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Abstract

In this report we present the so-called *Stadyc* model for the movement of a ship. The idea is the following: first compute the optimal positioning of the thrusters for the static (*Sta*) situation, then solve the dynamic (*dy*) model in order to predict the trajectory of the ship, and finally, the forces and positions of the thrusters are adapted in the control (*c*) model in order that the ship follows a given path as good as possible. The difference of the real position of the ship and the required position can be obtained from the model or from actual measurements.

Keywords. numerical model, Multi Thruster Control System, required trajectory, static model, dynamic model, control, physical constants

1 Introduction

In this report we present the so-called *Stadyc* model for the movement of a ship. The idea is the following: first compute the optimal positioning of the thrusters for the static (*Sta*) situation, then solve the dynamic (*dy*) model in order to predict the trajectory of the ship, and finally, the forces and positions of the thrusters are adapted in the control (*c*) model in order that the ship follows a given path as good as possible. The difference of the real position of the ship and the required position can be obtained from the model or from actual measurements. The dynamic model is given in Section 2. The static model is discussed in Section 3, and the control model is presented in Section 4.

2 The dynamic model for a ship

2.1 The dynamic model of the movement of a ship

In order to describe the dynamic model of the movement of a ship we note that a ship has only three degrees of freedom: the x and y position of the ship and the angle ϕ. To describe these degrees of freedom, we use the x, y coordinate system, which is fixed in space and oriented such that the positive y axis is pointing in the north direction. We use the following notations:

- m is the hydrodynamic mass of the ship. This constant may contain an extra term due to the movement of an amount of water, which is proportional to the underwater volume of the ship,
- I is the inertia for rotation of the ship,

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- $x_s(t)$ is the x-coordinate of the mass center of the ship as a function of t,
- $y_s(t)$ is the y-coordinate of the mass center of the ship as a function of t,
- $\phi(t)$ is the angle of the ship as a function of t with the positive x-axis.

Newton's law leads to the following dynamical system:

$$m\frac{d^2x_s}{dt^2} = R_x, \tag{1}$$

$$m\frac{d^2y_s}{dt^2} = R_y, \tag{2}$$

$$I\frac{d^2\phi}{dt^2} = M. \tag{3}$$

In order to describe the forces and the moment, we first give an expression for these quantities, with respect to the ξ, η coordinate system. This coordinate systems is connected to the ship. Its origin is in the mass center of the ship, whereas the positive η axis points into the direction of the bow. If n_t thruster are used, the forces are then given by:

$$R_\xi = R_\xi(1) + \ldots + R_\xi(nt) + R_{\xi,water} + R_{\xi,wind}, \tag{4}$$

$$R_\eta = R_\eta(1) + \ldots + R_\eta(nt) + R_{\eta,water} + R_{\eta,wind}, \tag{5}$$

and the moment

$$\begin{aligned} M &= -\eta pos(1) \times R_\xi(1) \ldots - \eta pos(1) \times R_\xi(n_t) + \\ &\quad +\xi pos(1) \times R_\eta(1) \ldots + \xi pos(n_t) \times R_\eta(n_t) + \\ &\quad +M_{water} + M_{wind}. \end{aligned} \tag{6}$$

Note that the moment is independent of the coordinate system, whereas the forces depends on the coordinate system. In the formula's given above the following notations are used:

- R_ξ is the sum of all the forces in the ξ direction,
- R_η is the sum of all the forces in the η direction,
- $R_\xi(j)$ and $R_\eta(j)$ are the components of the force from thruster j,
- $\xi pos(j)$ and $\eta pos(j)$ are the coordinates of the location of thruster j,
- $R_{\xi,water}$ and $R_{\eta,water}$ are the components of the water force,
- $R_{\xi,wind}$ and $R_{\eta,wind}$ are the components of the wind force,
- M is the sum of all the moments,
- M_{water} is the moment due to the water,
- M_{wind} is the moment due to the wind.

Note that the values of $R_\xi(j)$ and $R_\eta(j)$ for $j = 1, \ldots, nt$, are the only couplings between the static and the dynamic model. So if some $R_\xi(j)$ and $R_\eta(j)$ are required in the dynamic model, one should be able to compute them with the static model, or on the other hand, if the values of $R_\xi(j)$ and $R_\eta(j)$ are given by the static model, they can be used in the dynamic model to compute the form of the trajectory of the ship.

Now all forces are transformed to the fixed coordinate system, by the following transformation:

x-component

$$R_x = R_\xi \sin\phi + R_\eta \cos\phi,$$

y-component

$$R_y = -R_\xi \cos\phi + R_\eta \sin\phi.$$

Below we first study the influences of the water forces. The wind forces can modeled in the same way. We shall do this after the water forces are considered. In order to compute the water forces, we use the following definitions:

- $v_{x,water}$ is the x-velocity of the water (current) in the fixed coordinate system,
- $v_{y,water}$ is the y-velocity of the water (current) in the fixed coordinate system.

We assume that the water forces are a function of the velocity of the ship relative to the water. The relative velocities are given by:

$$v_x = \frac{dx_s}{dt} - v_{x,water},$$

$$v_y = \frac{dy_s}{dt} - v_{y,water}.$$

These velocities are decomposed into the ξ, η coordinates as follows:

$$v_\xi = v_x \sin\phi - v_y \cos\phi,$$

$$v_\eta = v_x \cos\phi + v_y \sin\phi.$$

We now use the following model to determine the water forces, due to the velocities v_ξ and v_η (for other models we refer to [3, 2]). A number of constants are unknown and should be determined, see Section 2.3.

$$R_{\xi,water} = -K_W v_\xi |v_\xi|, \tag{7}$$

$$R_{\eta,water} = -K_A v_\eta |v_\eta|, \tag{8}$$

where in general $K_W > K_A$. Finally, the moment originated by the water is modeled by:

$$M_{water} = -K_M v_\xi |v_\xi| - K_N \frac{d\phi}{dt}. \tag{9}$$

Now we consider the influence of the wind forces. In order to compute the wind forces, we use the following definitions:

- $v_{x,wind}$ is the x-velocity of the wind in the fixed coordinate system,
- $v_{y,wind}$ is the y-velocity of the wind in the fixed coordinate system.

We assume that the wind forces are a function of the velocity of the ship relative to the wind. The relative velocities are given by:

$$v_x = \frac{dx_s}{dt} - v_{x,wind}$$

$$v_y = \frac{dy_s}{dt} - v_{y,wind}$$

These velocities are decomposed into the ξ, η coordinates as follows:

$$v_\xi = v_x \sin\phi - v_y \cos\phi,$$

$$v_\eta = v_x \cos\phi + v_y \sin\phi.$$

We now use the following model to determine the wind forces, due to the velocities v_ξ and v_η (for other models we refer to [3, 2]). A number of constants are unknown and should be determined, see Section 2.3.

$$R_{\xi,wind} = -K_{W,wind} v_\xi |v_\xi|, \tag{10}$$

$$R_{\eta,wind} = -K_{A,wind} v_\eta |v_\eta|, \tag{11}$$

where in general $K_{W,wind} > K_{A,wind}$. Finally, the moment originated by the wind is modeled by:

$$M_{wind} = -K_{M,wind} v_\xi |v_\xi|. \tag{12}$$

We note that in general the coefficients due to the wind forces are one or two orders of magnitude less than the coefficients due to the water forces. However, the wind velocities can be one or two order larger than the water velocities. Finally, we expect that at sea the current is only a slowly varying function of space and time, whereas the wind velocities can be rapidly varying functions of time. Another difference between the wind and water model is that we assume that the wind resistance with respect to rotation is negligible. In Table 1 all the constants, which should be

constant	description	dimension
m	hydrodynamic mass of the ship	kg
I	inertia of the ship	$kg\ m^2$
K_W	coefficient of the water force in ξ-direction	$\frac{kg}{m}$
K_A	coefficient of the water force in η-direction	$\frac{kg}{m}$
K_M	coefficient of the water moment due to the velocity in η-direction	kg
K_N	coefficient of the water moment due to rotation	$\frac{kg\ m^2}{s}$
$K_{W,wind}$	coefficient of the wind force in ξ-direction	$\frac{kg}{m}$
$K_{A,wind}$	coefficient of the wind force in η-direction	$\frac{kg}{m}$
$K_{M,wind}$	coefficient of the wind moment due to the velocity in η-direction	kg

Table 1: The constant and their dimensions, which are used in the dynamic model

determined are summarized.

In order to solve the dynamic model, we first transform the system of second order equations (1), (2), and (3) to a system of first order equations. Therefore, we use the following unknowns:

$$p(1) = x_s,\ p(2) = \frac{dx_s}{dt},\ p(3) = y_s,\ p(4) = \frac{dy_s}{dt},\ p(5) = \phi,\ p(6) = \frac{d\phi}{dt}, \tag{13}$$

which leads to the following system:

$$\begin{aligned} \frac{dp(1)}{dt} &= p(2), \\ \frac{dp(2)}{dt} &= \frac{R_x}{m}, \\ \frac{dp(3)}{dt} &= p(4), \\ \frac{dp(4)}{dt} &= \frac{R_y}{m}, \\ \frac{dp(5)}{dt} &= p(6), \\ \frac{dp(6)}{dt} &= \frac{M}{I}. \end{aligned}$$

This together with suitable initial conditions leads to a unique solution, which describes the trajectory of the ship. In general it is impossible to solve this system by an analytical solution method. Therefor we solve this system by a numerical method, the so-called Euler Forward method. For the details of this method we refer to [1].

2.2 Numerical experiments

In this section we give some numerical experiments computed with the dynamic model in order to illustrate the power of this model. Most of the constants used in this model are chosen artificially. Below we give the constants, which are used in the first experiment:

```
% initial conditions

x_start = 0;
y_start = 0;

vx_start = 0;
vy_start = 5;

phi_start = 90;
vphi_start = 4;

% time constants

tend = 30;  % final time
tend = 90;  % final time
nend = 300;  % number of time steps
nplot = 30; % number of plots

% coefficients

mass = 10;
inertia = 10000;

% water resistance coefficients

Kw = 4;
Ka = 1;
Km = 100;
Kn = 100;

% velocity of the current

vx_water = 0;
vy_water = 0;

% wind resistance coefficients

Kw_wind = 0*0.4;
Ka_wind = 0*0.1;
Km_wind = 0*100;

% velocity of the wind

vx_wind = 0;
vy_wind = 0;


% thruster forces and positions
```

```
xpos(1) = -30;
ypos(1) = -60;

R1(1) = 10;
R2(1) = 50;

xpos(2) =  30;
ypos(2) = -60;

R1(2) = 10;
R2(2) = 50;
```

The resulting position of the ship and its orientation are given in Figure 1. The starting point of the ship is located in the origin. When a current is added, $v_{x,water}$ is -2, the trajectory is changed, see Figure 2. Below we compute again the position of the ship and its orientation, where the

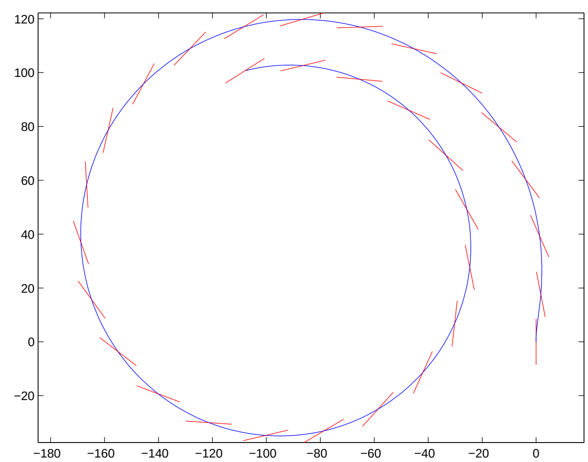

Figure 1: The trajectory of the ship without current

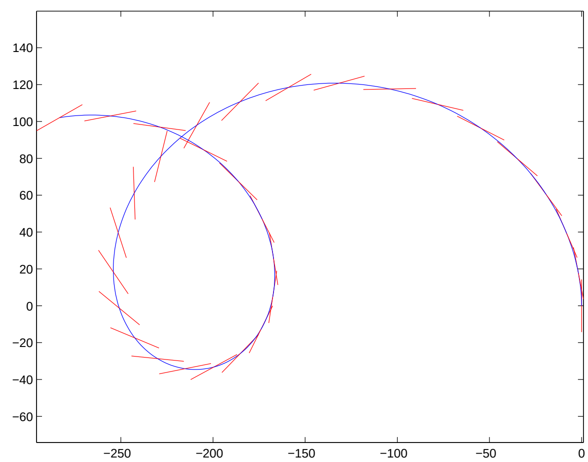

Figure 2: The trajectory of the ship with current

initial velocity of rotation is set equal to zero and both thrusters are only giving a force in the η direction. Without current the resulting trajectory is a straight line and the orientation of the ship is in the same direction (see Figure 3). When a current is added, $v_{x,water}$ is -2, the trajectory remains a straight line, but the orientation of the ship has a different angle (see Figure 4).

In our second experiment we investigate the influence of the wind forces. We start with a ship in

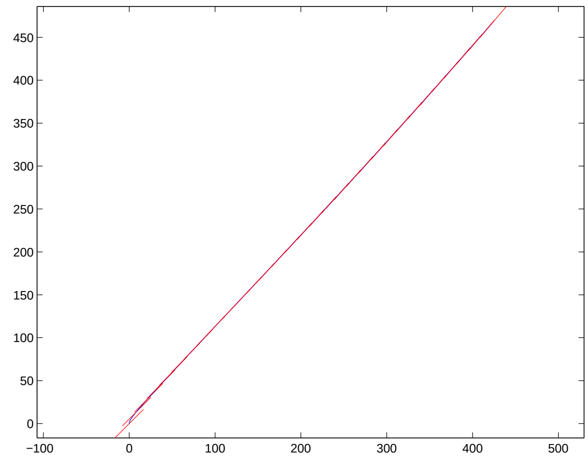

Figure 3: The trajectory of the ship without current

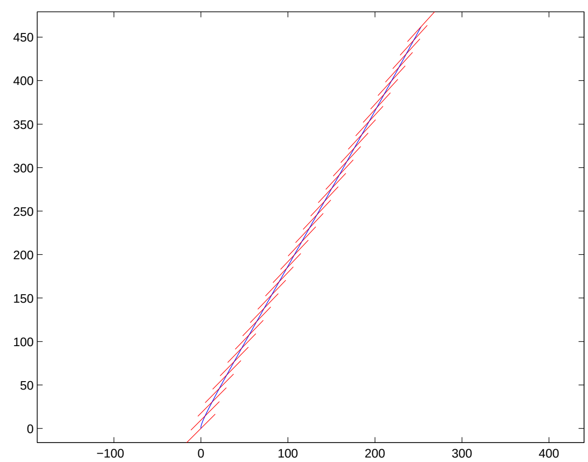

Figure 4: The trajectory of the ship with current

a stationary position, and the velocity of the current is equal to zero. The velocity of the wind is equal to $v_{x,wind} = 10$ and $v_{y,wind} = 10$. Below we give the constants, which are used in the second experiment:

```
x_start = 0;
y_start = 0;

vx_start = 0;
vy_start = 0;

phi_start = 90;
vphi_start = 0;

% time constants
```

```
tend = 270;  % final time
nend = 300;  % number of time steps
nplot = 60; % number of plots

% coefficients

mass = 10;
inertia = 10000;

% water resistance coefficients

Kw = 4;
Ka = 1;
Km = 100;
Kn = 400;

% velocity of the current

vx_water = 0;
vy_water = 0;

% wind resistance coefficients

Kw_wind = 0.04;
Ka_wind = 0.01;
Km_wind = 20;

% velocity of the wind

vx_wind = 10;
vy_wind = 10;


% thruster forces and positions

xpos(1) = -30;
ypos(1) = -60;

R1(1) = 0;
% R2(1) = 50;
R2(1) = 0;

xpos(2) =  30;
ypos(2) = -60;

R1(2) = 0;
R2(2) = 0;
```

The result of this simulation is given in Figure 5. Note that initially the ship is rotating such that the bow is pointing into the wind direction, furthermore the velocity increases to a limit value. Due to the damping and reaction forces of the water the rotation of the ships stops at a certain moment. This phenomenon depends on the choices of the coefficients. For other choices the ship keeps rotating.

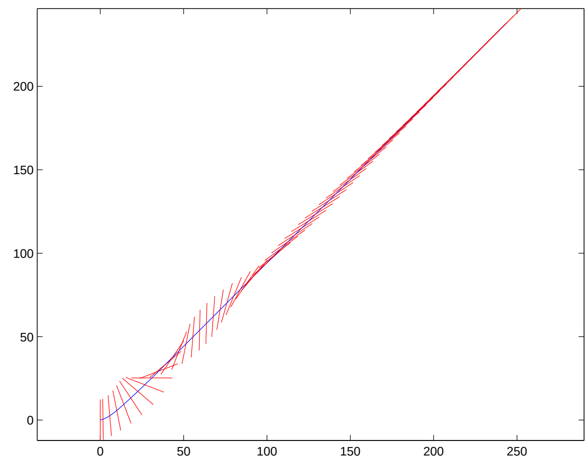

Figure 5: The trajectory of the ship with wind

2.3 How to find the constants?

Using a model, there is always the difficulty how to obtain the parameters, which are used in the model. There are three different ways to obtain a value for the parameters:

1. Ask the designer of the ship for the values.
2. Try to determine the coefficients by using formulas which describe the parameters as a function of the length depth etc. An advantage of this method is that the ship can be analyzed, before it is built. A disadvantage is that the formula's can be very complicated. This idea is described in Section 3.2 of [2].
3. Measure the 6 parameters by doing some maneuvers with the ship. An advantage is that this is simple to do. A disadvantage is that the parameters can only obtained when the ship is built. This idea is described in Section 3.3 of [2].

In this report we start by using the last method. We describe the maneuvers below.

Maneuver to determine the hydrodynamic mass m
Do this maneuver when the wind velocity is low.

- Measure the velocity of the current.
- Make the initial orientation of the ship such that the angle of the ship is equal to the angle of the velocity of the water.
- The velocity of the ship should be equal to the velocity of the water, so the relative velocity is equal to zero. Since the relative velocity is zero, the water forces are zero.
- Start the thrusters at $t = 0$ with a given power, where the ξ-forces are equal to zero, after the position $(x_s(0), y_s(0))$ is measured. From these assumptions it follows that

$$R_\eta = R_\eta(1) + R_\eta(2).$$

  For a short time t_{end} (so that the resulting velocities remain small) the water forces are negligible.

- The trajectory of the ship is now given by:

$$x_s(t) = x_s(0) + v_{x,water}t + \frac{R_\xi \sin\phi}{2m}t^2,$$

$$y_s(t) = y_s(0) + v_{y,water}t - \frac{R_\xi \cos\phi}{2m}t^2.$$

- Measuring $x_s(t_{end})$ and $y_s(t_{end})$ one can estimate m by

$$m = \frac{0.5R_\xi \sin\phi t_{end}^2}{x_s(t_{end}) - x_s(0) - v_{x,water}t_{end}},$$

  or

$$m = -\frac{0.5R_\xi \cos\phi t_{end}^2}{y_s(t_{end}) - y_s(0) - v_{y,water}t_{end}}.$$

Maneuver to determine the inertia I of the ship
Do this maneuver when the wind velocity is low.

- Make the initial orientation of the ship such that the angle $\phi = 0$.
- The velocity of the ship should be equal to the velocity of the water, so the relative velocity is equal to zero. Since the relative velocity is zero, the water forces are zero.
- Start the thrusters at $t = 0$ with a given power and compute the resulting moment M by using Equation (6). For a short time t_{end} (so that the resulting velocities remain small) the water forces are negligible.
- The angle ϕ is now given by:

$$\phi(t) = \frac{M}{2I}t^2.$$

- Measuring $\phi(t_{end})$ one can estimate I by

$$I = \frac{0.5M}{\phi(t_{end})}t_{end}^2.$$

Maneuver to determine the coefficient K_A
Do this maneuver when the wind velocity is low.

- Start the thrusters with a given power, where the ξ-forces are equal to zero. From these assumptions it follows that

$$R_\eta = R_\eta(1) + R_\eta(2).$$

- Wait until the velocity of the ship is stationary. We assume that $v_{x,water}$ and $v_{y,water}$ remain constant during this maneuver. Measure v_x and v_y, the relative velocities. Compute $v_\eta = v_x \cos\phi + v_y \sin\phi$. Since the velocities are stationary it appears that the sum of the outer forces is zero:

$$R_\eta = K_A v_\eta |v_\eta|.$$

  From this equation the value of K_A can be estimated by

$$K_A = \frac{R_\eta}{v_\eta |v_\eta|}.$$

Maneuver to determine the coefficient K_W
Do this maneuver when the wind velocity is low.

- Start the thrusters with a given power, where the η-forces are equal to zero and M is equal to zero. From these assumptions it follows that

$$R_\xi = R_\xi(1) + R_\xi(2).$$

- Wait until the velocity of the ship is stationary. We assume that $v_{x,water}$ and $v_{y,water}$ remain constant during this maneuver. Measure v_x and v_y the relative velocities. Compute $v_\xi = v_x \sin\phi - v_y \cos\phi$. Since the velocities are stationary it appears that the sum of the outer forces is zero:

$$R_\xi = K_W v_\xi |v_\xi|.$$

From this equation the value of K_A can be estimated by

$$K_W = \frac{R_\xi}{v_\xi |v_\xi|}.$$

Maneuver to determine the coefficient K_N
Do this maneuver when the wind velocity is low.

- Give the ship a rotation velocity $\frac{d\phi}{dt}(0)$ and a relative velocity $v_\xi = 0$.
- Switch off the thrusters at $t = 0$.
- The resulting equation is:

$$I \frac{d^2\phi}{dt^2} = -K_N \frac{d\phi}{dt}.$$

This differential equation can be solved. The solution is given by the following expression:

$$\frac{d\phi}{dt}(t) = \frac{d\phi}{dt}(0) \exp(-\frac{K_N}{I} t).$$

- Measure $\frac{d\phi}{dt}(t_{end})$. Then the value of K_N can be estimated by

$$K_N = \frac{I}{t_{end}} \left(\ln(\frac{d\phi}{dt}(0)) - \ln(\frac{d\phi}{dt}(t_{end})) \right)$$

Maneuver to determine the coefficient K_M
Since all other constants are known the coefficient K_M can be estimated as follows. Make a curve with the ship. Use the same data in the simulation and adjust the coefficient K_M such that the simulated trajectory matches the measured trajectory.

Maneuver to determine the wind relates coefficients
The same type of maneuvers can be done in order to determine $K_{A,wind}$, $K_{W,wind}$ and $K_{M,wind}$. At this moment it is not necessary to specify these maneuvers.

3 The static model of the thrusters

A number of algorithms has already be described in [4]. In this section we shall describe algorithms for more complicated situations.

3.1 Static model for two thrusters with a non constant power

In this subsection we assume that the x, y coordinates are connected with the ship, where the origin is equal to the mass center and the positive y-axis is pointing in the direction of the bow. We start again with two thrusters. However, now the forces are no longer given in Cartesian components, but in polar coordinates. As usual we assume that R is the size of the required force, Q is the angle of the force and M is the moment. Furthermore, we assume that:

- the coordinates of the thrusters are: $(xpos(1), ypos(1))$ and $(xpos(2), ypos(2))$.
- the thrusters are further characterized by:
  - angle Φ_1 and Φ_2 given in radials;
  - power P_1 and P_2.

The resulting forces are defined as follows:

$$R_1 = R_1(P_1, \Phi_1, \Phi_2)$$

and

$$R_2 = R_2(P_2, \Phi_1, \Phi_2).$$

This forces and angles should be chosen such that the usual equations are satisfied. These equations are summarized below.

$$R_1(P_1, \Phi_1, \Phi_2)\cos\Phi_1 + R_2(P_1, \Phi_1, \Phi_2)\cos\Phi_2 = R\cos(\frac{Q\pi}{180^o}), \tag{14}$$

$$R_1(P_1, \Phi_1, \Phi_2)\sin\Phi_1 + R_2(P_1, \Phi_1, \Phi_2)\sin\Phi_2 = R\sin(\frac{Q\pi}{180^o}), \tag{15}$$

and

$$-ypos(1)R_1(P_1, \Phi_1, \Phi_2)\cos\Phi_1 - ypos(2)R_2(P_1, \Phi_1, \Phi_2)\cos\Phi_2 +$$
$$+xpos(1)R_1(P_1, \Phi_1, \Phi_2)\sin\Phi_1 + xpos(2)R_2(P_1, \Phi_1, \Phi_2)\sin\Phi_2 = M. \tag{16}$$

The idea is the following: take $\Phi_1 = 0, h, ..., 2\pi$ and determine for each Φ_1 the value of Φ_2, P_1, and P_2 such that the 3×3 non linear system given by (14), (15), and (16) is satisfied. Determine then $\Phi_{1,min}$ such that $|P_1| + |P_2|$ is minimal.

As an example we consider the following function:

$$R_1(P_1, \Phi_1, \Phi_2) = P_1 \times f_1(\Phi_1) \times g_1(\Phi_1, \Phi_2), \tag{17}$$

where f_1 and g_1 are chosen as:

$$f_1(\Phi_1) = \begin{array}{l} 1, \ \frac{\pi}{4} \leq \Phi_1 \leq 2\pi, \\ 1 - \frac{4}{\pi}(\frac{\pi}{4} - \Phi_1), \ \frac{\pi}{8} \leq \Phi_1 \leq \frac{\pi}{4}, \\ 1 - \frac{4}{\pi}\Phi_1, \ 0 \leq \Phi_1 \leq \frac{\pi}{8}, \end{array}$$

and

$$g_1(\Phi_1, \Phi_2) = \begin{array}{l} 1, \ \frac{\pi}{8} \leq \Phi_1 \leq 1\frac{7}{8}\pi, \\ \\ \frac{1}{2}, \ -\frac{\pi}{8} \leq \Phi_1, \ \ \Phi_2 \leq \frac{\pi}{8}. \end{array}$$

In the same way the function $R_2(P_2, \Phi_1, \Phi_2)$ can be defined. In this advanced two thruster model, we use the following algorithm, where we assume that the required forces and momentum are given:

1. compute the solution with the standard model,
2. compute P_1, P_2, Φ_1, and Φ_2 from this model,

3. use these values as a start for the advanced model,
4. step $\Phi_1^0, \dots \Phi_1^N$ where $N \times h = 2\pi$,
5. compute Φ_1 such that $|P_1| + |P_2|$ is minimal.

As a first model we take $R_1 = P_1$ and $R_2 = P_2$. Assume that Φ_1 is given, and determine P_1, P_2, and Φ_2 such that

$$f_1(P_1, P_2, \Phi_2) = 0,$$
$$f_2(P_1, P_2, \Phi_2) = 0,$$
$$f_3(P_1, P_2, \Phi_2) = 0,$$

where

$$f_1(P_1, P_2, \Phi_2) = P_1 \cos \Phi_1 + P_2 \cos \Phi_2 - F_x,$$
$$f_2(P_1, P_2, \Phi_2) = P_1 \sin \Phi_1 + P_2 \sin \Phi_2 - F_y,$$

and

$$f_3(P_1, P_2, \Phi_2) = -ypos(1) P_1 \cos \Phi_1 - ypos(2) P_2 \cos \Phi_2 + xpos(1) P_1 \sin \Phi_1 + xpos(2) P_2 \sin \Phi_2 - M.$$

In order to solve this non-linear system we use the Newton-Raphson method [1]:

$$P_{new} = P_{old} - (F'(P_{old}))^{-1} F(P_{old}),$$

where P_{new} and P_{old} are vectors with components (P_1, P_2, Φ_2) for the new and for the previous iteration. Furthermore F' is the Jacobian of the non-linear vector function $(f_1(P_1, P_2, \Phi_2), f_2(P_1, P_2, \Phi_2), f_3(P_1, P_2, \Phi_2))$. The iteration is stopped if

$$\frac{\|F(P_{new})\|_2}{\|(F_x, F_y, M)\|_2} \leq \varepsilon.$$

Below the coefficients of the 3×3 Jacobian matrix are given:

$$\frac{\partial f_1}{\partial P_1} = \cos \Phi_1, \quad \frac{\partial f_1}{\partial P_2} = \cos \Phi_2, \quad \frac{\partial f_1}{\partial \Phi_2} = -P_2 \sin \Phi_2,$$
$$\frac{\partial f_2}{\partial P_1} = \sin \Phi_1, \quad \frac{\partial f_2}{\partial P_2} = \sin \Phi_2, \quad \frac{\partial f_2}{\partial \Phi_2} = P_2 \cos \Phi_2,$$
$$\frac{\partial f_3}{\partial P_1} = -ypos(1) \cos \Phi_1 + xpos(1) \sin \Phi_1,$$
$$\frac{\partial f_3}{\partial P_2} = -ypos(2) \cos \Phi_2 + xpos(2) \sin \Phi_2,$$
$$\frac{\partial f_3}{\partial \Phi_2} = ypos(2) P_2 \sin \Phi_2 + xpos(2) P_2 \cos \Phi_2.$$

This method works fine. The results for this simple model and the previous model are identical. For the more advanced model, where the resulting force R_1 is given by (17), we use the same algorithm where the Jacobian is based on the simple model. In a certain sense this can be interpreted as a pseudo Newton-Raphson method.

3.2 Numerical experiments

We consider two thrusters, where the power of the thrusters depends on the angle. The results are given in Figure 6 for the standard situation and in Figure 7 for the more advanced model. Note that there are considerable differences in the angle and power of the thrusters.

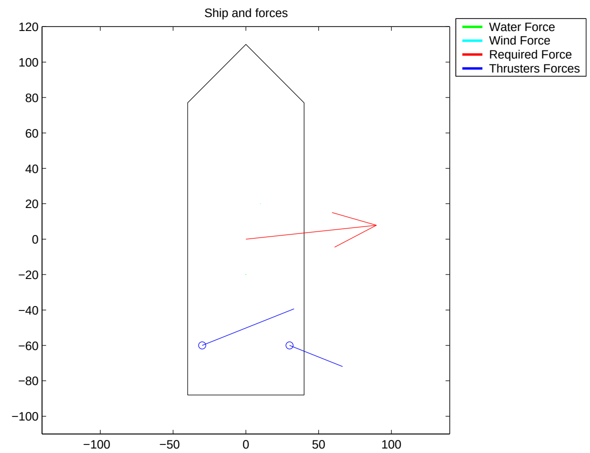

Figure 6: The orientation and power of the thrusters in a standard situation

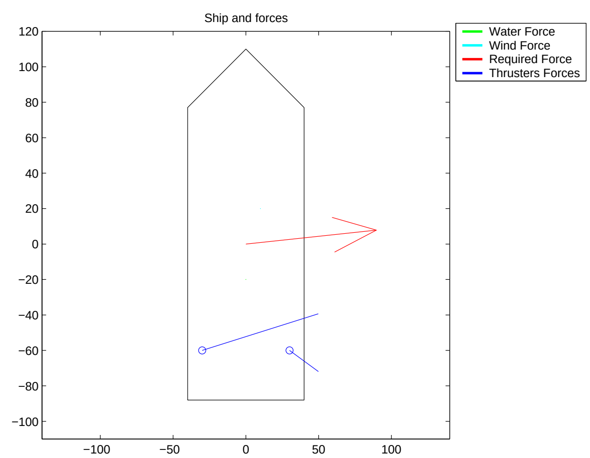

Figure 7: The orientation and power of the thrusters in a more complicated situation

4 Control of the position of a ship

In this section we consider a simple situation. The ship should follow a straight line with angle of 45^o and the angle of the ship should also be equal to 45^o. Below we give the control options to adapt the moment and the forces in order to arrive at the required line with the required orientation. We assume three constants, which can be chosen to influence the control: ω which is a relaxation parameter, ε, which is used as a tolerance, and $t_{control}$, which is the time needed to arrive at the required line with the required orientation.

Control for the momentum
First, we try to find the moment such that after $t_{control}$ the ship has the correct orientation. Therefor we solve the simplified dynamic momentum equation

$$\frac{d^2\phi}{dt^2} = \frac{M}{I}.$$

Integration in time leads to

$$\frac{d\phi}{dt} = \frac{M}{I}t + \phi'(t).$$

A second integration from time t to time $t + t_{control}$ yields

$$\phi(t + t_{control}) = \phi(t) + \frac{M}{2I} t_{control}^2 + \phi'(t) t_{control}.$$

This implies that the required control of the momentum is equal to (including the relaxation parameter):

$$M = \omega \frac{2I}{t_{control}^2} (\frac{\pi}{4} - \phi(t) - \phi'(t) t_{control}).$$

Control for the forces

The distance from the ship to the line with angle 45^o is equal to $\sqrt{\frac{1}{2}} |x_s - y_s|$, where x_s and y_s are the coordinates of the ship. We only consider the x-direction, because the y-direction can be done in a similar way. Again we solve the simplified dynamic equation (unless the difference is less than ε):

$$m \frac{d^2 x_s}{dt^2} = R_x.$$

Integration in time leads to

$$\frac{dx_s}{dt} = \frac{R_x t}{m} + x_s'(t).$$

So the distance after $t + t_{control}$ is

$$distance = \frac{R_x}{2m} t_{control}^2 + x_s'(t) t_{control}.$$

This implies that the required force is

$$R_x = \frac{2m}{t_{control}^2} (distance - x_s'(t) t_{control}).$$

The choice for the sign of the distance depends on position of the ship above the required line or below the required line.

In Figure 8 a trajectory of the ship is given for a typical situation.

5 Conclusions

It appears that the *Stadyc* model for the movement of a ship works very well. The various difficulties are subdivided into different problems, which can be solved separately. First the optimal positioning of the thrusters can be computed for the static (*Sta*) situation. Then the dynamic (*dy*) model can be used to predict the trajectory of the ship. Finally, the forces and positions of the thrusters are adapted in the control (*c*) model in order to make that the ship follows a given path as good as possible.

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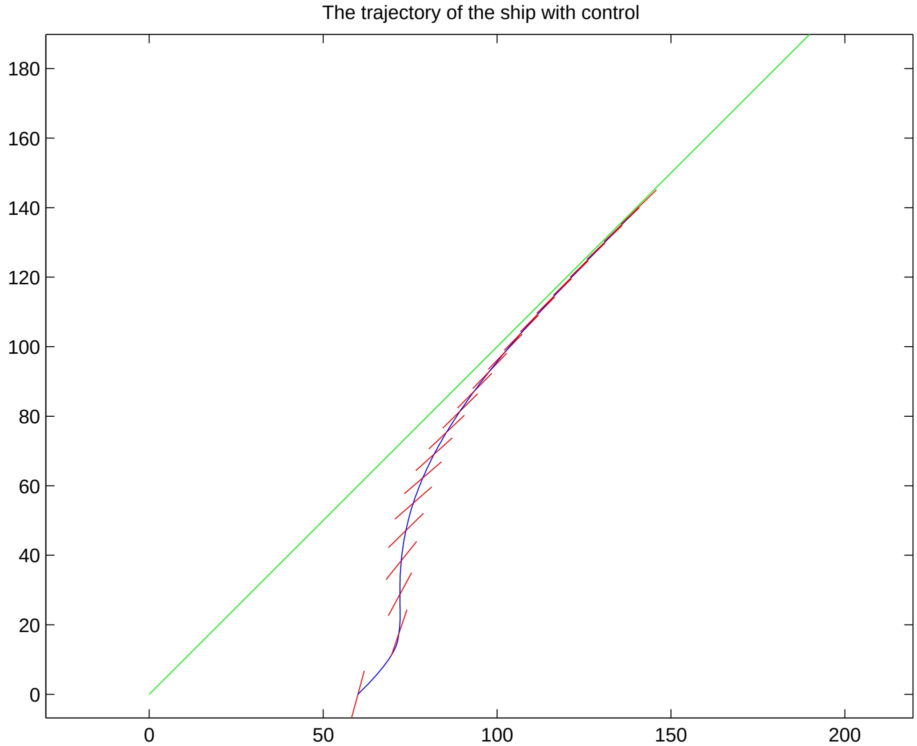

Figure 8: The trajectory of the ship using control

[4] C. Vuik and I. Benchev. The static model for the multi thruster control system without constraints. Report 07-02, Delft University of Technology, Department of Applied Mathematical Analysis, Delft, 2007.