Modeling of three-dimensional bubbly flows with a mass-conserving level-set method

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Abstract

In this work incompressible two-phase flows are considered. The aim is to model high density-ratio flows with arbitrary complex interface topologies, such as occur in air/water flows. Between the phases a sharp front exists, where density and viscosity change abruptly.

The computational method used in this paper is the mass conserving level-set method. It is based on the level-set methodology, using a volume-of-fluid (VOF) function to conserve mass. This function is advected without the necessity to reconstruct the interface. The ease of the method is based on an explicit relationship between the VOF function and the level-set function. The method is straightforward to apply to arbitrarily shaped interfaces, which may collide and break up.

Keywords: Level-set; Volume-of-fluid; Incompressible; Multi-phase; Navier–Stokes; Bubbles

1. Introduction

Various methods have been put forward to treat bubbly flows. The level-set method has been chosen as the basis of our work. However, mass-conservation is not an intrinsic property and is considered the major drawback of the level-set method. Our work focuses on a mass-conserving way to advect the interface, resulting in what we will call the mass-conserving level-set method (MCLS, [1,2,3]). We use a volume-of-fluid function Ψ as a help variable to conserve mass, without applying the difficult convection (namely interface reconstruction) that makes the VOF so elaborate. We propose a simple relationship between the level-set function Φ and volume-of-fluid function Ψ. It makes the advection of the volume-of-fluid function Ψ easy (i.e. without interface reconstruction) and finding Φ from Ψ a straightforward task.

2. Governing equations

Consider two fluids '0' and '1' in domain $\Omega \in \mathbb{R}^3$, which are separated by an interface S. Both fluids are assumed to be incompressible, i.e.

$$\nabla \cdot \mathbf{u} = 0 \tag{1}$$

where $\mathbf{u} = (u, v, w)^t$ is the velocity vector. The flow is governed by the incompressible Navier–Stokes equations:

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho}\nabla p + \frac{1}{\rho}\nabla \cdot \mu(\nabla \mathbf{u} + \nabla \mathbf{u}^t) + \mathbf{g} \tag{2}$$

where ρ, p, μ and $\mathbf{g}$ are the density, pressure, viscosity and gravity vector respectively. The density and viscosity are constant within each fluid. We have

$$\mu = \mu_0 + (\mu_1 - \mu_0)H(\Phi) \tag{3}$$

and similar for ρ, where Φ is the level-set function describing the interface S, and H is the Heaviside step function.

The viscosity μ is made continuous at the interface by smoothing Eq. (3). This makes the derivatives of the velocity components continuous at the interface [4,5] and decouples the interface conditions, which is advantageous for the discretization. Following Chang et al. [6], the viscosity is smoothed over three mesh widths.

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For the surface tension forces the continuous surface force/stress (CSF, [7]) methodology is adopted.

3. Computational approach

The Navier–Stokes equations are solved on a Cartesian grid in a rectangular domain by the pressure-correction method [8]. The unknowns are stored in a marker-and-cell (staggered) layout [9]. For the interface representation the Level-Set methodology is adopted. The interface conditions are satisfied by means of the continuous surface force (CSF) methodology. The discontinuous density field is dealt with similarly to the GhostFluid method for incompressible flow [4]. Further information about the flow-field computations can be found in Van der Pijl et al. [1,2,3].

The strategy of modeling two-phase flows is to compute the flow with a given interface position and subsequently evolve the interface in the given flow field. The interface is implicitly defined by a level-set function Φ. More precisely, the interface, say S, is the zero level-set of Φ:

$$S(t) = \left\{x \in \mathbb{R}^2 | \Phi(x,t) = 0\right\} \tag{4}$$

The interface is evolved by advecting the Level-Set function in the flow field as if it were a material constant:

$$\frac{\partial \Phi}{\partial t} + \mathbf{u} \cdot \nabla \Phi = 0 \tag{5}$$

A homogeneous Neumann boundary condition for Φ is imposed at the boundaries.

3. MCLS

The difficulty with the level-set method is that conservation of Φ does not imply conservation of mass. On the other hand, with the volume-of-fluid method, mass is conserved when Ψ is conserved. In order to conserve mass with the level-set method, corrections to the Level-Set function are made by considering the fractional volume Ψ of a certain fluid within a computational cell. First the usual level-set advection is performed. Since the obtained level-set function $\Phi^{n+1,*}$ will certainly not conserve mass, corrections to $\Phi^{n+1,*}$ are made such that mass is conserved. This requires three steps:

(i) the relative volume of a certain fluid in a computational cell (called 'volume-of-fluid' function Ψ) is to be computed from the level-set function Φ^n: $\Psi = f(\Phi, \nabla\Phi)$;

(ii) the volume-of-fluid function has to be advected conservatively during a time step towards Ψ^{n+1};

(iii) with this new volume-of-fluid function Ψ^{n+1}, corrections to $\Phi^{n+1,*}$ are sought such that $\int (\Phi^{n+1}, \nabla\Phi^{n+1}) = \Psi^{n+1}$ holds.

More details can be found in Van der Pijl et al. [1,2,3].

4. Applications

The behavior of the MCLS approach is shown by a

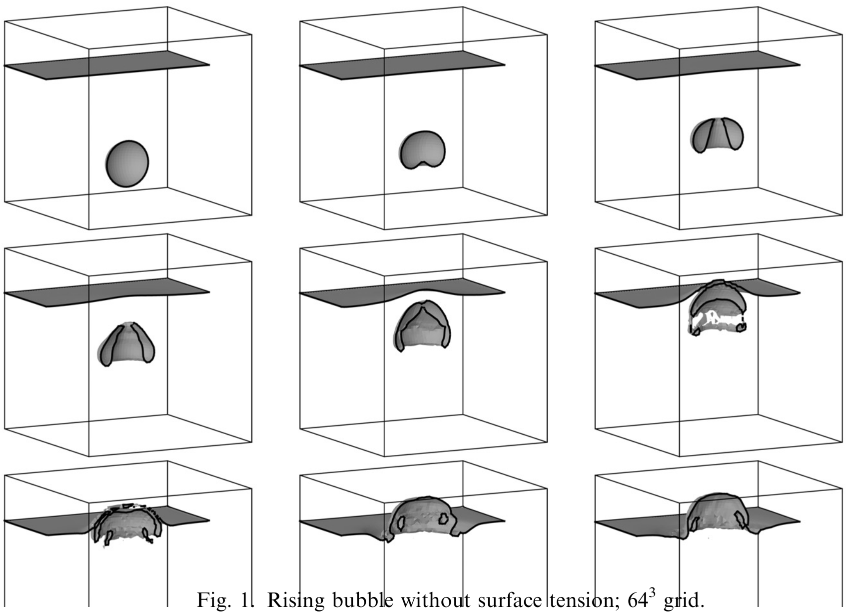

Fig. 1. Rising bubble without surface tension; 64^3 grid.

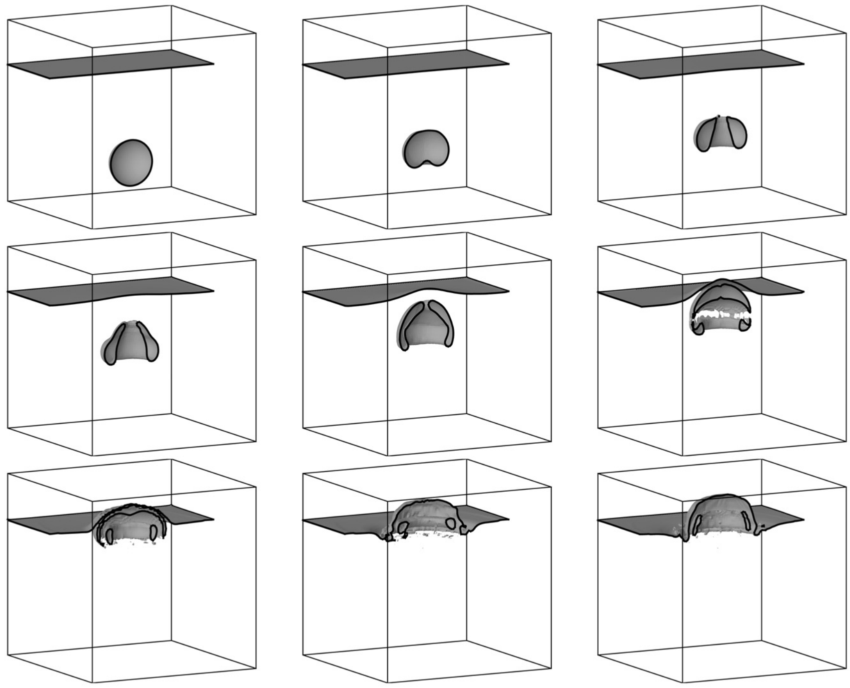

Fig. 2. Rising bubble without surface tension; 96^3 grid.

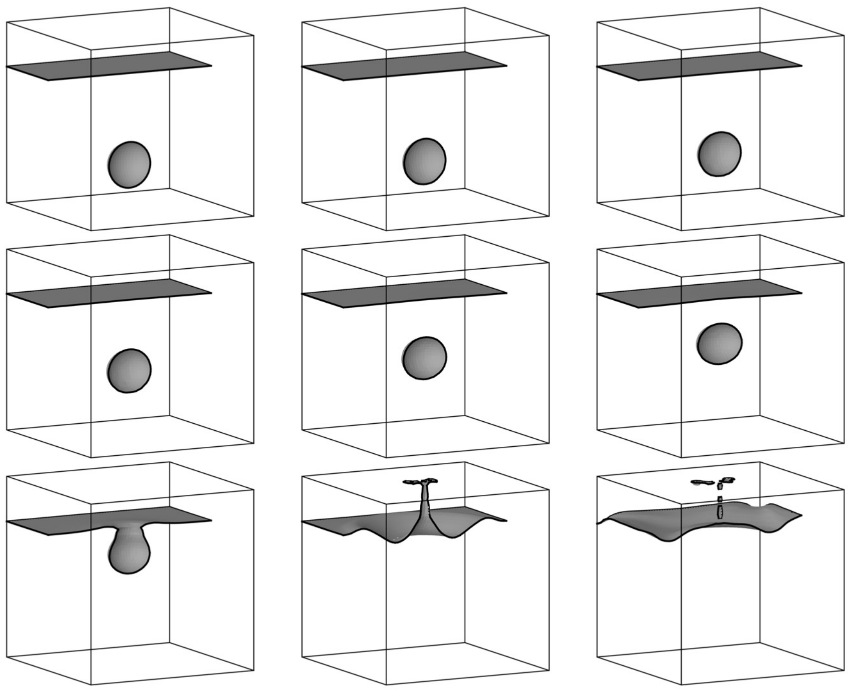

Fig. 3. Rising bubble without surface tension; 64^3 grid.

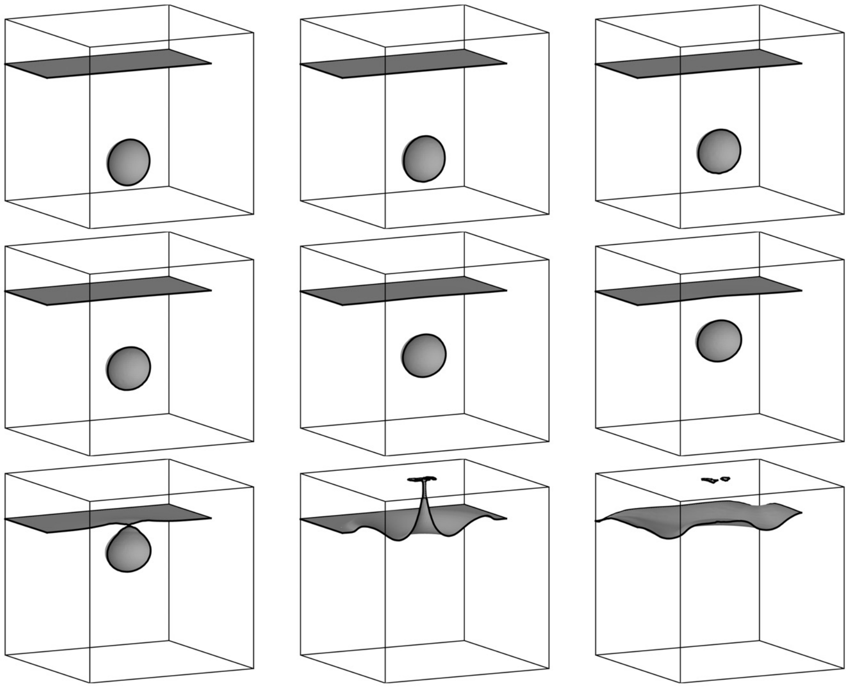

Fig. 4. Rising bubble with surface tension; 96^3 grid.

rising bubble in three dimensions. The gravity and material constants are: $g = 9.8\ \mathrm{m/s^2}$, $\sigma = 0.0728\ \mathrm{kg/s^2}$, $\rho_w = 10^3\ \mathrm{kg/m^3}$, $\rho_a = 1.226\ \mathrm{kg/m^3}$, $\mu_w = 1.137\ 10^{-3}\ \mathrm{kg/ms}$ and $\mu_a = 1.78\ 10^{-5}\ \mathrm{kg/ms}$. where subscripts *w* and *a* indicate water and air respectively. In Fig. 1 and 2 a three-dimensional rising bubble bursting through a free surface without surface tension is is shown for a $64 \times 64 \times 64$ and a $96 \times 96 \times 96$ grid respectively. The domain is a cube with a width, length and height of 0.01 m. The bubble is initially placed at 1/4th height. The snapshots are taken at equal time differences of 0:005 s. For the ease of visualization, only $y < 1/2L_y$ is plotted. Also, the interface position in the plane $y = 1/2L_y$ is plotted. It can be seen that the bubble deforms and breaks up to form a bell-like and ring-like structure, just before it breaks through the free surface. In Fig. 3 and 4 results with surface tension are presented. The deformation of the bubbles is significantly reduced due to the surface tension effects.

5. Conclusion

The application of the mass conserving level-set (MCLS) to three dimensional bubbly flows has been presented. The method is based on the level-set methodology, where mass is conserved by considering the fractional volume of a certain fluid within a computational cell. Mass is conserved up to a specified (vanishing) tolerance. The MCLS method combines the attractiveness of the level-set method with the mass-conserving properties of the volume-of-fluid methods, without adopting the latter. This makes the implementation much easier than for a volume-of-fluid (based) method, especially in three-dimensional space. The applicability of the MCLS method was illustrated by the application to a bubble flow in three dimensions.

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References

[1] van der Pijl SP, Segal A, Vuik C. A mass-conserving Level-Set method for modeling of multi-phase flows. AMA report 03–03, Delft University of Technology, 2003. http://ta.twi.tudelft.nl

[2] van der Pijl SP, Segal A, Vuik C. Modelling of multi-phase flows with a Level-Set method. In: M Feistauer, V Dolejší, P Knobloch, K Najzar, editors, Numerical Mathematics

and Advanced Applications Berlin: Springer, 2004 pp. 698–707.

[3] van der Pijl SP, Segal A, Vuik C. Modelling of three-dimensional bubbly flows with a mass-conserving Level-Set method. In P Neittaanmäki, T Rossi, S Korotov, E Oñate, J Périaux, D Knörzer, editors, Proc of the European Congress on Computational Methods in Applied Sciences and Engineering, ECCOMAS 2004, Jyväskylä, Finland, July 24–28, 2004.

[4] Kang M, Fedkiw RP, Liu X-D. A boundary condition capturing method for multiphase incompressible flow. J Sci Comput 2000;323–360.

[5] Li Z, Lai M-C. The immersed interface method for the Navier-Stokes equations with singular forces. J Comput Phys 2001;171:822–842.

[6] Chang YC, Hou TY, Merriman B, Osher S. A level set formulation of Eulerian interface capturing methods for incompressible fluid flows. J Comput Phys 1996;124:449–464.

[7] Brackbill JU, Kothe DB, Zemach C. A continuum method for modeling surface tension. J Comput Phys 1992;100:335–354.

[8] van Kan JJIM. A second-order accurate pressure correction method for viscous incompressible flow. SIAM J Sci Stat Comput 1986;7:870–891.

[9] Harlow FH, Welch JE. Numerical calculation of time-dependent viscous incompressible flow of fluid with free surfaces. Phys Fluids 1965;8:2182–2189.