

Convergence of Newton's Method for Steady-State Load Flow Problems in Multi-Carrier Energy Systems

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Abstract—Coupling single-carrier networks into multi-carrier energy systems (MESs) has recently become more important. Various formulations of the single-carrier load flow problem (LFP) are used. Moreover, different coupling models lead to different integrated systems of equations for the LFP of MESs. Both could affect the convergence of the Newton-Raphson method (NR) used to solve the nonlinear system of equations. This paper considers the steady-state LFP for example MESs of varying size, with various coupling models and topologies, and various formulations in the single-carrier parts. Based on numerical experiments, this paper compares the convergence behavior of NR for the various single- and multi-carrier systems. For these examples, NR of the steady-state LFP of the MESs is independent of the size of the network and of the coupling, and NR requires at most as many iterations as the slowest single-carrier network.

Index Terms—Integrated energy systems, Load flow analysis, Multi-carrier energy networks, Numerical analysis, Power flow analysis

I. INTRODUCTION

Multi-carrier energy systems (MESs) have become more important over the years, as the need for efficient, reliable and low carbon energy systems increases. In these energy systems, different energy carriers, such as electricity and heat, interact with each other leading to one integrated energy network. An important tool for the design and operation of energy systems is steady-state load flow analysis. Load flow models for single-carrier networks (SCNs) have been widely studied, but only recently load flow models for MESs have been proposed. For instance, in [1] and [2] the energy hub concept (EH) is used, in [3]–[5] a more ad hoc approach, and in [6] a graph-based framework. All these approaches lead to one integrated system of nonlinear equations. Generally, both for the SCNs and for the MESs, the Newton-Raphson method (NR) is used to solve the system of nonlinear load flow equations.

However, different couplings lead to different integrated systems of equations for the load flow problem (LFP). Moreover, various formulations of the single-carrier (SC) load flow equations are used (e.g. [7] for gas). To the best of the authors knowledge, the effect of different couplings, and of different formulations of the systems of equations, on the convergence

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behavior of NR for the integrated system of equations has not been discussed.

In this paper, we investigate the effect of coupling and the effect of the formulation of the LFP in the SC parts, on the convergence behavior of NR for the steady-state LFP of MESs. For the SCNs, we consider two versions of the flow equation in a pipe combined with two formulations of the LFP in the gas network, and two types of boundary conditions (BCs) for a load combined with two formulations of LFP in the heat network. Furthermore, various components, models, and topologies for coupling are used to combine the SCNs into a multi-carrier network (MCN).

We use a graph-based formulation for general MESs [6], to solve the steady-state LFP for several example MESs. We consider a base network and a larger extended network, both consisting of gas, electricity, and heat. Four different topologies for coupling are used. Based on numerical experiments for the example MESs, we compare the convergence behavior of NR between the MESs and the SCNs, between the different coupling models, and between the different formulations of the LFP for the SCNs.

II. LOAD FLOW EQUATIONS

Energy systems are represented by a network, which is a collection of nodes, connected by (directed) links. Flow enters or leaves the network through sources or sinks, both called loads, represented by terminal links. For steady-state load flow, the variables and the elements they are associated with are given in Tab. I. Conservation of energy holds for all the SC nodes. All SC links representing a physical component have a link equation that relates link and nodal variables. We use the same load flow equations as used in [6], with the following changes. All equations and variables are in SI units.

A. Single-carrier networks

For a transmission line in the electrical network, represented by a link k from node i to node j , we use a π -line model (e.g. [8]).

In a gas network, the general steady-state flow equation of a pipe [7], represented by a link k from node i to node j , can either express the link flow as a function of pressures, denoted as $f^q(\Delta p)$, or express the pressure drop as a function

TABLE I
VARIABLES FOR A GAS, HEAT, AND ELECTRICAL NETWORK.

Network	Node	Link	Terminal node
Gas	pressure p^g	flow q	injected flow q
Heat	pressure p^h supply temperature T^s return temperature T^r	flow m	injected flow m outflow temperature T^o heat power φ
Electricity	voltage V	current I	injected current I injected complex power S

of link flow, denoted by $f_k^{\Delta p(q)}$. Defining the pressure drop by $\Delta p_k^g := (p_i^g)^2 - (p_j^g)^2$, we have

$$f_k^{g(\Delta p)} := q_k - C_k^g \text{sign}(\Delta p_k^g) (f_k^g)^{-\frac{1}{2}} |\Delta p_k^g|^{\frac{1}{2}} = 0 \quad (1a)$$

$$f_k^{\Delta p(q)} := \Delta p_k^g - (C_k^g)^{-2} f_k^g |q_k| q_k = 0 \quad (1b)$$

with q_k is the link flow, p_i^g the nodal pressure, f_k^g the friction factor, and C_k^g the pipe constant.

In a heat network, a heat source or sink, represented by a terminal link l connected to node i , has a heat power equation $C_p m_{i,l} \Delta T_{i,l} - \varphi_{i,l} = 0$, with a temperature difference defined by:

$$\Delta T_{i,l} := \begin{cases} T_i^s - T_{i,l}^o & , \text{ if } l \text{ is a sink} \\ T_{i,l}^o - T_i^r & , \text{ if } l \text{ is a source} \end{cases} \quad (2)$$

Here, $\varphi_{i,l}$ is the heat power, $m_{i,l}$ is the mass flow, $T_{i,l}^o$ is the outflow temperature of the load, and C_p the specific heat of the water. We assume that a heat node can have only sink or only source terminal links connected to it, such that we can call the node a sink or a source respectively.

B. Coupling

The SCNs are connected through a coupling node to form a MCN [6]. The coupling variables are associated with the (terminal) links connected to the coupling node. The coupling node can represent various coupling units, and can be used with various models. We consider a combined heat and power plant (CHP), and a gas boiler (GB) combined with a gas-fired generator (GG). For the models, we use a linear model for the CHP and the GB, a linear model for a basic GG and a nonlinear model for a gas-fired generator with valve-point effect (GG VP). Since all these units convert gas to electricity and heat, we also consider the energy hub (EH) concept as coupling model [1]. Every coupling node has at least one coupling equation. Furthermore, every coupling unit that produces heat has a heat power equation.

For the linear coupling models, we use $\varphi^c = \eta_{\text{GB}} \text{GHV} q^c$ for the GB, $\text{GHV} q^c = (\eta_{\text{CHP}}^{ge})^{-1} P^c + (\eta_{\text{CHP}}^{gh})^{-1} \varphi^c$ for the CHP, and $P^c = \eta_{\text{GG}} \text{GHV} q^c$ for the GG, with GHV the gross-heating value of the gas, and η the efficiency. For the nonlinear model of the GG, we use the model as stated in [3].

For the EH we use $P^c = c^{ge} \text{GHV} q_c$ and $\varphi^c = c^{gh} \text{GHV} q_c$, where we take $c^{ge} = \frac{1}{2} \eta_{\text{GG}}$ and $c^{gh} = \frac{1}{2} \eta_{\text{GB}}$, to model a GG and a GB as an EH.

III. SYSTEM OF EQUATIONS

Typically, collecting all the load flow equations for the SCNs still leaves more variables than equations. Therefore, some variables are assumed known, which we will call the boundary conditions (BCs) of the network. A node type is assigned to every node based on the known variables. Moreover, the coupling models generally introduce more variables than equations, such that additional BCs are needed for the MCN [6]. The load flow equations and the BCs can be combined in various ways to form the system of equations for the SCNs and MCNs. We consider three formulations in the gas network, and four in the heat network.

A. Various Formulations

For the gas network, we consider the nodal formulation [7], and the full formulation. In the nodal formulation, the link equations (1a) are substituted in conservation of mass. In the full formulation, the link equations (either (1a) or (1b)) are not substituted, and the link flows are added as unknowns.

For the heat network, we use two different types of BCs for the load nodes, and two different formulations of the hydraulic-thermal system of equations, giving a total of four formulations. For the BCs at the load nodes, $\varphi_{i,l}$ and either $T_{i,l}^o$ or $\Delta T_{i,l}$ are assumed known. The first formulation, which we call the terminal link formulation, does not substitute terminal flows. If $\Delta T_{i,l}$ is known for loads, $T_{i,l}^o$ is added as a variable, and (2) is added to the system of equations. The system of nonlinear equations for the terminal link formulation is given by:

$$\begin{aligned} \mathbf{F}^h &= (\mathbf{F}^m \quad \mathbf{F}^L \quad \mathbf{F}^{\text{T}^s} \quad \mathbf{F}^{\text{T}^r} \quad \mathbf{F}^\varphi \quad \mathbf{F}^{\Delta \text{T}})^T = \mathbf{0}, \\ \mathbf{x}^h &= (\mathbf{m}^L \quad \mathbf{m}^{\text{T}^L} \quad \mathbf{p}^h \quad \mathbf{T}^s \quad \mathbf{T}^r \quad \mathbf{T}^o)^T \end{aligned} \quad (3)$$

with \mathbf{m}^L , \mathbf{m}^{T^L} , \mathbf{p}^h , \mathbf{T}^s , \mathbf{T}^r , and \mathbf{T}^o the vectors of unknown link mass flows, terminal mass flows, nodal pressures, supply and return temperatures, and terminal outflow temperatures, and with \mathbf{F}^m , \mathbf{F}^L , \mathbf{F}^{T^s} , \mathbf{F}^{T^r} , \mathbf{F}^φ , and $\mathbf{F}^{\Delta \text{T}}$ the vectors of conservation of mass, link equations, supply line mixing rules, return line mixing rules, heat power equations, and temperature difference equations.

The second formulation, which we call the standard formulation, is generally used (see e.g. [5] or [6]). This system of equations is smaller than (3), but \mathbf{F}^m is now nonlinear, and the supply line mixing rule \mathbf{F}^{T^s} depends on \mathbf{T}^r and vice versa.

For the electrical network we use the commonly used complex power formulation in polar coordinates (e.g. [8]).

For the coupling part, we use the system of equations as stated in [6]. Note that assuming one or more of the coupling (energy) flows known effectively decouples the system of equations, such that there is no need to model the system as one integrated system. Therefore, we assume P^c , Q^c , q^c , φ^c and m^c unknown. However, if a coupling unit produces heat, $T^{o,c}$ or ΔT^c can be assumed known as a BC. Additionally required BCs must be imposed in the SC parts of the network.

The load flow equations for the SCNs and the coupling part can be combined into one system of nonlinear equations:

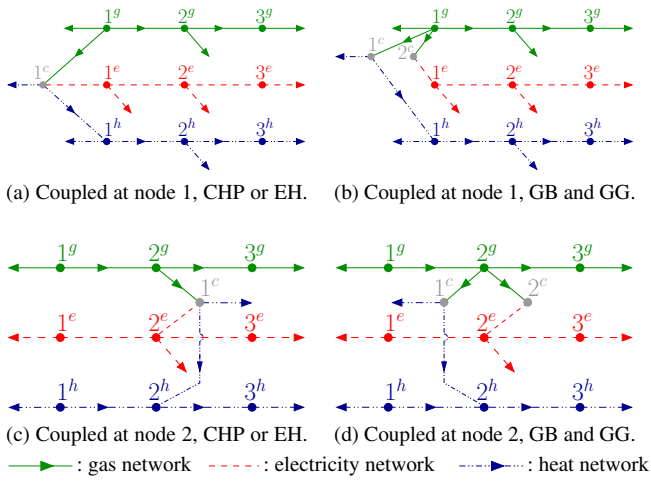


Fig. 1. MES network topology. Arrows show defined direction of flow.

$$\mathbf{F} = (\mathbf{F}^g \quad \mathbf{F}^e \quad \mathbf{F}^h \quad \mathbf{F}^c)^T = \mathbf{0},$$

$$\mathbf{x} = (\mathbf{x}^g \quad \mathbf{x}^e \quad \mathbf{x}^h \quad \mathbf{x}^c)^T \quad (4)$$

with \mathbf{F}^g , \mathbf{F}^e , \mathbf{F}^h , \mathbf{F}^c and \mathbf{x}^g , \mathbf{x}^e , \mathbf{x}^h , \mathbf{x}^c the system of equations and variables for the gas, electricity, heat, and coupling part of the MES, respectively.

B. Newton-Raphson

We scale the integrated system of equations (4) by matrix multiplication, and solve this scaled system using the Newton-Raphson method (NR). The equations and variables are scaled by diagonal matrices $D_F = \text{Diag}(1/\mathbf{F}_b)$ and $D_x = \text{Diag}(1/\mathbf{x}_b)$ respectively, with \mathbf{F}_b and \mathbf{x}_b the vectors with base values for each equation and base values for each variable. The scaled system is then given by $\hat{\mathbf{F}} := D_F \mathbf{F}$, with scaled variables $\hat{\mathbf{x}} := D_x \mathbf{x}$. The iteration scheme of NR in multiple dimensions, for the scaled system, is given by:

$$\hat{\mathbf{x}}^{k+1} = \hat{\mathbf{x}}^k - \hat{J}(\mathbf{x}^k)^{-1} \hat{\mathbf{F}}(\mathbf{x}^k) \quad (5)$$

with $\mathbf{x}^k = D_x^{-1} \hat{\mathbf{x}}^k$, and $\hat{J}(\mathbf{x}^k) = D_F J(\mathbf{x}^k) D_x^{-1}$. The Jacobian matrix J is given by:

$$J = \begin{pmatrix} J_{gg} & J_{ge} & J_{gh} & J_{gc} \\ J_{eg} & J_{ee} & J_{eh} & J_{ec} \\ J_{hg} & J_{he} & J_{hh} & J_{hc} \\ J_{cg} & J_{ce} & J_{ch} & J_{cc} \end{pmatrix} = \begin{pmatrix} J_{gg} & 0 & 0 & J_{gc} \\ 0 & J_{ee} & 0 & J_{ec} \\ 0 & 0 & J_{hh} & J_{hc} \\ 0 & 0 & J_{ch} & J_{cc} \end{pmatrix} \quad (6)$$

where the submatrices are defined as $J_{\alpha\beta} = \frac{\partial \mathbf{F}^\alpha}{\partial \mathbf{x}^\beta}$ for $\alpha, \beta \in \{g, e, h, c\}$. Since the BCs additionally required due to the coupling are imposed in the SC parts of the network, these submatrices will generally not be square.

NR is stopped when the error $\|D_F \mathbf{F}(\mathbf{x}^k)\|_2 < 10^{-6}$, or when the maximum number of iterations is reached.

IV. EXAMPLES

To investigate the convergence of NR for SCNs and MCNs, we consider a gas, electrical, and heat SCN. We consider two cases, a base case with a fixed small topology, and an extended case, which takes the base case and increases the SCNs to various sizes. The coupling part is the same for both cases.

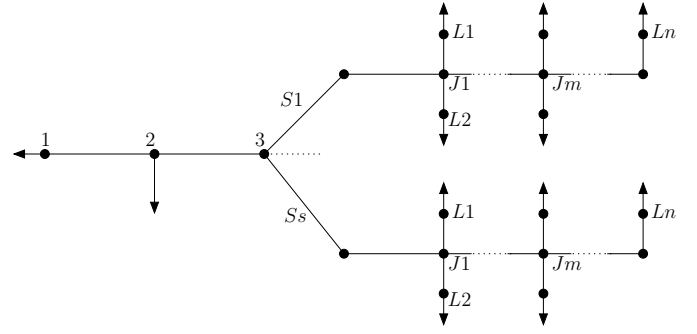


Fig. 2. Network topology of a SCN, extended with s streets S , each consisting of n loads L . In every street, m junctions J are connected to two loads.

TABLE II
BC NODE SETS FOR THE EXAMPLE SCNs

Node	Gas		Electricity		Heat	
	Type	Specified	Type	Specified	Type	Specified
1	ref.	p^g	slack	$ V , \delta$	source ref. slack	T^s, p^h
2	load	q	gen.	$P, V $	load (source)	T^o, φ
3	load	q	load	P, Q	load (sink)	T^o, φ

A. Topology

Each SCN in the base case consist of three nodes. For all three carriers, node 1 is a source, and node 3 is a sink. For the electrical network and the heat network, node 2 is an additional source. For coupling, we consider components that convert gas to electricity, heat, or both, see Sec. II-B. One electrical and one heat source are replaced with a coupling, such that the SCNs are coupled at node 1 or at node 2. The networks are coupled by a single node representing a CHP or an EH, or by two nodes representing a GB and a GG. Fig. 1 shows the possible topologies for the base case MES.

For the extended case, the base network from Fig. 1 is extended by ‘streets’ of additional loads in every SC part, see Fig. 2. There are s streets, $S_1 - S_s$, which are all connected to node 3 of the base SCN through a junction node. The streets consists of n loads, $L_1 - L_n$, connected to the main street links by junctions, m of which, $J_1 - J_m$, are connected to two loads. Fig. 2 shows the topology of such an extended SCN, consisting of $3+s(2n-m+1)$ nodes and $2+s(2n-m+1)$ links. The extended MES is created by coupling the SCNs in the same way as for the base network, shown in Fig. 1.

B. Boundary Conditions

Tab. II gives the node types used for the LFP in the SCNs, based on which variables are specified. Note that the electrical and heat network do not necessarily have a physical solution. For instance, if $|\varphi_2| > \varphi_3 > 0$, the source slack node 1 would have to behave as a sink, which is unphysical.

Node 1 is the slack node in all three SCNs, such that load flow analysis determines the amount of injected flow or energy entering node 1. Replacing the slacks of the electrical and heat network with a coupling is then straightforward. The electrical and heat SC parts of the MES will determine φ^c and P^c , after

TABLE III
BC NODE SETS FOR THE EXAMPLE MESS, COUPLING AT NODE 1.

Node	CHP or GB + GG		EH	
	node set 1	node set 2	node set 1	node set 2
1 ^g	p^g	p^g	p^g	p^g
2 ^g	q	q	q	q
3 ^g	q	q	q	q
1 ^e	P, Q, V , δ	P, Q, V , δ	Q, V , δ	P, Q, V , δ
2 ^e	$P, V $	$P, V $	$P, V $	$P, V $
3 ^e	P, Q	P, Q	P, Q	P, Q
1 ^h	$T^s, p^h, m = 0$	$p^h, m = 0$	$T^s, p^h, m = 0$	T^s, p^h
2 ^h	T^o, φ	T^o, φ	T^o, φ	T^o, φ
3 ^h	T^o, φ	T^o, φ	T^o, φ	T^o, φ
1 ^c	-	T^o	-	T^o
2 ^c	-	-	-	-

TABLE IV
BC NODE SETS FOR THE EXAMPLE MESS, COUPLING AT NODE 2.

Node	CHP or GB + GG			EH		
	node set 1	node set 2	node set 3	node set 1	node set 2	node set 3
1 ^g	p^g, q	p^g, q	p	p^g, q	p	p
2 ^g	q	q	q	q	q	q
3 ^g	q	q	q	q	q	q
1 ^e	$P, V $	$ V , \delta$	$P, V $	$ V , \delta$	$P, V $	$ V , \delta$
2 ^e	P, Q, V , δ	$P, Q, V $	P, Q, V , δ	$P, Q, V $	P, Q, V , δ	$P, Q, V $
3 ^e	P, Q	P, Q	P, Q	P, Q	P, Q	P, Q
1 ^h	T^s, p^h	p^h, T^o, φ	p^h, T^o, φ	T^s, p^h	p^h, T^o, φ	p^h, T^o, φ
2 ^h	$m = 0$	$m = 0$	$m = 0$	$m = 0$	$m = 0$	$m = 0$
3 ^h	T^o, φ	T^o, φ	T^o, φ	T^o, φ	T^o, φ	T^o, φ
1 ^c	T^o	T^o	T^o	T^o	T^o	T^o
2 ^c	-	-	-	-	-	-

which the coupling equations for a CHP, or for a GB and a GG, uniquely determine q^c . For the EH, the coupling equation needs only P^c or φ^c to uniquely determine q^c and φ^c or P^c . Hence, either the electrical or the heat network will need an additional slack.

Conversely, node 2 was a generator or a source node in the SC electrical and heat network, such that P and φ were given. Replacing those sources with the coupling, after which P^c and φ^c are unknown, means BCs must be chosen such that the SC parts can determine q^c , P^c , or φ^c for the coupling equation(s) to be able to determine the others.

Tab. III and Tab. IV give the various sets of BCs used in the MCNs, for the base case. The node sets for the extended case are the same, the additional nodes are junction or load nodes.

C. Numerical Experiments

For the numerical experiments, we consider the base case MESS with the topologies as shown in Figs. 1a-1b and the node sets given in Tab. III when coupled at node 1, and the topologies shown in Fig. 1c-1d and the nodes sets given in Tab. IV when coupled at node 2. The extended case uses the same node sets, and 30 nodes per SCN ($n = 5, m = 2, s = 3$) for a medium network, or 323 nodes per SCN ($n = 10, m = 5, s = 20$) for a large network.

We use a flat initial guess of NR, except for a linear profile for \mathbf{p}^g , \mathbf{p}^h , and \mathbf{T}^s , where the nodes furthest from the source have the lowest value.

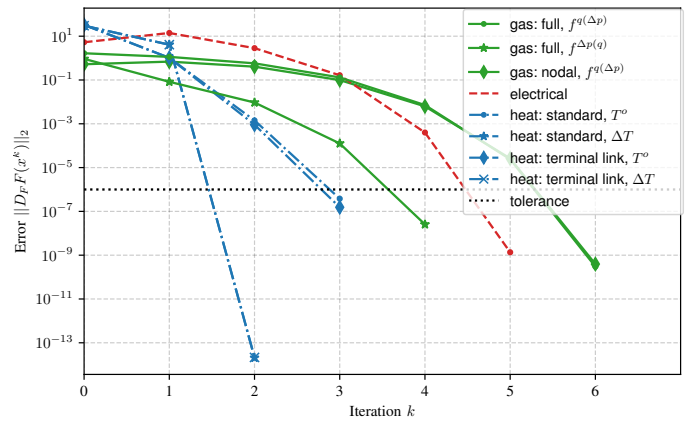


Fig. 3. Convergence of NR for the extended SCNs, with $n = 10$, $m = 5$, and $s = 20$, using various formulations in gas and in heat.

TABLE V
NR ITERATIONS FOR LFP OF SCNS.

Carrier	Formulation	Case (number of nodes per SCN)		
		Base	Ext. medium (30)	Ext. large (323)
Gas	nodal, $f^q(\Delta p)$	4	6	6
	full, $f^q(\Delta p)$	4	6	6
	full, $f^{\Delta p(q)}$	3	5	4
Electricity		2	3	5
Heat	standard, T^o	3	3	3
	standard, ΔT	2	2	2
	terminal link, T^o	3	3	3
	terminal link, ΔT	2	2	2

The convergence of NR for the SCNs shows very similar behavior for all network sizes. However, the convergence is different for the various formulations in the gas network and in the heat network. Fig. 3 shows the convergence behavior for the largest network, as an example. Tab. V summarizes the iterations needed by NR to converge. For the gas network, link equation $f^{\Delta p(q)}$ results in slightly faster convergence than $f^q(\Delta p)$. With $f^q(\Delta p)$, the nodal and full formulation give the same results. For the heat network, convergence shows no difference between the terminal link or the standard formulation. Assuming $\Delta T_{i,l}$ known for a heat load instead of $T_{i,l}^o$ gives better convergence. Hereafter, we only show the results of the LFP for the MESS with the full formulation in gas and the terminal link formulation, with $\Delta T_{i,l}$ known, in heat. Using the nodal formulation in gas, and standard formulation or terminal link formulation with $T_{i,l}^o$ known in heat, give similar results.

Fig. 4 shows typical convergence of NR for the base case MES coupled at node 2, using different topologies, coupling components, and all node sets. Numerical experiments show similar convergence for the extended case coupled at node 2, and for the base and extended case coupled at node 1, for all considered topologies, coupling components, and node sets.

For comparison, we give the results for the node sets in which the coupling component functions as a slack for the heat network. That is, we use node set 1 when coupling at node 1, and node set 3 when coupling at node 2. Tab. VI shows the

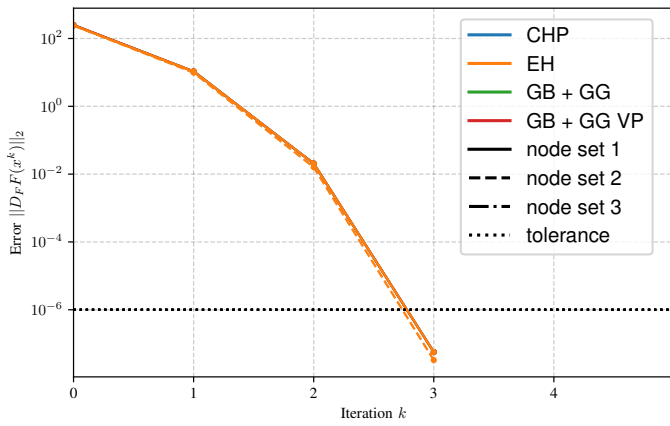


Fig. 4. Convergence of NR for the base case MES, coupled at node 2, using full formulation with $f^{\Delta p(q)}$ in gas, and terminal link formulation with $\Delta T_{i,l}$ known in heat. Most of the curves are indistinguishable at this scale.

TABLE VI
NR ITERATIONS FOR LFP OF THE MESS.

Size	Form. gas	Coupled at	CHP	GB & GG	GB & GG VP	EH	Max. SC
base	full, $f^q(\Delta p)$	node 1	4	4	4	4	4
		node 2	4	4	4	4	4
	full, $f^{\Delta p(q)}$	node 1	3	3	3	3	3
		node 2	3	3	3	3	3
medium $n = 5$ $m = 2$ $s = 3$	full, $f^q(\Delta p)$	node 1	6	6	6	6	6
		node 2	6	6	6	6	6
	full, $f^{\Delta p(q)}$	node 1	5	5	5	5	5
		node 2	5	5	5	5	5
large $n = 10$ $m = 5$ $s = 20$	full, $f^q(\Delta p)$	node 1	6	6	6	6	6
		node 2	6	6	6	6	6
	full, $f^{\Delta p(q)}$	node 1	4	4	4	4	5
		node 2	4	4	4	4	5

number of iterations needed by NR for the base and extended case MESSs, for these two node sets, to converge. Numerical experiments show similar results for the other node sets, and other formulations. Given a formulation of the SC parts, the NR iterations for MESSs follow a similar pattern for both points of coupling, all four coupling components and models, and all node sets. Hence, for this example, the convergence behavior of the MES is determined by the SCNs, and not by the coupling. Moreover, we find that for the extended case, the number of iterations barely increases when the size of the network increases, both for the SCNs and for the MESSs.

For these examples, NR for the integrated system of load flow equations requires a number of iterations roughly equal to the maximum number of iterations of NR for the corresponding SCNs. Hence, the convergence of NR is independent of the coupling. This could be due to the topology and the choice of BCs. All node sets are chosen such that the steady-state LFP can be solved uniquely for one or two SCNs, with the coupling energy as unknown. The coupling equations can then be used to compute the other coupling energies or energy, which serve as a BC for the other SCNs. Hence, the LFP for the examples MESSs could be solved by sequentially solving the SC LFPs, instead of solving one integrated system of equations. In other words, the Jacobian matrix (6) used in NR could easily be reordered into block upper triangular form

where the coupling part is included with the SC parts. This may induce the solution paths of the subsystems to be very similar to those of the individual subsystems when solved separately, so that the number of iterations of the integrated systems is automatically near the maximum of the iteration numbers of the individual subsystems.

We expect similar convergence behavior for other MESSs where the integrated LFP can easily be decomposed into solvable SC subsystems, but further research is required.

V. CONCLUSION

We modeled steady-state load flow for MESSs by solving the integrated system of nonlinear equations for various small and large example networks. We compared the convergence behavior of NR for the various MESSs and SCNs.

Various formulations of the system of equations were used in the gas and heat SC parts. For the example networks, expressing flow as a function of pressure drop for the link equations in the gas network resulted in slower convergence compared with expressing pressure drop as a function of flow. There is no difference in convergence behavior between the nodal and the full formulation in the gas network. Assuming the temperature difference for a heat load known, instead of the outflow temperature, results in faster convergence for the heat network. There is no difference in convergence behavior between the standard and the terminal link formulation in the heat network. Furthermore, increasing the size of the SCNs barely increases the number of NR iterations.

To couple the SCNs, we considered an EH, a CHP, and a gas-boiler combined with a gas-fired generator. The networks were either coupled at their slack nodes (node 1), or at a heat source and electrical generator (node 2). The convergence of NR for the MESSs is independent of the coupling component, point of coupling, coupling model, and node set.

For these examples, NR for LFP of the MES requires at most as many iterations as the slowest SCN. Moreover, the number of NR iterations are independent of the coupling and almost independent of the size of the network.

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