Pricing options under Merton jump–diffusion dynamics with deep-learning based numerical methods

Supervisors: Shuaiqiang Liu and Cornelis Vuik

Student: Yufan Zhou

Introduction:

Pricing options can be formulated as an optimal stopping problem, for instance, in the context of Bermudan options, the holder chooses between immediate exercise and continuation at a finite set of exercise dates. Under the risk-neutral measure, the value function satisfies a dynamic programming principle in which the continuation value is a conditional expectation of future discounted payoffs. In the pure-diffusion setting, this expectation admits a partial differencial equation (PDE) characterization via the Feynman–Kac representation; Jump–diffusion dynamics provide a more realistic description of asset behavior but it leads to a partial integro-differential equation (PIDE) and an obstacle-type formulation in continuous time, which complex the siutation. In high dimensions, traditional numerical PDE methods (e.g., the finite difference method) suffer the curse of dimensionality. Monte-Carlo simulation-based algorithms [1] can be used in the case of multiple dimensional problems but may still become time-consuming in very high dimensions.

Recently, machine learning methods have emerged as practical numerical tools for high-dimensional stochastic control and optimal stopping. Related approaches include deep dynamic programming methods for optimal stopping [2], deep backward stochastic differential equation solvers [3], and reinforcement learning formulations that learn exercise policies [4], [5]. Some works have begun extending such schemes to jump dynamics by incorporating jump increments into the network inputs [6] and adapting the discrete-time backward recursion to jump–diffusion settings [7].

This MSc thesis will explore machine learning techniques to solve high-dimensional option pricing problem under the Merton jump–diffusion model [8].

Objectives:

1. Study the theoretical foundations, including dynamic programming, Ito lemma, Feynman-Kac theorem, etc.
2. Review recent deep learning-based schemes for solving optimal stopping problem, e.g., dynamic programming, deep backward stochastic differential equation, by applying Feynman-Kac theorem and deep neural networks, etc.
3. Develop and adapt the methodology to Merton jump-diffusion asset dynamics.
4. Conduct numerical experiments of pricing European/Bermudan/American options.

References:

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