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The static model for the Multi Thruster Control System without constraints

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Abstract

In this report we give a simple model for the determination of the best position of the thrusters if a required force and moment are specified by the position and orientation of the joystick. We start with two thrusters and generalize the model to multiple thrusters. Some numerical experiments are given to illustrate the power of the developed algorithm. In this research we assume that there are no constraints imposed on the power of the thrusters.

Keywords. numerical model, Multi Thruster Control System, ship maneuvering

1 Introduction

In this section we give a short description of the company HRP Thruster Systems. Thereafter we specify the Multi Thruster Control System designed by HRP.

1.1 HRP Thruster Systems

HRP Thruster Systems is a company, which has been making every effort to develop thruster solutions to satisfy customer requirements. HRP is concentrating on total solutions for steerabel thrusters to be used for any application.

Cost-effective marine transportation has been, and always will be, an important issue. Developments in shipbuilding technology are constantly leading to new insights, using new techniques, shapes, energy, power and electronics for the purpose of staying competitive. Today's vessels are being manned with crews that are increasing in knowledge, thus allowing a decrease in the number of crew members. Ease in operations, reliability of systems and quick maintenance, while maintaining low costs are of vital importance to the customers of HRP.

Steerable thrusters are composed of different parts that integrate with each other for a total solution. The performance of the system is determined by the power and the tuning of the thruster, the hydraulic steering system, the transmission and electronic control. There are different combinations that lead to a total solution that is tailored to meet the needs of each client.

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Figure 1: Two examples of vessels equipped with HRP thrusters

1.2   The Multi Thruster Control System

Features
The HRP MTC System can be used in different modes which can be selected subject to the circumstances on the river or operation area.

- Maneuvering mode - to be used during mooring, or maneuvering in narrow passages;

- Auto Pilot mode - to be used for long journeys;

- Steering mode - to be used for short transits.

Maneuvering mode
Side Stepping: The vessel is moving sideways, maintaining the heading. Optimal setting and working angle of the thrusters is calculated by the system. The side step action can be combined by either moving forward or backward. The direction of the joystick determines the direction of the move.

Rotation: The vessel will rotate around its center of rotation without forward speed. The location of the center point can be moved from mid-ship towards aft-ship or fore ship on the vessel. This action is accomplished by rotation of the joystick.

Rotation combined with longitudinal movement: The vessel will make a combined move going both forward (or backward) and change heading. The forward tail function of the system predicts what move the ship will be making by projecting time steps on the display. This action is accomplished by forward stroke and rotation of the joystick.

Auto Pilot mode: Used for long-term straight-forward sailing. Both rate-of-turn course keeping or way-point steering functions are available. In this mode, side stepping or rotation at zero vessel speed has been disabled. Course changes are possible by rotation of the joystick to a new heading.

Steering mode
This function is for manual steering in forward or backward direction. Instead of "heading selection" with an Auto Pilot, a "rate-of-turn" is selected. All thrusters are responding to the movement of the joystick simultaneously, pointing in the same direction at all time.

Individual thruster configuration: Each thruster within the program can be configured individually, independent from the other thrusters. This can be done by double clicking on the thruster symbol on the right side of the screen which shows a thruster control menu.

In case of any failure on the engine, drive line or the thruster, that particular thruster can either be reduced in power or even completely canceled. This means that the system will neglect this thruster for further operation of the vessel and re-configure its algorithm to make the remaining thrusters compensate for the changes.

In this menu, it is also possible to set minimum and maximum drive speeds for each prime move (diesel engine or electric motor). Particularly for engines running on heavy fuel, it is recommended not to run the engine under an e.g. 35% power loading to avoid pollution. To cope with this, the system will be programmed such that the minimum speed will not be less than for example 60%. If reduced forward thrust is required, i.e. less than what is achieved with 60% of the power of the engine, the program will oppose the thrusters accordingly.

Figure 2: Some snapshots of the HRP multi thruster control system

2   The mathematical model

To derive the mathematical model we note that the complete description of the movement of a vessel combined with the angle and power of the thrusters is too complicated. In principle two models can be used: the dynamic model and the static model. In the dynamic model we use Newton's law, which shows that the mass times the acceleration is equal to the sum of the forces. A comparable description can be given for the moment equation. This leads to a set of differential equations, which can be solved if all forces are known. To keep things simple we first restrict ourselves to the static model.

In the static model we make the assumption that the forces and moments are restricted to 2 dimensions. We use the following definitions:

- R is the force requested by the position of the joystick,

- Rex_i is the ith exterior force, possibilities are: wind, water resistance force etc., we take $i = 1, \ldots, m$,

- R_j is the jth interior force, or force provided by the thrusters, $j = 1, \ldots, n$.

In our static model we assume that

$$R = \sum_{i=1}^{m} Rex_i + \sum_{j=1}^{n} R_j. \tag{1}$$

For the moments we have the same notations:

- M is the moment requested by the position of the joystick,

- Mex_i is the ith exterior moment, $i = 1, \ldots, m$,

- M_j is the jth interior moment, $j = 1, \ldots, n$.

We again assume that

$$M = \sum_{i=1}^{m} Mex_i + \sum_{j=1}^{n} M_j. \tag{2}$$

We define the x and y axis of our coordinates such that the origin is located in the center of mass of the vessel, and the positive y-axis is pointing in the direction of the bow. The angle with the positive x-axis is denoted by ϕ, which is positive in the counter clock wise direction. All moments are calculated with respect to the center of mass of the vessel.

3 Mathematical model for 2 thrusters

In this section we consider the simple situation of a vessel with 2 thrusters. In Subsection 3.1 we consider the mathematical model and give some numerical experiments in Subsection 3.2.

3.1 Mathematical model

We take the following notations:

- $a1$, $b1$ are the coordinates of Thruster 1,

- $a2$, $b2$ are the coordinates of Thruster 2,

- R is the size of the force,

- Q is the angle of the force given in degrees,

- M is the moment.

We can decompose the force in its x and y components as follows:

$$R_x = R \times \cos(\frac{Q\pi}{180^o}),$$

$$R_y = R \times \sin(\frac{Q\pi}{180^o}).$$

Using the force and moment balance one obtains the following equations:

$$R_{1x} + R_{2x} = R_x, \tag{3}$$

$$R_{1y} + R_{2y} = R_y, \tag{4}$$

and

$$-b1 \times R_{1x} + a1 \times R_{1y} - b2 \times R_{2x} + a2 \times R_{2y} = M. \tag{5}$$

This is a system with 3 equations and 4 unknowns. To simplify things we express R_{1y}, R_{2x} and R_{2y} as a function of R_{1x}. From (3) and (4) it easily follows that

$$R_{2x} = R_x - R_{1x}, \tag{6}$$

$$R_{2y} = R_y - R_{1y}. \tag{7}$$

Substituting (6) and (7) in (5) leads to

$$-b1 \times R_{1x} + a1 \times R_{1y} - b2 \times (R_x - R_{1x}) + a2 \times (R_y - R_{1y}) = M.$$

Rewriting this equation shows that

$$(-b1 + b2) \times R_{1x} + (a1 - a2) \times R_{1y} = M + b2 \times R_x - a2 \times R_y.$$

From now on we assume that $a1 \neq a2$. This implies that R_{1y} and R_{2y} can be written as:

$$R_{1y} = \frac{M + b2 \times R_x - a2 \times R_y + (b1 - b2) \times R_{1x}}{a1 - a2}, \tag{8}$$

$$R_{2y} = R_y - \frac{M + b2 \times R_x - a2 \times R_y + (b1 - b2) \times R_{1x}}{a1 - a2}. \tag{9}$$

Define the following constants: $c1 = \frac{M + b2 \times R_x - a2 \times R_y}{a1 - a2}$ and $c2 = \frac{b1 - b2}{a1 - a2}$. We obtain from (6), (8) and (9) the following expressions:

$$R_{2x} = R_x - R_{1x},$$

$$R_{1y} = c1 + c2 \times R_{1x},$$

$$R_{2y} = R_y - c1 - c2 \times R_{1x}.$$

The total power of the thrusters is given by

$$g = \sqrt{(R_{1x})^2 + (R_{1y})^2} + \sqrt{(R_{2x})^2 + (R_{2y})^2}.$$

The function g is called the energy function. Note that g can be written as a function of R_{1x}:

$$g(R_{1x}) = \sqrt{(R_{1x})^2 + (c1 + c2 \times R_{1x})^2} + \sqrt{(R_x - R_{1x})^2 + (R_y - c1 - c2 \times R_{1x})^2}. \tag{10}$$

We now determine the value of R_{1x} such that the function g is minimal. Thereafter the other components of the forces are computed by the given formulas.

In order to show that the energy has only one minimum, we plot g as a function of R_{1x} in Figure 3 for a typical choice of parameters. It appears that g is minimal in the neighborhood of $R_{1x} = 30$. One method to compute the minimal value of g is the so-called 'brute force' method. In this
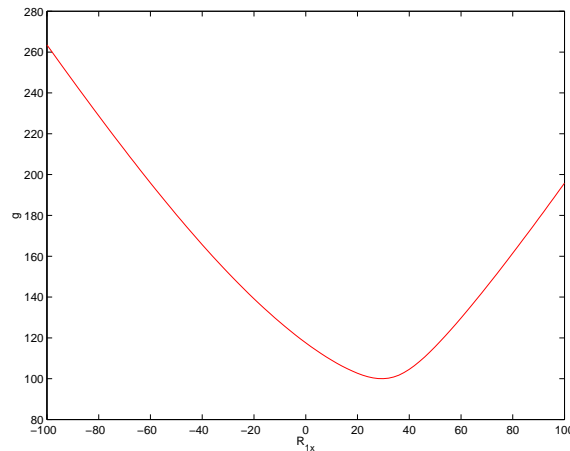

Figure 3: The energy function g as a function of R_{1x}

method one divides the interval $[-100, 100]$ in N subintervals. The boundaries of the subintervals are called grid points. Then an array is made of length N, which contains the values of g in these grid points. Thereafter the minimal value of this array is determined and one obtains a good approximation of the value of R_{1x} such that $g(R_{1x})$ is minimal. For two thrusters, this is a feasible method. Choosing $N = 100$ one needs 100 function evaluations to obtain an approximation of the

optimal value of R_{1x}. For present day personal computers this takes only a fraction of second to compute this value. However, when more thrusters are used one get serious problems with this 'brute force' method. The reason for this is as follows: increasing the number of thrusters with one extra thruster leads to an increase of two extra variables. So for 10 thrusters one has 17 variables, which can be chosen in order to minimize the energy function. Using the 'brute force' method one needs 100^{17} function evaluations. This leads to unacceptable computing times even on a modern supercomputer. This problem is known as the 'curse of dimensionality'. In the next paragraph we try to find a better solution method.

It is well known that to find a minimum value of a function, one can also compute the zero value of the derivative of this function. Hopefully, this solves the 'curse of dimensionality' problem for more thrusters. The first derivative of g is given by:

$$g'(R_{1x}) = \frac{R_{1x} + c2 \times (c1 + c2 \times R_{1x})}{\sqrt{(R_{1x})^2 + (c1 + c2 \times R_{1x})^2}} - \frac{(R_x - R_{1x}) + c2 \times (R_y - c1 - c2 \times R_{1x})}{\sqrt{(R_x - R_{1x})^2 + (R_y - c1 - c2 \times R_{1x})^2}} \quad (11)$$

In Figure 4 we plot the derivative of g. Note that indeed this function crosses the R_{1x}-axis in the neighborhood of $R_{1x} = 30$, which is a good approximation of the optimal value of R_{1x}.

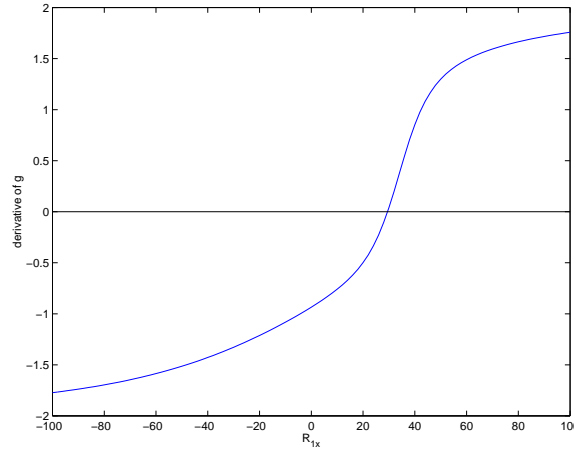In order to compute the value of R_{1x} such that $g(R_{1x})$ is minimal we compute R_{1x} such that

Figure 4: The derivative of the energy function g' as a function of R_{1x}

$g'(R_{1x}) = 0$. Various numerical methods can be used to approximate the zero of a function: bi-section, a fixed point iteration and the Newton Raphson method. As a first try, we take the simple Koorden Newton method. This method requires two starting values [2]. Our first starting value is $p_0 = 0$. If $g'(p_0) = 0$ we stop, if $g'(p_0) > 0$ we take $p_1 = -0.1 \times R$, whereas if $g'(p_0) < 0$ we take $p_1 = 0.1 \times R$. Thereafter we use the following algorithm:

$k = 1$

while $|g'(p_k)| \geq 10^{-4} \times R$

$\quad p_{k+1} = p_k - g'(p_k) \times \frac{p_k - p_{k-1}}{g'(p_k) - g'(p_{k-1})}$

$\quad k = k + 1$

end

$R_{1x}(min) = p_{k+1}$

The result is close to the result with the first 'brute force' method. It appears that using the first

method 1908 flops are used and for the second method only 1057 flops are used. So we see already for the two thruster model, that there is a gain in work by using a more advanced mathematical solution method.

3.2 Numerical experiments

We have computed a number of examples with both methods: the Matlab method provided by HRP and the method discussed Section 3.1. In most cases the results are more or less the same. In Figure 5 and 6 the results are given for: $a1 = -30, b1 = -60, a2 = 30, b2 = -60, Q = 70, R = 100$, and $M = 10$. For the present model we have the following results:

$R1x = 30.00$
$R1y = 81.02$
$R1 = 86.40$
$phi1 = 69.68$
$R2x = 4.20$
$R2y = 12.95$
$R2 = 13.61$
$phi2 = 72.02$

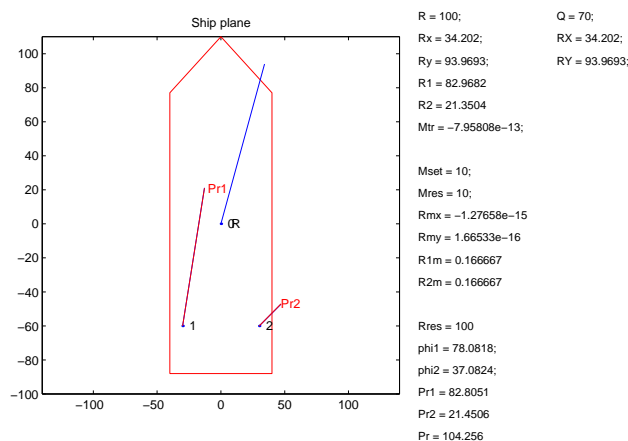Note that in this example the results are slightly different.

Figure 5: The results with the HRP model

4 The mathematical model for multiple thrusters

We make the following assumptions:

- the number of thrusters is equal to n_t,

- the coordinates of the thrusters are given by

$$(xpos(1), ypos(1)), (xpos(2), ypos(2)), \ldots, (xpos(n_t), ypos(n_t)),$$

- R is the size of the force, Q is the angle of the force given in degrees and M is the moment.
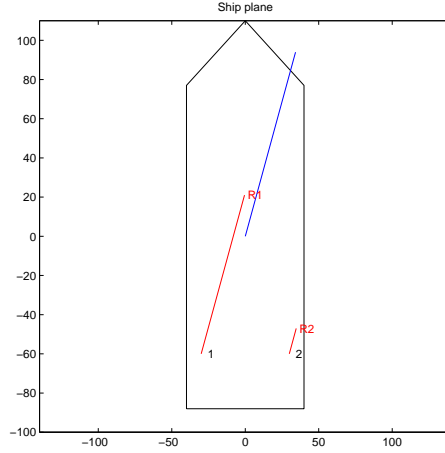
As before we define:

$$R_x = R \times \cos(\frac{Q\pi}{180^o}),$$

Figure 6: The results with the present model

$$R_y = R \times \sin(\frac{Q\pi}{180^o}).$$

From the balance of force we obtain:

$$R_1(1) + R_1(2) + \ldots + R_1(n_t) = R_x,$$

$$R_2(1) + R_2(2) + \ldots + R_2(n_t) = R_y.$$

The momentum rule leads to

$$-ypos(1) \times R_1(1) \ldots - ypos(n_t) \times R_1(n_t) + xpos(1) \times R_2(1) \ldots + xpos(n_t) \times R_2(n_t) = M. \quad (12)$$

This is a system with 3 equations and $2 \times n_t$ unknowns, so $2 \times n_t - 3$ variables can be chosen arbitrarily. We take the same approach as for two thrusters, we express $R_1(n_t)$ and $R_2(n_t)$ as a function of the other forces, so

$$R_1(n_t) = R_x - R_1(1) - R_1(2) - \ldots - R_1(n_t - 1), \quad (13)$$

$$R_2(n_t) = R_y - R_2(1) - R_2(2) - \ldots - R_2(n_t - 1). \quad (14)$$

Substituting (13) and (14) into (12) leads to

$$(ypos(n_t) - ypos(1)) \times R_1(1) + \ldots + (ypos(n_t) - ypos(n_t - 1)) \times R_1(n_t - 1) - ypos(n_t) \times R_x +$$

$$(xpos(1) - xpos(n_t)) \times R_2(1) + \ldots + (xpos(n_t - 1) - xpos(n_t)) \times R_2(n_t - 1) + xpos(n_t) \times R_y = M.$$

Now we assume that $xpos(n_t - 1) \neq xpos(n_t)$. Using this assumption, one can also describe $R_2(n_t - 1)$ as

$$
\begin{aligned}
R_2(n_t - 1) \quad = \quad & \frac{M + ypos(n_t) \times R_x - xpos(n_t) \times R_y}{xpos(n_t - 1) - xpos(n_t)} + \\
& + \frac{ypos(1) - ypos(n_t)}{xpos(n_t - 1) - xpos(n_t)} R_1(1) + \ldots + \frac{ypos(n_t - 1) - ypos(n_t)}{xpos(n_t - 1) - xpos(n_t)} R_1(n_t - 1) + \\
& + \frac{xpos(n_t) - xpos(1)}{xpos(n_t - 1) - xpos(n_t)} R_2(1) + \ldots + \frac{xpos(n_t) - xpos(n_t - 2)}{xpos(n_t - 1) - xpos(n_t)} R_2(nt - 2).
\end{aligned}
$$

Define

$$s \; = \; \frac{M + ypos(n_t) \times R_x - xpos(n_t) \times R_y}{xpos(n_t - 1) - xpos(n_t)},$$

$$c_1(j) \; = \; \frac{ypos(j) - ypos(n_t)}{xpos(n_t - 1) - xpos(n_t)}, \;\; j = 1, \ldots, n_t - 1,$$

$$c_2(j) \; = \; \frac{xpos(n_t) - xpos(j)}{xpos(n_t - 1) - xpos(n_t)}, \;\; j = 1, \ldots, n_t - 2,$$

then

$$R_2(n_t - 1) = s + \sum_{j=1}^{n_t - 1} c_1(j) \times R_1(j) + \sum_{j=1}^{n_t - 2} c_2(j) \times R_2(j). \tag{15}$$

Finally, we substitute (15) into (14) and obtain:

$$R_2(n_t) = R_y - s - \sum_{j=1}^{n_t - 1} c_1(j) \times R_1(j) - \sum_{j=1}^{n_t - 2} c_2(j) \times R_2(j).$$

The total power of all thrusters is defined as:

$$g(R_1(1), \ldots, R_1(n_t - 1), R_2(1), \ldots, R_2(n_t - 2)) = \sum_{j=1}^{n_t} \sqrt{(R_1(j))^2 + (R_2(j))^2}.$$

A natural candidate for the minimal value of g is to compute the solution of the following non-linear system:

$$\frac{\partial g}{\partial R_1(1)} \; = \; 0,$$

$$\vdots \;\; \vdots \;\; \vdots$$

$$\frac{\partial g}{\partial R_1(n_t - 1)} \; = \; 0,$$

$$\frac{\partial g}{\partial R_2(1)} \; = \; 0,$$

$$\vdots \;\; \vdots \;\; \vdots$$

$$\frac{\partial g}{\partial R_2(n_t - 2)} \; = \; 0.$$

So, now we have to calculate the partial derivatives of g and use a zero point algorithm to compute the solution of the resulting non-linear system. For the partial derivatives we obtain the following formulas:

$$\frac{\partial g}{\partial R_1(j)} \; = \; \frac{R_1(j)}{\sqrt{(R_1(j))^2 + (R_2(j))^2}} + \frac{R_2(n_t - 1) \times c_1(j)}{\sqrt{(R_1(n_t - 1))^2 + (R_2(n_t - 1))^2}}$$

$$- \; \frac{R_1(n_t) + R_2(n_t) \times c_1(j)}{\sqrt{(R_1(n_t))^2 + (R_2(n_t))^2}}, \;\; j = 1, \ldots, n_t - 1,$$

$$\frac{\partial g}{\partial R_2(j)} \; = \; \frac{R_2(j)}{\sqrt{(R_1(j))^2 + (R_2(j))^2}} + \frac{R_2(n_t - 1) \times c_2(j)}{\sqrt{(R_1(n_t - 1))^2 + (R_2(n_t - 1))^2}}$$

$$- \; \frac{R_2(n_t) \times (1 + c_2(j))}{\sqrt{(R_1(n_t))^2 + (R_2(n_t))^2}}, \;\; j = 1, \ldots, n_t - 2.$$

When we have computed the solution, we can compute $R_1(n_t)$, $R_2(n_t - 1)$, and $R_2(n_t)$ by (13), (14) and (15), respectively.

Efficiency

Let us define the efficiency eff of the thrusters by

$$eff(n_t, R, Q, M) = \frac{R}{g_{min}}.$$

If the position of the thrusters is given, then the following inequality should hold:

$$eff(n_t + 1, R, Q, M) \geq eff(n_t, R, Q, M).$$

In other words: if a thruster is added the efficiency should be increased. It would be nice to check this by using our program.

Since it is not so easy to find a standard Matlab method to solve the non-linear system, we use a slightly different approach. We use the so-called steepest descent method, see for instance Section 1.5 of [1] to find the minimal energy. The idea is the following: the graph of the energy can be seen as a mountainous region. The direction of the steepest descent can be computed by mathematical means using the gradient vector:

$$-\nabla g = - \begin{pmatrix} \frac{\partial g}{\partial R_1(1)} \\ \vdots \\ \frac{\partial g}{\partial R_1(n_t-1)} \\ \frac{\partial g}{\partial R_2(1)} \\ \vdots \\ \frac{\partial g}{\partial R_2(n_t-2)} \end{pmatrix}.$$

So, now we know the direction to look for the minimum of the energy. The question is: how far should we go into this direction? To answer this question we use a line-search method. We take as a new approximation:

$$R^{new} = R^{old} - \omega \nabla g$$

and determine the value of ω_{opt}, such that the energy for R^{new} is minimal. The search for ω_{opt} is splitted into two parts.

In the first part we start with $\omega_{min} = 0$ and $\omega_{max} = 1$, together with g_{min} and g_{max}. We take $\omega_{min} = \omega_{max}$ and multiply ω_{max} with a factor 2 until g_{max} is larger than g_{min}. At this moment we know that ω_{opt} is between ω_{min} and ω_{max}.

In the second part we halve the interval such that ω_{opt} remains between ω_{min} and ω_{max}. We stop the iteration if the distance between ω_{min} and ω_{max} is less than some prescribed value. Finally, the resulting forces are plotted.

5 Numerical results

We consider the same example as before where $R = 100$, $Q = 0$, and $M = 10$. With this advanced method it is possible to compute the optimal solution with an arbitrary number of thrusters. The result is given in Figure 7.

We also consider the example of a vessel with three thrusters. The location of two thruster is the usual position (-30, -60) and (30, -60). The location of the third thruster is varied between (0,-60) until (0,40). The resulting efficiency is given in Figure 8. Note that the efficiency of the configuration varies between 44 % and 100 %. So our program can also be used to find an optimal configuration of the location of the thrusters on the vessel.
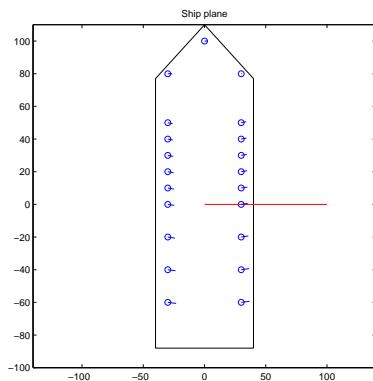
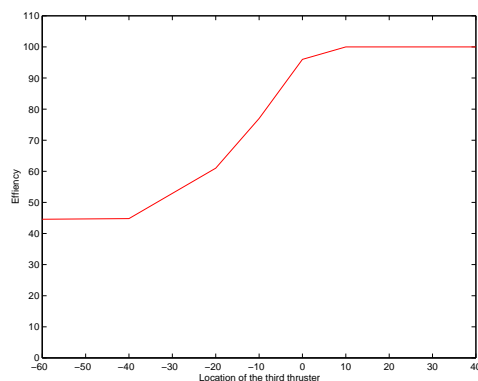Figure 7: Example of a vessel equipped with 21 HRP thrusters

Figure 8: Effiency of the thrusters as percentage of the required force

6   Executive summary

We start this report by describing the Multi Thruster Control System of HRP. This is a very powerful tool to maneuver a vessel with many thrusters by using one joystick. In this report we restrict ourselves to the static model: the required force and momentum are given and constant. The question is to determine the optimal position of the thrusters, which means that the required energy is minimal. In this report we assume that there are no upper nor lower bounds on the power of the thrusters. An algorithm is described, which can be used to determine the position of an arbitrary number of thrusters. This algorithm can also be used to optimize the location of the thrusters. Some examples are given to illustrate the power of the algorithm. In the future a study should be made to investigate the influence of constraints on the resulting output.

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