# A Mass-Conserving Level Set method for pipe flows in cylindrical coordinates

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## ABSTRACT

Multiphase flows through pipelines play an important role in many industrial processes, such as oil and gas recovery, chemical processing, power generation and various civil engineering applications, such as free surface flows and sediment transport. For the design of multiphase pipeline systems the industry uses computationally fast, but simplified, time-dependent, one-dimensional models, which are unable to accurately predict the onset and propagation of unstable flow, often characterized by roll waves and slug formation. As a result, industry has the desire to gain more insight into this flow transition by considering the onset of turbulent behaviour in 3D flows.

Correct modelling of two-phase flows requires accurate prediction of the interface between the media involved. Several techniques exist for this, the most important being the Level Set (LS) method and the Volume-Of-Fluid (VOF) method. Using the LS method, the interface in some domain $\Omega^n$ of dimension $n$ is elegantly represented by a level set $\phi(\mathbf{x}) = c$ (with $c$ some constant) of a particular function $\phi$ of dimension $(n+1)$. Even though the level set function $\phi$ can be advected along with the underlying flow field in a conservative way, mass is generally not conserved and this immediately shows the largest drawback of this method.
The VOF method comprises a discontinuous color function $\Psi$ that for every computational cell provides the volume fraction of one of the two media present. The function $\Psi$ can be advected in a conservative way as well, and in this case mass is conserved by construction. The reconstruction of the interface is done based on the value of $\Psi$ in each cell and can be laborious, especially in 3D.

Several hybrid methods have been introduced that aim to combine the elegant and efficient interface reconstruction of the LS method, while at the same time trying to conserve mass using the color function concept from the VOF method. One example is the Mass Conserving Level Set (MCLS) method, developed by S. van der Pijl[1], which uses both the LS method (for interface representation) and the VOF method (to remain conservative) by explicitly expressing the color (VOF) function $\Psi$

in terms of the level set function $\phi$. The MCLS method is computationally very efficient because, in contrast to many other hybrid methods, the elaborate interface reconstruction of the VOF method is completely avoided.

The MCLS method is highly dependent on the geometry of the cartesian grid it was originally defined on. When modelling pipe flows however, the transition to cylindrical coordinates is considered for better approximation of the pipe geometry. This requires a partial reformulation of the original MCLS method to make it function in cylindrical domains. Additionally, solving the flow field equations in cylindrical domains requires treatment of the induced singularity at the core of the pipe.

The results of the extended MCLS method will be validated using certain classic examples (Taylor bubble, Benjamin bubble, Kelvin-Helmholtz waves) of which in some cases even analytical solutions exist. Also experimentally obtained values will be used for comparison.

The next step will be to look at turbulent behaviour of two-phase flows. A closer look will be necessary on existing two-phase turbulence models and the boundary conditions required at the interface. We will try to expand the extended MCLS method to ultimately be able to study the onset of unstable flow regimes.

## References

[1] S.P. van der Pijl, A. Segal, C. Vuik, P. Wesseling: A mass-conserving level set method for modelling of multi-phase flows. *International Journal for Numerical Method in Fluids*, 47 (2005), 339–361.