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EFFICIENT COMPUTATION OF FLOW WITH CAVITATION BY COMPRESSIBLE PRESSURE CORRECTION

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Abstract. *We describe the application of a staggered scheme with a compressible pressure correction method to the computation of flow with cavitation modeled by the homogeneous equilibrium model, resulting in a nonconvex hyperbolic system. Adaptation of the standard pressure correction algorithm to improve the stability for fully compressible flow is discussed. Results are presented for a flow around a two dimensional hydrofoil, in which the Mach number varies between* 10^{-3} *and* 30.

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1 HOMOGENEOUS EQUILIBRIUM MODEL FOR FLOW WITH CAVITATION

A rather simple way of modeling liquid/vapor two phase flow is the homogeneous equilibrium model (HEM). Based on the assumption of thermodynamic equilibrium and the neglect of velocity slip between both phases it is possible to derive single phase equations for the two-phase mixture completed with a mixture equation of state. The equation of state $p = p(\rho)$ makes the density of the mixture equal to the density of the liquid phase when the pressure is above the vapor pressure, and equal to the density of the vapor phase below the vapor pressure, with a smooth artificial transition in between. When the pressure of the mixture is either well below or above the vapor pressure, the speed of sound is large but finite and the flow will be weakly compressible. In the phase transition region the speed of sound will retain a very small value of $\mathcal{O}(1$ m/s). In practical computations this means that the Mach number will vary from 10^{-3} to an artificial value well in the range of 10-30. For efficient computation of two phase flow, with the HEM it is therefore required that the time-integration method is accurate and stable uniformly in the Mach number for $0 < \mathrm{Ma} < 30$.

2 UNSTEADY SHEET CAVITATION

The HEM has been applied to model unsteady sheet cavitation on hydrofoils in [2, 3, 5, 7, 9, 12, 13, 16]. Unsteady sheet cavitation is the formation of thin vapor filled pockets on the suction side of hydrofoils, when the pressure at the leading edge of the foil drops below the vapor pressure. Unsteady sheet cavitation is a cyclic process, that is illustrated in Figure 1.

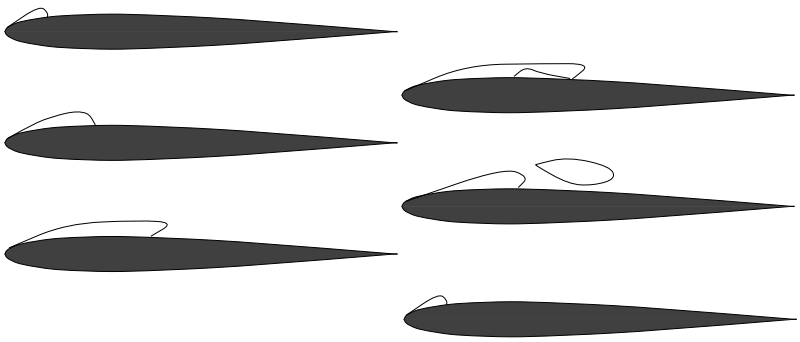


Figure 1: Cyclic behavior of unsteady sheet cavitation on hydrofoil.

In each cycle the cavity grows from its initial length to a maximal length. Meanwhile a re-entrant jet develops at the aft end of the cavity, that moves forward and upward. At a certain instant the forward moving re-entrant jet touches the upper liquid/vapor interface and the aft part of the cavitation bubble is shed. The periodical behavior commences after a large number of cycles has occured, starting from the initial condition. Therefore

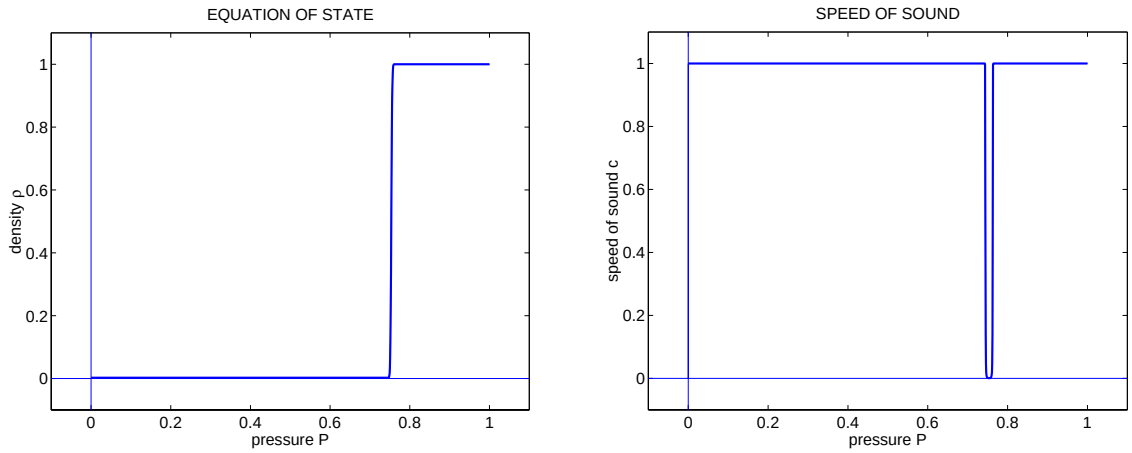
a time-integration scheme is required that allows for large time-steps to bridge this initial phase efficiently.

3 GOVERNING EQUATIONS

Under the assumption of inviscid isothermal flow, the application of the HEM leads to the standard single phase Euler equations for the mixture and a barotropic mixture equation of state:

$$\frac{\partial \rho}{\partial t} + \frac{\partial m}{\partial x} = 0,$$
$$\frac{\partial m}{\partial t} + \frac{\partial}{\partial x}\left(\frac{m^2}{\rho}\right) = -\frac{\partial p}{\partial x},$$
$$\rho = \rho(p).$$

The equation of state and the speed of sound as a function of pressure are shown schematically in Figures 2a and 2b. It is to be noted that the equation of state is nonconvex, so that the governing equations consitute a nonconvex hyperbolic system.



(a) The equation of state.

(b) Speed of sound c as function of pressure.

Figure 2: Properties of the liquid/vapor mixture.

4 DISCRETISATION

4.1 Current trends

A number of different methods are currently used for discretisation of the equations for the HEM:

- MacCormack [11],

- Real gas extensions of flux splitting schemes: AUSM+ [4],
- Real gas extensions of approximate Riemann solvers: Roe [14],
- Jameson type schemes [5],
- Staggered schemes [3],

In the last years a number of approximate Riemann solvers and flux vector splitting schemes, like the Roe and the AUSM+ scheme have been extended to handle the VanderWaals equation of state for real gases. This equation of state is very similar to the equation of state of the HEM. The difference lies in the fact that the variation in density is much smaller for the VanderWaals equation of state. In [17] an extension of the Osher scheme to handle a general equation of state is presented. In this case the fluxes have to be numerically evaluated from integral relations, rendering the scheme computationally much more expensive than for the perfect gas case.

Another choice is a Jameson type scheme. This can be trivially extended for the HEM, because no use is made of the complicated eigen-structure due to the nonconvex equation of state. The difficulty lies in the adjustment of the artificial viscosity parameters for a very large range of Mach numbers and shocks an order of magnitude stronger than generally encountered in aerospace engineering.

Altough these schemes are well suited for applications where the freestream Mach number is $\mathcal{O}(1)$, e.g. cavitation in fuel line nozzles or in the wake of detonation driven underwater projectiles [11], a major drawback of these schemes is the behavior for low Mach number flow. To handle the stiffness of the system for $\mathrm{Ma} << 1$, some form of artificial compressibility/time-derivative preconditioning is needed, that will destroy the temporal accuracy of the scheme. Because in this case the solution is time-dependent, each physical time-step has to be solved for by stepping in pseudo-time until steady state. The latter makes the use of both types of schemes computationally expensive, the more so because there is still limited knowledge of optimal preconditioners for the HEM.

As opposed to the last two schemes, the staggered discretisation of [1], combined with a compressible pressure correction algorithm, has computational cost and accuracy uniform in the Mach number. Altough this approach may be more costly to apply when the Mach number allows the use of the standard compressible schemes, in this application the bulk of the computing time is incurred in the liquid phase with a low Mach number of $\mathcal{O}(0.001)$. Of course it makes sense to choose a method that treats the large low Mach number region very efficiently and the very small high Mach number region a little less efficiently, than the other way around.

4.2 The staggered scheme

For sake of brevity we will discuss only the one-dimensional case. We use a staggered placement of the unknowns, illustrated in Figure 3. Application of a first order upwind

scheme in the momentum and mass conservation equation leads to the following discretisation:

$$\frac{m_{j+\frac{1}{2}}^{n+1}-m_{j+\frac{1}{2}}^{n}}{\delta t}+\frac{1}{\delta x}\left(u^n m^{n+1}\right)\Big|_{j-\frac{1}{2}}^{j+\frac{1}{2}}=\frac{1}{\delta x}\,p^{n+1}\Big|_{j}^{j+1},$$
$$\frac{\rho_j^{n+1}-\rho_j^n}{\delta t}+\frac{1}{\delta x}\left(\frac{2\rho_j^{n+1}}{\rho_j^{n+1}+\rho_{j+1}^{n+1}}m_{j+\frac{1}{2}}^{n+1}-\frac{2\rho_{j-1}^{n+1}}{\rho_{j-1}^{n+1}+\rho_j^{n+1}}m_{j-\frac{1}{2}}^{n+1}\right)=0. \tag{1}$$

Von Neumann stability has shown the necessity of the application of the density upwind bias in the mass conservation to retain stability for fully compressible flow. The pressure gradient is centrally discretised irrespective of the Mach number.

5 SOLUTION PROCEDURES

5.1 Compressible pressure correction (CPC)

The solution procedure we use is a modification of the compressible pressure correction algorithm [1] for a perfect gas. Based on Von Neumann analysis the original time-stepping method was modified to have more favorable stability properties for highly compressible flow. The solution procedure consists of the following 4 stages:

1. solve density predictor equation

$$\frac{\rho_j^*-\rho_j^n}{\delta t}+\frac{1}{\delta x}\left(u_k^n\rho_{k-\frac{1}{2}}^*\right)\Big|_{k=j-\frac{1}{2}}^{k=j+\frac{1}{2}}=0, \tag{2}$$

2. solve momentum predictor equation

$$\frac{m_{j+\frac{1}{2}}^*-m_{j+\frac{1}{2}}^n}{\delta t}+\left(\left(u^n+\frac{m^n}{\rho^*}\right)m^*-u^n m^n\right)\Big|_{j-\frac{1}{2}}^{j+\frac{1}{2}}=-\,p^n\big|_j^{j+1}, \tag{3}$$

3. solve pressure correction equation

$$\frac{\rho(p_j^n+\delta p_j)-\rho_j^n}{\delta t}+\frac{1}{\delta x}\left(s_k^{n+1}\left(m_k^*-\frac{1}{\delta x}\left(\delta p_{k+\frac{1}{2}}-\delta p_{k-\frac{1}{2}}\right)\right)\right)\Big|_{k=j-\frac{1}{2}}^{k=j+\frac{1}{2}}=0, \tag{4}$$
$$s_{j+\frac{1}{2}}^{n+1}=\frac{2\rho(p_j^n+\delta p_j)}{\rho(p_{j+1}^n+\delta p_{j+1})+\rho(p_j^n+\delta p_j)}.$$

4. update the tentative momentum and old pressure field:

$$m_{j+\frac{1}{2}}^{n+1} = m_{j+\frac{1}{2}}^*-\delta t\left(\delta p_{j+1}-\delta p_j\right) \tag{5}$$
$$p_j^{n+1} = p_j^n+\delta p_j \tag{6}$$

Some remarks concerning this method are:

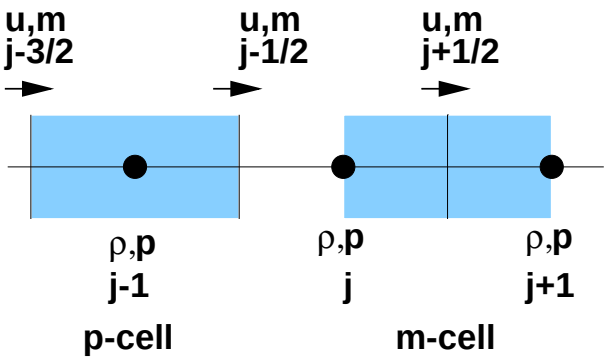


Figure 3: Staggered placement of unknowns

- Newton linearisation is used for the convective terms in the momentum equation.
- A Crank-Nicolson type scheme is used for the velocity in the convective terms in the pressure correction equation (4). This gives a more favorable spectrum of the eigenvalues of the time-integration methods, resulting in increased damping in the high frequency modes, enhancing (almost) unconditional stability for $0 < \mathrm{Ma} < 30$ [15].
- The pressure correction equation is solved by a nonlinear Gauss-Seidel method accelerated by intermediate linearized steps with preconditioned GMRES. Due to the density upwind bias together with the spatial variations in the speed of sound, the Jacobian of the pressure correction equation is far from diagonally dominant.
- The Newton linearized formulation of the convective terms enables extension of the method to second order temporal accuracy, with the θ-method [15].
- Higher order spatial discretisation are achieved by introducing intermediate deferred/ defect correction steps.

5.2 Compressible pressure correction and SIMPLE

In the SIMPLE method in its generic form the following set of equations is solved until convergence of $\| \left(p^{(k+1)} - p^{(k)}\right) \|$:

$$\frac{m^{*(k+1)} - m^n}{\delta t} + \left(u^{(k)} m^{*(k)}\right) = -p_x^{(k)} \tag{7}$$

$$\frac{(\rho(p^{(k+1)}) - \rho(p^{(k)})}{\delta t} - \delta t \left(p^{(k+1)} - p^{(k)}\right)_{xx} = -m_x^{*(k+1)} - \frac{(\rho(p^{(k)}) - \rho(p^n)}{\delta t} \tag{8}$$

$$m^{(k+1)} = m^{*(k+1)} - \delta t \left(p^{(k+1)} - p^{(k)}\right)_x \tag{9}$$

as opposed to the single step solution procedure of CPC.

The main difference between the pressure correction method and SIMPLE lies in the possibility to iteratively solve a nonlinear scheme with a large stencil implicitly. Within

the framework of compressible pressure correction we distinguish between the *target*, *actual* and *resolved* discretisation. The *target* discretisation is obtained after finite volume discretisation, can be nonlinear, and can only be solved for in an iterative manner. The *actual* discretisation follows from the *target* discretisation after linearisation, and the introduction of further approximations, such as deferred or defect correction. Finally, the *resolved* discretisation includes the segregated (pressure correction) solution procedure.

Altough SIMPLE and compressible pressure correction have the same *target* discretisation, the *resolved* discretisation is totally different. In the former case, due to the iterative nature of the algorithm, the *target* and *resolved* discretisation are identical, whereas in the latter case this is definitely not the case. The stability properties of the method are determined by the stability properties of the resolved discretisation.

In practice this means that the CFL-condition for the compressible pressure correction algorithm is more severe than for SIMPLE. However, the application of SIMPLE is much more expensive than CPC, requiring multiple accurate solutions of the nonlinear pressure correction equation.

For this time-dependent application it was found that the CPC algorithm is much more efficient, the more so because considerable underrelaxation has to be applied to the SIMPLE algorithm to achieve convergence within each iteration.

6 RESULTS

For high to moderate values of the cavitation number σ defined as:

$$\sigma = \frac{p_\infty - p_{\text{vapor}}}{\frac{1}{2}\rho_\infty V_\infty}$$

the cavitation bubble on a hydrofoil will remain steady. For this simulation the momentum and mass conservation equations are discretised with the third order ISNAS-scheme [18], applied in a deferred correction manner. We have chosen the density ratio $\rho_{\text{liquid}}/\rho_{\text{vapor}} = 100$.

Figure 4 shows the density distribution on a NACA66 hydrofoil, as computed with the current method. Darker shading corresponds to lower density. The cavitation bubble is captured as a low density region . In Figure 4 the density varies between 10 (liquid phase)

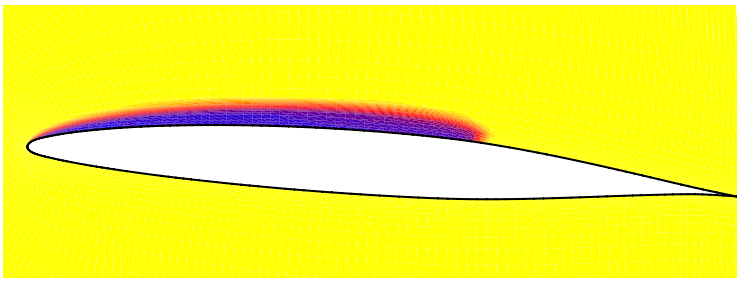


Figure 4: NACA66, $\alpha = 4^o$, $\sigma = 0.87$

and 0.1 (vapor phase). The cavity extend is predicted with an accuracy, comparable to the accuracy as obtained by interface capturing methods.

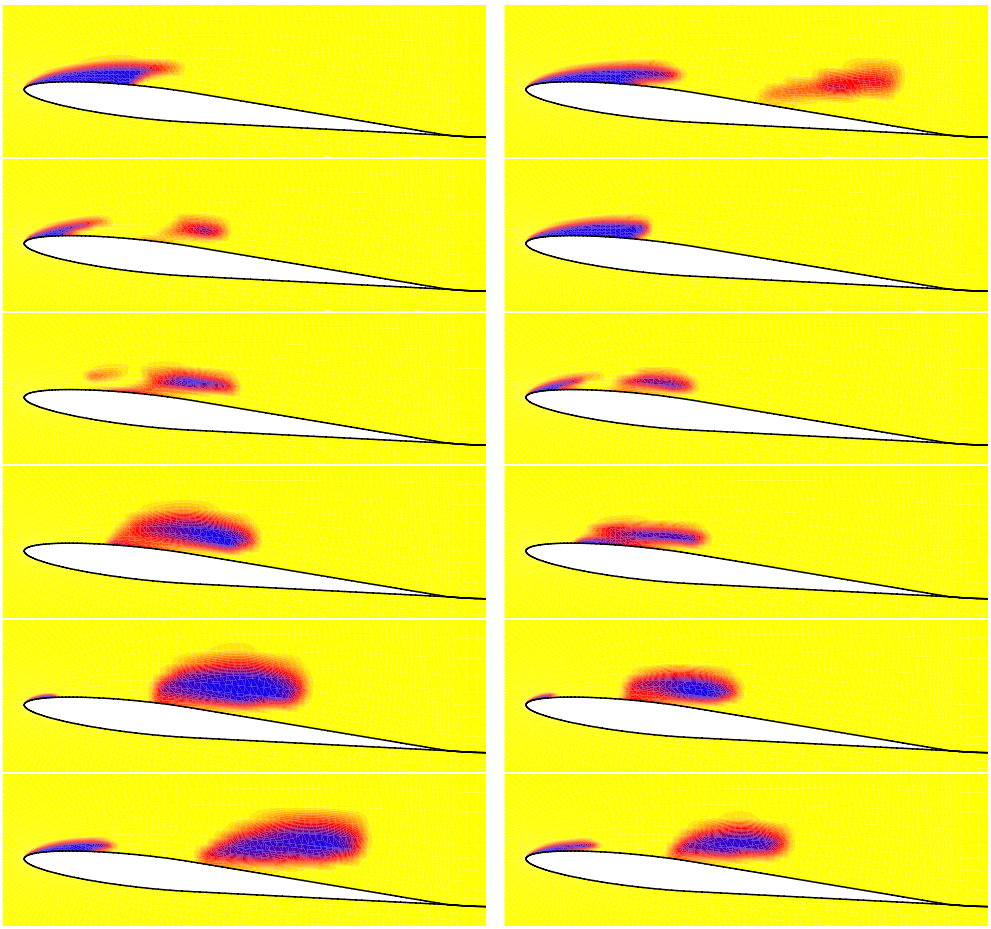


Figure 5: EN-wing, $\alpha = 6.2^o$, $\sigma = 1.2$

For a larger angle of attack or lower σ the cavity becomes unsteady and shows the behavior as discussed in Section 2. In Figure 5 results are shown for the EN-hydrofoil, that has been studied extensively experimentally in [6] and numerically in [8]. In [8] σ was lowered from the experimental value of $\sigma = 1.2$ to $\sigma = 1.1$ so that the numerically predicted average cavity length matched the experimentally found value. We have retained $\sigma = 1.2$. Figure 5 shows in a sequence of snapshots of the density field, how a vapor cloud is convected downstream and collapses, after the aft part of the fixed cavity has been shed. To obtain these unsteady results use was made of a first order upwind scheme, for both the mass conservation equation and the momentum equation. The results show good agreement with the experimental results of [6], with respect to shedding frequency and average cavity length.

7 CONCLUSIONS

The Homogeneous Equilibrium Model for two phase flow is a numerically demanding application due to the simultaneous occurrence of almost incompressible ($\mathrm{Ma} < 0.001$) as well as highly compressible ($\mathrm{Ma} = 10 - 20$) regions in the flow domain. Standard schemes for compressible flow will have to be preconditioned to handle the weakly compressible flow of the liquid phase. Standard schemes for weakly compressible flow are either inefficient for time-dependent flow or suffer from a severe stability induced restriction on the time-step for highly compressible flow. Based on Von Neumann analysis a noniterative pressure correction time-stepping method is developed that has stability properties almost uniform in the Mach number. The method has been applied to the computation of steady and unsteady sheet cavitation on hydrofoils.

8 FUTURE RESEARCH

To further verify the results we intend to adapt the method for axi-symmetric flows to compare with the experimental results in [10]. Furthermore, an energy equation will be incorporated to be able to handle cavitating flow of cryogenic fluids, where the strong dependence of the vapor pressure on the local temperature has to be taken into account to correctly predict the extent of the cavitating region.

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