomgraphy

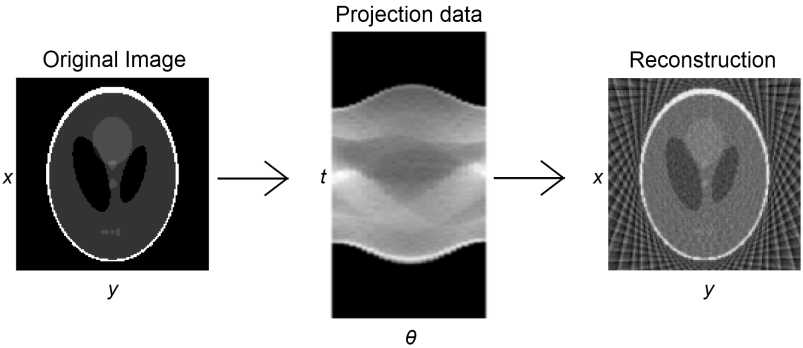
- tomos: slice/part
- graphein: to write
- Invention X-ray 1895 by Wilhelm Röntgen
- Non-invasive way to see inside of an object

Applications:

- Medical
- Geophysics
- Astrophysics
- Material Science
- Many others...


$P_\theta(t)$
t
y
$f(x,y)$
θ
x


P_1
y
P_2
$f(x,y)$
x

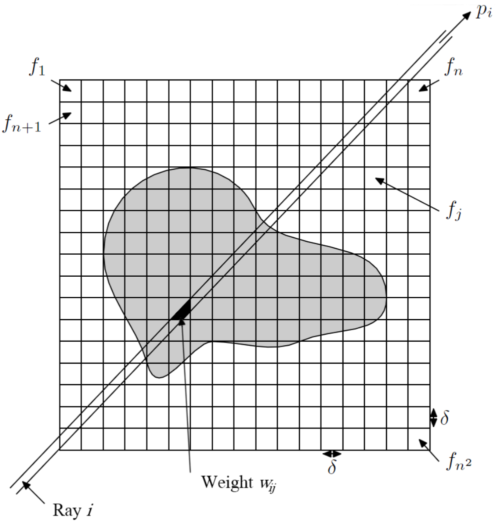

Original Image
x
y
Projection data
t
θ
Reconstruction
x
y

Roughly two ways to reconstruct an object from projections:

- *Analytical*: Using Fourier transforms
- *Algebraic*: Formulating problem as system of linear equations

Layout

Model Description



Pixels / cells: $f_j, j = 1, 2, \ldots, N$
Rays: $p_i, i = 1, 2, \ldots, M$
Contribution (weight) cell j to ray i: w_{ij}, assume $w_{ij} \geq 0$

$$\begin{aligned} w_{11} f_1 + w_{12} f_2 + \cdots + w_{1N} f_N &= p_1 \\ w_{21} f_1 + w_{22} f_2 + \cdots + w_{2N} f_N &= p_2 \\ &\vdots \\ w_{M1} f_1 + w_{M2} f_2 + \cdots + w_{MN} f_N &= p_M. \end{aligned}$$

$$W\mathbf{f} = \mathbf{p}$$

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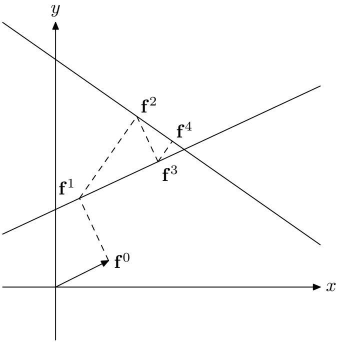
$$W\mathbf{f} = \mathbf{p}$$

Kaczmarz's Method / ART

By Stefan Kaczmarz (1937). Rediscovered (1970) as Algebraic Reconstruction Technique (ART) by Gordon, Bender and Herman.

Idea: Subsequently project approximation onto hyperplanes



Let $\mathbf{w}_i = (w_{i1}, w_{i2}, \ldots, w_{iN})^T$, i-th row of W
And $\mathbf{r}^k = \mathbf{p} - W\mathbf{f}^k$, k-th residual
The k-th approximation is found as[1]

$$\mathbf{f}^k = \mathbf{f}^{k-1} + \frac{\langle \mathbf{r}^{k-1}, \mathbf{w}_i \rangle}{\langle \mathbf{w}_i, \mathbf{w}_i \rangle} \mathbf{w}_i, \quad i = (k-1) \mod (M) + 1.$$

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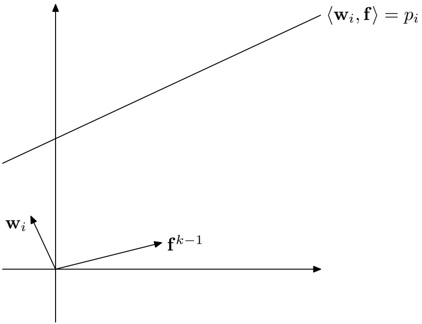

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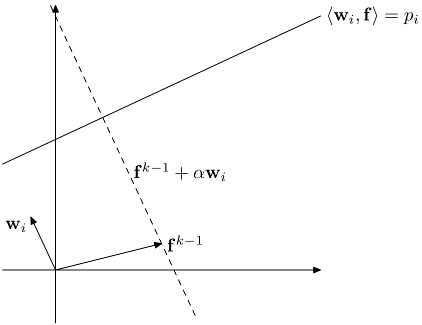

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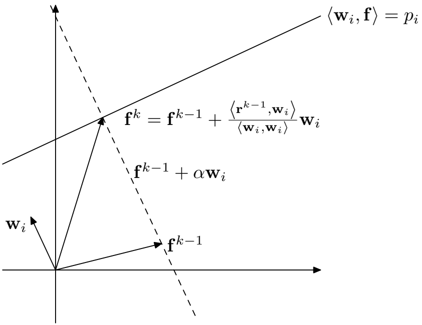

SIRT

Simultaneous Iterative Reconstruction Technique (1979) by Dines and Lyttle.

ART: Project successively onto hyperplanes.

SIRT[1]:

i. First compute correction for all rows using current approximation.
ii. Average over all corrections.

$$f_j^k = f_j^{k-1} + \frac{1}{\sum_{i=1}^{M} w_{ij}} \sum_{i=1}^{M} \frac{w_{ij} r_i^{k-1}}{\sum_{h=1}^{N} w_{ih}}.$$

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SART[2]: Update per projection angle

i. Compute correction for all rays with angle θ_l.

ii. Avarage over these corrections.

R: No. rays per angle

$$f_j^k = f_j^{k-1} + \frac{1}{\sum_{i=R\cdot(l-1)+1}^{R\cdot l} w_{ij}} \sum_{i=R\cdot(l-1)+1}^{R\cdot l} \frac{r_i^{k-1} w_{ij}}{\sum_{h=1}^{N} w_{ih}}.$$

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Convergence of SIRT

Recall SIRT

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Let C, R be diagonal matrices containing inverse column (C) and row (R) sums.

$$C = \begin{pmatrix} \ddots & & \\ & \frac{1}{\sum_{i=1}^M w_{ij}} & \\ & & \ddots \end{pmatrix}, R = \begin{pmatrix} \ddots & & \\ & \frac{1}{\sum_{j=1}^N w_{ij}} & \\ & & \ddots \end{pmatrix},$$

Then SIRT can be written as

$$\mathbf{f}^k = \mathbf{f}^{k-1} + CW^T R \mathbf{r}^{k-1}$$

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$$\begin{aligned}\mathbf{f}^k &= \mathbf{f}^{k-1} + CW^T R \mathbf{r}^{k-1} \\ &= \mathbf{f}^{k-1} + CW^T R \left(\mathbf{p} - W\mathbf{f}^{k-1}\right) \\ &= (I - CW^T RW)\mathbf{f}^{k-1} + CW^T R\mathbf{p}.\end{aligned}$$

$(I - CW^T RW)$ is *iteration matrix*.

Definition

The *spectral radius* of $A \in \mathbb{R}^{n\times n}$, denoted $\rho(A)$, is defined as

$$\rho(A) = \max_{\lambda_i, i=1,\dots,n} |\lambda_i|$$

where λ_i are the eigenvalues of A.

If $\rho(I - CW^T RW) < 1$, then convergence is guaranteed[3].

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Let λ be an eigenvalue of $CW^TRW \Rightarrow 1-\lambda$ eigenvalue of $(I - CW^TRW)$

To prove

$$\rho\left(I - CW^TRW\right) = \max_{\lambda} |1-\lambda| < 1 \Leftrightarrow 0 < \lambda < 2$$

Unfortunately, if W is not of full rank, one can only show $\lambda \geq 0$. Then *stagnation* may occur: error does not change.

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Remains to show that $\lambda < 2$.

The spectral radius of a matrix is less or equal to any operator norm[4]. Thus:

$$\rho(CW^TRW) \leq \|CW^TRW\|_\infty \leq \|CW^T\|_\infty \|RW\|_\infty.$$

Recall: c_{jj} are inverse column sums of $W \rightarrow$ inverse rows sums of $W^T \Rightarrow \|CW^T\|_\infty = 1$. Equivalently $\|RW\|_\infty = 1$

Thus $0 \leq \lambda \leq 1 < 2$

Hence SIRT either converges or stagnates.

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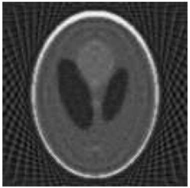
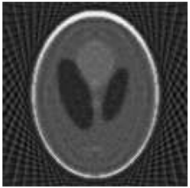
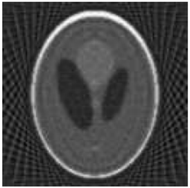
ARM Experiments

Used image was the *Shepp-Logan head phantom*. The image was 128 by 128 pixels and scanned using 32 projection angles with 192 rays per projection.

Without Noise

ART: 5 iter.	SIRT: 200 iter.	SART: 200 iter.
		

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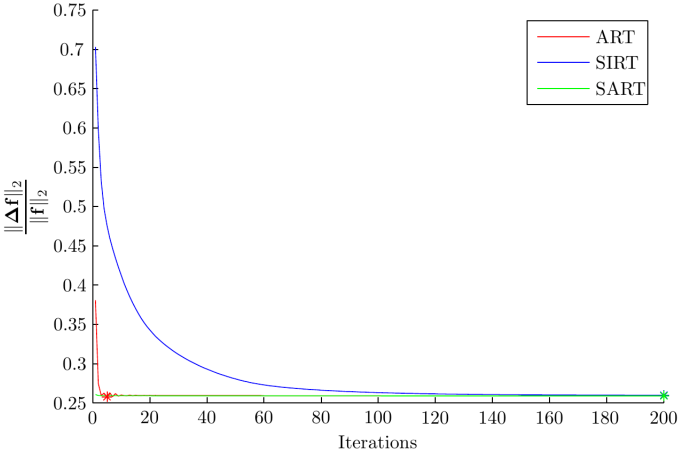
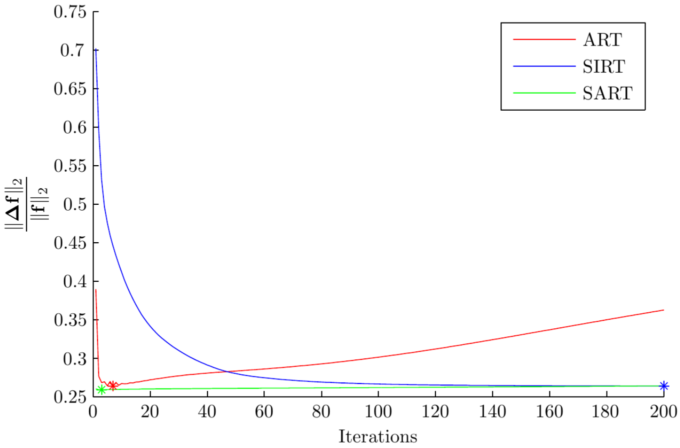
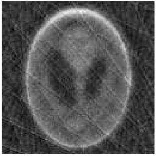


With Noise

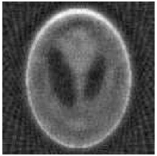

0.75
0.7
0.65
0.6
0.55
0.5
0.45
0.4
0.35
0.3
0.25
$\frac{\|\Delta \mathbf{f}\|_2}{\|\mathbf{f}\|_2}$
0
20
40
60
80
100
120
140
160
180
200
Iterations
ART
SIRT
SART

With Noise

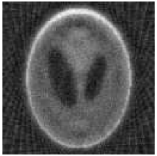
ART: 7 iterations



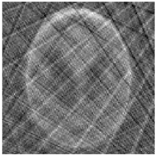
SIRT: 200 iterations



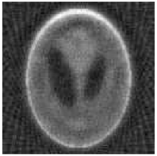
SART: 3 iterations



ART: 200 iterations



SIRT: 200 iterations



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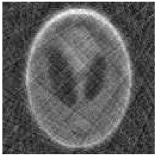

Layout

Discrete Tomography

Discrete tomography

- Object consists of some finite set of densities $\{\rho_1, \rho_2, \ldots, \rho_l\}$.
- In general very few projections angles (< 15) resulting from a small angular range;
- Different strategies for solving:
  - Combinatorial
  - Statistical
  - Continuous optimisation
  - **Continuous with discretisation step** $\Rightarrow$ DART[5]

[5] Kees Joost Batenburg and Jan Sijbers, "DART: A practical reconstruction algorithm for discrete tomography", *IEEE Transactions on Image Processing* 20(9) (September 2011) 2542–2553

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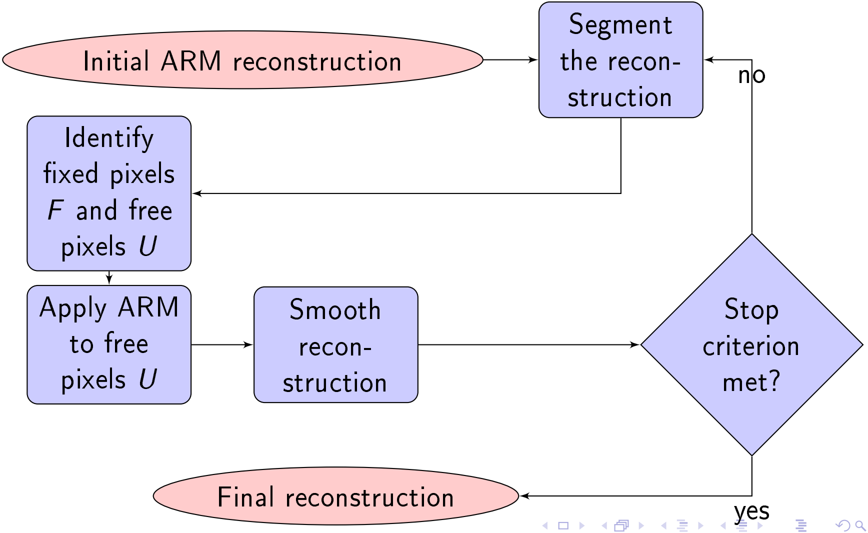
Discrete Algebraic Reconstruction Technique (DART)



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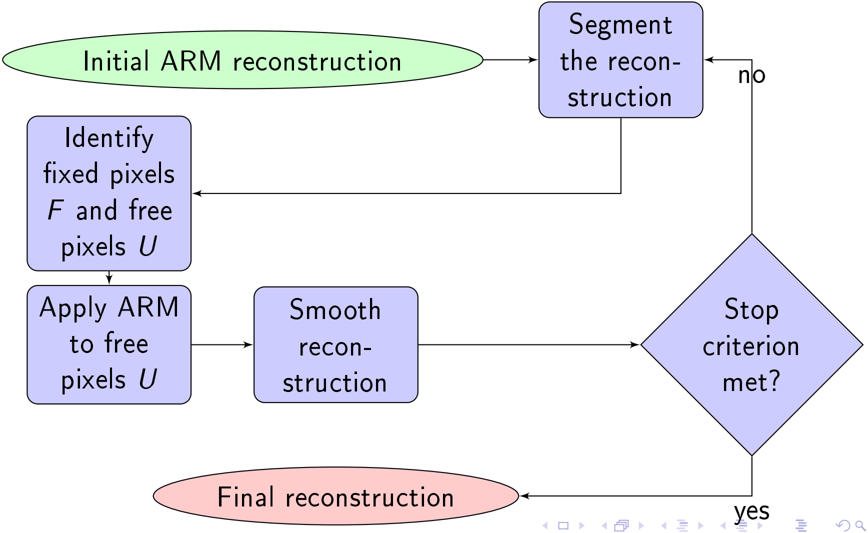

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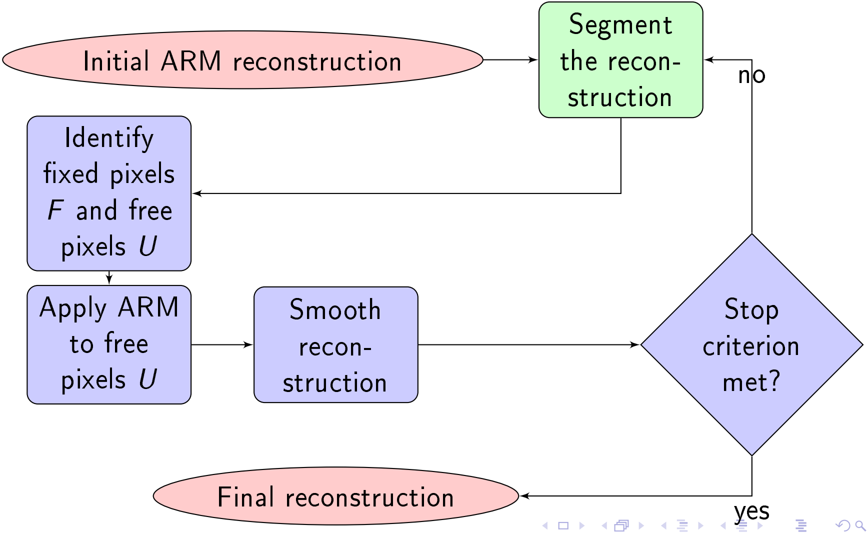

Segmentation

Segmentation is setting the values of the pixels to one of the admitted grey values $\rho \in \{\rho_1, \rho_2, \ldots, \rho_l\}$.

Most intuitive segmentation, rounding values to nearest grey value:

$$\tau_i = \frac{\rho_i + \rho_{i+1}}{2},$$

$$r(v) = \begin{cases} \rho_1, & (v < \tau_1) \\ \rho_2, & (\tau_1 \leq v < \tau_2) \\ \vdots & \\ \rho_l, & (\tau_{l-1} \leq v) \end{cases}.$$

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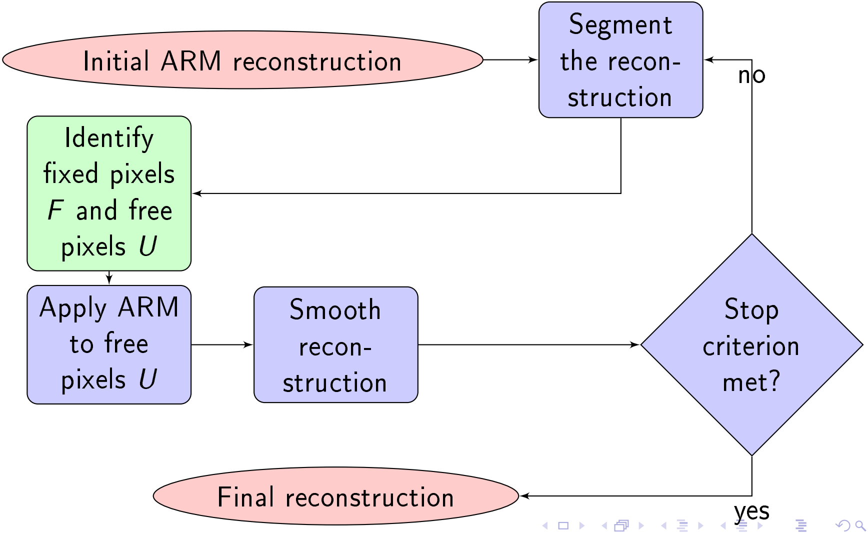

Fixed and Free Pixels

Set of fixed pixels F: Pixels surrounded by pixels with the same grey value.
Free (boundary) pixels U: At least one neighbour with a different grey value.

Every pixel in F is freed with probability $1 - p$
p: The fix probability

Needed to find overlooked holes in the image.

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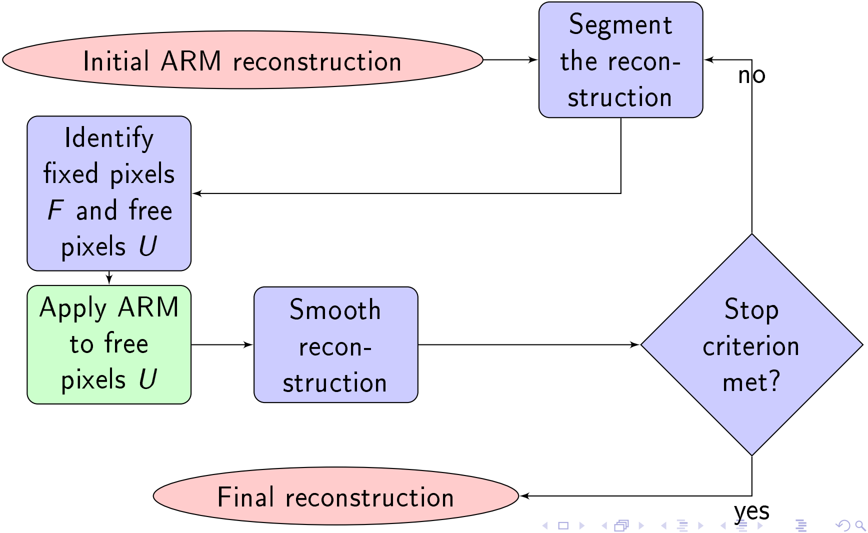

Apply ARM to Free Pixels

Original system:

$$\begin{pmatrix} | & & | \\ \mathbf{w}_{:,1} & \cdots & \mathbf{w}_{:,N} \\ | & & | \end{pmatrix} \begin{pmatrix} f_1 \\ \vdots \\ f_N \end{pmatrix} = \begin{pmatrix} p_1 \\ \vdots \\ p_M \end{pmatrix}.$$

Suppose pixel j is fixed, new system:

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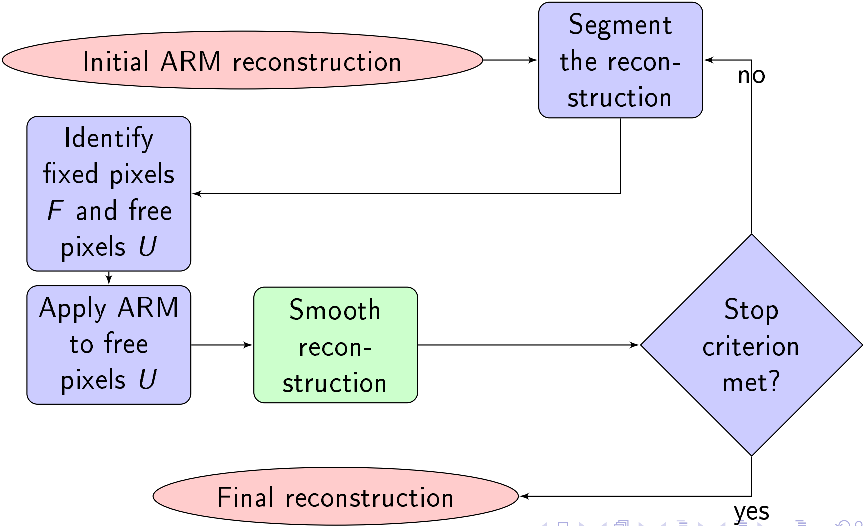

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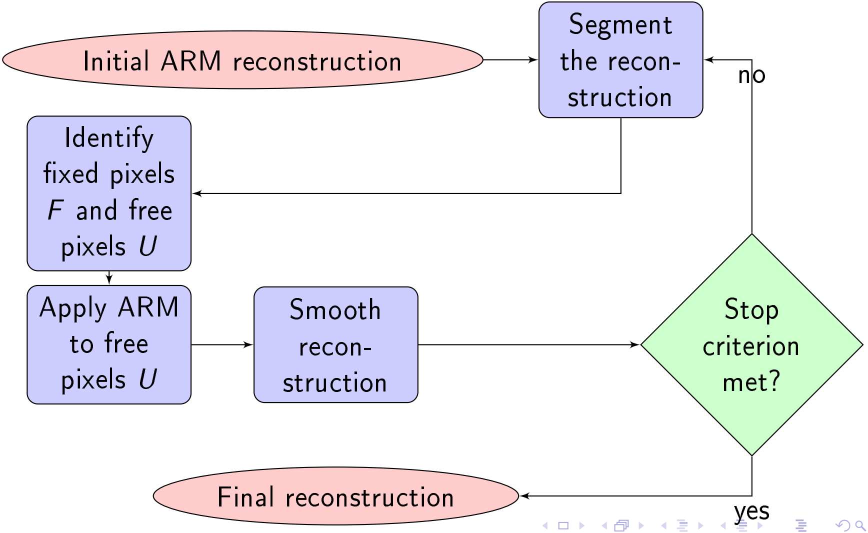

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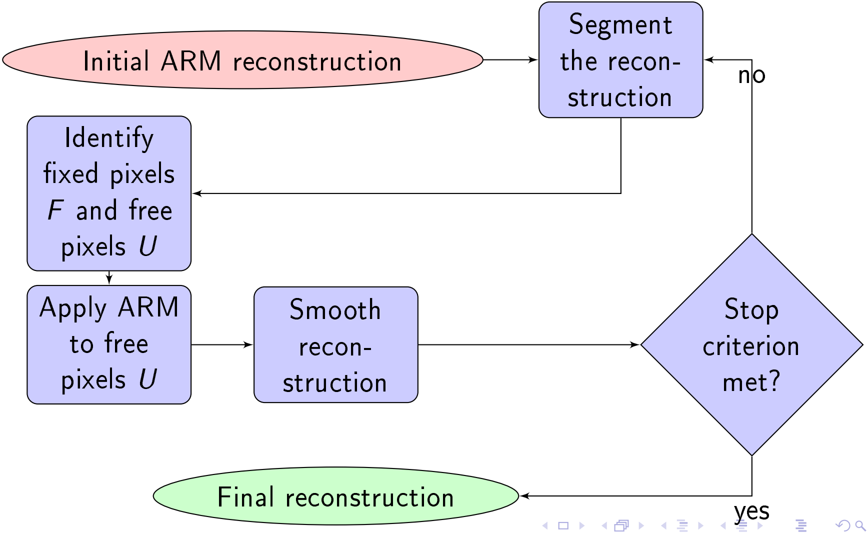
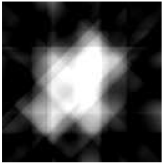


DART

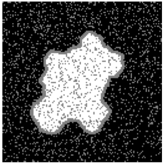
DART

Initial ARM Reconstruction



Segmentation

Free Pixels

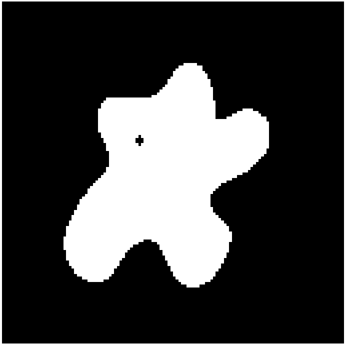


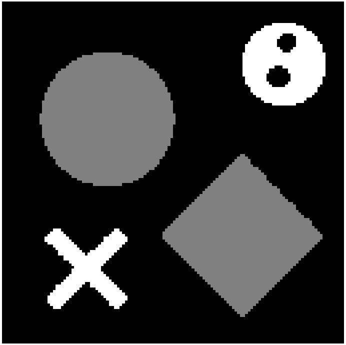
ARM on Free Pixels

Smoothed Image

Segmentation,
First DART reconstruction

DART Experiments





Layout

Research Questions

The approach of DART is rather heuristic at the moment:

- The smoothing operation;
- The random subset construct.

Can the DART algorithm be improved?

- Which algorithm should be used as ARM in DART and does it matter?
- Can better results be obtained by introducing *regularization* directly onto the set of free pixels U?
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Regularization

Regularization is the use of additional information to make an ill-posed problem well-posed.
The segmentation is a form of regularization.

Methodology

To answer the questions one could solve the system

$$\begin{pmatrix} W \\ D \end{pmatrix} \mathbf{f} = \begin{pmatrix} \mathbf{p} \\ D\mathbf{v} \end{pmatrix},$$

D diagonal matrix,
$\mathbf{v}$ vector.

Questions