Numerical Modelling of a Pulse Combustion Burner: Limiting Conditions of Stable Operation

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Summary. Numerical modelling of pulse combustors may give important guidelines on how design parameters should be chosen. This paper gives a mathematical analysis of a simple model for thermal pulse combustion and determines conditions under which this model can describe stable pulse operation.

1 Introduction

Compared to conventional combustion, pulse combustion has significant advantages in terms of thermal efficiency, energy savings and environmental impact. The high heat transfer rate makes it particularly attractive for applications such as heating, particle drying, waste incineration, etc. Areas for which industrial application of pulse combustion can be beneficial include heating, drying, calcinating, gasification, and waste incineration.

The operation of a pulse combustor is based on a coupling between intermittent (pulse) combustion and resonant acoustics in the burner system. Self-sustained pulse combustion and high-intensity sound waves result if the system's acoustics and the combustion process are in phase (i.e. if *Rayleigh's criterion* [2] is satisfied).

The pulse combustion characteristics are determined by complex interactions between physical and chemical processes, which depend on many parameters (e.g. fuel supply, mixing processes, reaction rates, tailpipe length). This severely complicates the design process.

Numerical modelling may give important guidelines on how the design parameters should be chosen in order to achieve an optimal performance of the pulse combustion process. To gain insight into the role of various design parameters, we study a simple model of a so-called thermal pulse combustor. By integrating the model equations in time it is possible to predict whether

A.D. Fitt et al. (eds.), *Progress in Industrial Mathematics at ECMI 2008*,
Mathematics in Industry 15, DOI 10.1007/978-3-642-12110-4_140,

stable pulse operation for a given set of design parameter values is possible. Such an analysis, however, is very time consuming if many combinations of design parameter values have to be considered. In order to make a computationally less demanding analysis we perform a stability analysis on the model equations. We will show that the stability analysis provides insightful information by comparing it with the results of a time integration analysis.

2 Thermal Pulse Combustion: A Mathematical Model

Richards et al. [3] introduced a mathematical model that describes pulse combustion in a system with a continuous fuel supply, which they call *thermal pulse combustion*. Figure 1 gives a schematic representation of such a thermal pulse combustor. Thermal pulse combustion is different from ordinary pulse combustion, where fuel periodically enters the combustion chamber because of time-dependent pressure differences over valves. However, Richards et al. show that pulsating combustion can occur even in the case of a continuous fuel supply.

Richards et al. model this device with a simple lumped parameter model, taking the combustion chamber as a control volume. The amount of energy in the combustion chamber is changed by inflow of reactants, combustion, outflow of combustion products, and heat transfer to the chamber wall. The combustion process is modelled by a one-step Arrhenius law for a bimolecular reaction between fuel and oxidizer. The gases in the combustion chamber are assumed to be well-stirred. The tailpipe flow is modelled as a plug flow, i.e. with a uniform density over its volume and driven by the pressure difference over the tailpipe. Flow from the combustion chamber into the tailpipe is assumed to be isentropic. Wall friction of the tailpipe gases is taken into account. It is assumed that the gases are perfect, and that all mixtures of reactants and products have the same (constant) specific heats. By applying conservation of mass, energy and species to the control volume, and coupling this to the tailpipe dynamics by conservation of momentum, Richards et al. derived a system of four ordinary differential equations. It can be written as

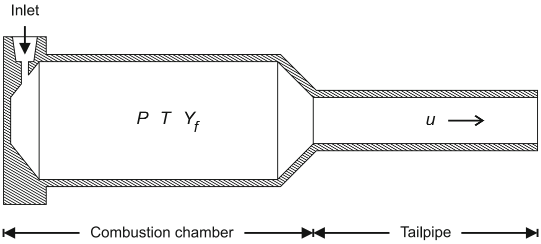

Fig. 1. Control volumes and variables of the thermal pulse combustor model

$$\frac{dP}{dt} = \gamma(A + B \cdot RR + CD - (C + GZ_e)T), \tag{1}$$

$$\frac{dT}{dt} = \gamma(A + B \cdot RR + CD)\frac{T}{P} - (A + \gamma C + (\gamma - 1)GZ_e)\frac{T^2}{P}, \tag{2}$$

$$\frac{du}{dt} = E(P_e - 1)\,\frac{T_e}{P_e} - F\,u\,|u|, \tag{3}$$

$$\frac{dY_f}{dt} = (A(Y_{f,i} - Y_f) - RR)\frac{T}{P}. \tag{4}$$

where RR, P_e, T_e and Z_e are functions of P, T, u and Y_f, and $A, \ldots, G$, γ and $Y_{f,i}$ are constants that depend on the system's design parameters and the fluid properties. The (non-dimensionalized) variables are: the pressure (P), temperature (T), and fuel mass fraction (Y_f) in the combustion chamber and the fluid velocity (u) in the tailpipe. The system of equations can be expressed in vector form by

$$\frac{d\mathbf{y}}{dt} = \mathbf{f}(\mathbf{y}), \quad \text{where} \quad \mathbf{y} = (P, T, u, Y_f)^\top.$$

3 Parameter Study

3.1 Numerical Time Integration

Through numerical time integration it is possible to study the stability of the combustion process for given sets of parameters.

Figure 2 shows how the pressure evolves in time for three different values of the wall temperature (T_w) of the combustion chamber. The top figure shows steady combustion for T_w = 1,200 K. The middle figure shows pulse combustion for T_w = 1,000 K. The bottom figure shows that flame extinction occurs for T_w = 750 K.

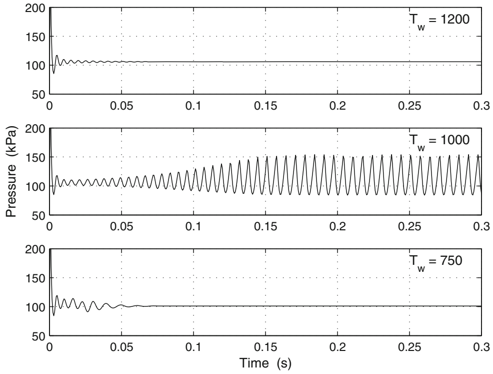

Fig. 2. Pressure signals in combustion chamber from numerical simulations for three wall temperatures

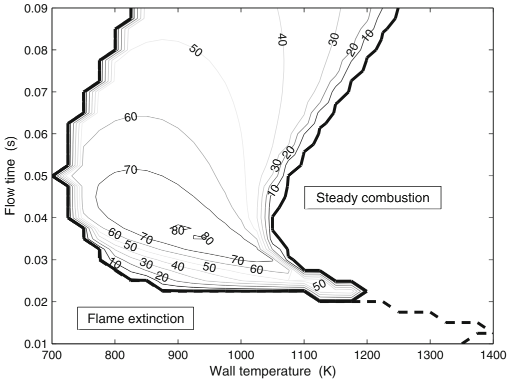

Fig. 3. Peak-to-peak pressure amplitudes (in kPa) obtained from numerical simulation for various combinations of wall temperature and flow time

Figure 3 shows how the peak-to-peak amplitude of the pressure oscillations depends on the wall temperature T_w and the flow time τ_f, which is inversely proportional to the fuel mass inflow rate. The figure indicates for which combinations of these two parameters stable pulse operation occurs.

3.2 Stability Analysis

This two parameter analysis is already very time consuming. In order to explore the parameter space in an insightful and computationally inexpensive way, we perform a stability analysis of the steady-state solutions. These are found by solving $\mathbf{f}(\mathbf{y}) = \mathbf{0}$.

Several steady-state solutions may exist for any given set of parameter values. Figure 4 shows the steady-state temperature(s) for given wall temperatures. Clearly either one, two, or three steady-state solutions exist, depending on the wall temperature.

The stability of the steady states can be determined by calculating the four eigenvalues of the corresponding Jacobian matrices. If all four eigenvalues have a negative real part, the steady state is stable and no pulse combustion can occur in its neighbourhood.

Figure 5 shows the largest of the real parts of the eigenvalues corresponding to the steady-state solution with the highest temperature. It indicates for which pairs of the parameters T_w and τ_f the steady-state solution is stable or unstable. The boundary for flame extinction results from bifurcation of the steady state, while the boundary for steady combustion results from a sign change of the real part of the eigenvalue. Note the qualitative correspondence with Fig. 3.

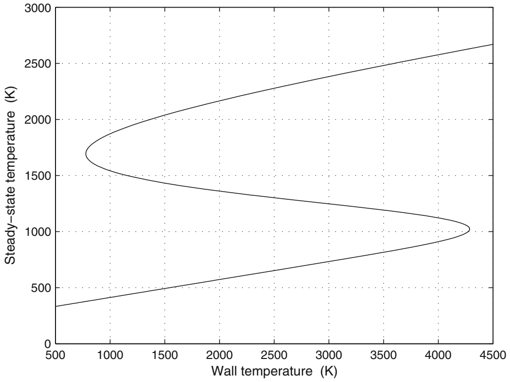

Fig. 4. Steady-state temperatures as function of the wall temperature

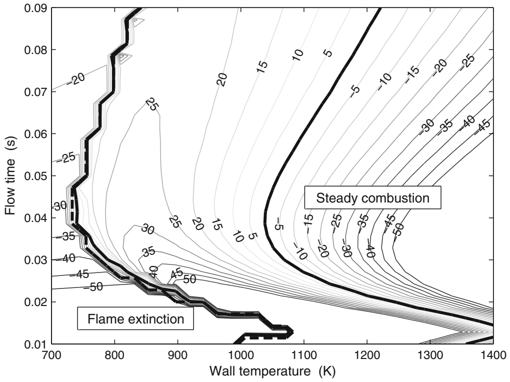

Fig. 5. Largest of the real parts of the four eigenvalues corresponding to the steady-state solution with the highest temperature

4 Concluding Remarks

4.1 Discussion

The stability analysis provides a useful tool for the parameter study. For state variables close enough to a stable steady state, the stability analysis gives us definitive information that pulse combustion is not possible. In general, the stability analysis gives a strong indication for which system parameters pulse combustion is not possible. For other system parameter values the stability analysis is not conclusive. Figure 3, for example, shows regions where flame extinction occurs, while the corresponding regions in Fig. 5 show unstable steady states. However, the stability analysis always gives good guidelines to determine which values of the design parameters should be investigated further for obtaining stable pulse operation.

4.2 Validity

The model in [3] describes *thermal* pulse combustion. The model can be adapted for valved pulse combustion, see for example [1] and [4], but then only one steady state exists: flame extinction. The model of Richards et al. also has serious limitations, see our analysis in [6]. Despite its limitations, experiments with different heat release rates in the model, and extending it with variable air/fuel ratio and stochastic noise, suggest that good agreement with experimental data for valved combustion can be obtained.

4.3 Future Research

The stability analysis has provided useful insight into the behaviour of a pulse combustor as modelled by Richards et al. As a next step in our research, we want to extend this stability analysis to more advanced models, and we also hope to gain more insight into its relation with Rayleigh's criterion. Furthermore, the model of Richards et al. can be improved by including additional physics (non-stoichiometric and pressurized combustion, combustion noise). Finally, we want to use the resulting model to study chaotic behaviour in pulse combustion operation, as for example in [5].

Acknowledgement

The authors thank H. Corstens for his stimulating comments on many aspects of this work. This joint research project of DLF Sustainable and the Delft University of Technology is sponsored by SenterNovem.

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