Assessment of multi class kinematic wave models

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1 Introduction

In the last decade many multi class kinematic wave (MCKW) traffic flow models have been proposed. MCKW models introduce heterogeneity among vehicles and drivers. For example, they take into account differences in (maximum) velocities and driving style. Nevertheless, the models are macroscopic and the flow is modeled as a continuum flow, without tracing individual vehicles. The first MCKW models were simple extensions of the mixed class kinematic wave model [1, 2]. For example, Wong and Wong introduced a kinematic wave model with unequal velocities for all classes [3]. More recent MCKW models take into account that some vehicle classes use more road space per vehicle than others and that this space occupancy may change if the velocity changes [4, 5, 6, 7].

Recently, frameworks were proposed to assess fundamental relations [8] and to assess car following traffic flow models [9, 10]. Some important properties of MCKW models have been analyzed before [3, 4, 5, 11, 12, 13], but no consistent assessment framework has been developed yet.

Our main contribution is the introduction of a framework for the assessment of MCKW models. It is applied to analyze whether MCKW models have certain important properties. The framework consists of a set of requirements (see Section 2) and a generalized MCKW model (see Section 3). In the full paper we show that all MCKW models known from literature fit in the generalized model. In Section 4 we apply the framework and assess all MCKW models. We conclude that only few models have all desirable properties. Finally, in Section 5 we give an outlook for the full paper.

2 Requirements

We propose two types of requirements: fundamental relation requirements and model dynamics requirements. The first type of requirements relate to the statics of the model: if there is a certain number of vehicles of each class, what is their velocity? What happens if the number of vehicles changes (slightly)? The second type of requirements relate to the velocity of characteristics carrying information: do vehicles react on each other and if so, which vehicle reacts on which vehicle and how quickly? To our opinion, all requirements are important properties of MCKW models and thus a model should satisfy them.

Fundamental relation requirements are:

1. In free flow the velocities of each class may differ.
2. In free flow the velocity of at least one class may decrease with increasing density.
3. In congestion the velocity of each class is equal.
4. If the density reaches a certain threshold (jam density), vehicle velocity is zero.
5. If the density of only one class increases, while all other class specific densities remain constant, vehicle velocities do not increase.

Model dynamics requirements are:

6. Characteristics have finite velocity.
7. Characteristics do not have a larger velocity than vehicles.

Requirement 1, 2 and 3 are in line with observed decreasing car velocity even at low densities, unequal velocities of cars and trucks at low densities and equal velocities at high densities [14, Chapter 8]. Requirement 3 is also in line with observed low velocity variance at high densities [15, 16]. Requirement 4 may seem trivial: if there are too many vehicles they will come to a complete stop. However, as we will see later, not all fundamental relations applied in MCKW models satisfy this requirement. Requirement 5 implies the following: if one car is added and the number of trucks and any other class remains the same, the vehicle velocity will decrease (or remain the same). This is only trivial if there is a simple relation between the densities and velocities, which is not the case in all models.

Requirement 6 implies that if a vehicle reacts on an other vehicle, it only does so after a certain nonzero time. Therefore, any disturbance will only have influence on the traffic state upstream or downstream after a certain time, if it has influence at all. Requirement 7 implies that the model is anisotropic and vehicles and drivers only react on their leaders and not on their followers. Therefore, disturbances can only downstream with at most the velocity of the vehicles. They can also travel upstream with any velocity.

3 Generalized MCKW model

The MCKW models known from literature are based on different principles and assumptions, see Table 1. However, they can all be fitted into a generalized MCKW model. The generalized model consists of the following equations. The multi class conservation of vehicles equation:

$$\frac{\partial \rho_u}{\partial t} + \frac{\partial \rho_u v_u}{\partial x} = 0 \tag{1a}$$

with ρ_u the class specific density of class u (vehicle/m), v_u its class specific velocity (m/s) and t and x time and space coordinates, respectively. The velocity v_u is determined by the class specific fundamental relation:

$$v_u = v_u(\rho) \tag{1b}$$

with ρ the effective density:

$$\rho = \rho(\rho_1, \dots, \rho_U) \tag{1c}$$

and U the total number of classes.

Table 1: Classification of multi-class models.

<table>
<tr><th>Principle/ assumption</th><th>Nr. of classes</th><th>Model reference</th><th>Fundamental relation</th><th colspan="2">Effective density</th></tr>
<tr><td>user equilibrium</td><td>2</td><td>Logghe & Immers [5]</td><td>Daganzo (bi-linear)</td><td colspan="2">depends on regime: free flow, semi congestion, congestion</td></tr>
<tr><td>road fractions</td><td rowspan="3">U</td><td>Ngoduy & Liu [4]</td><td rowspan="2">Smulders (quadratic-linear)</td><td colspan="2">$\rho = \bar{\eta} \sum_u \rho_u$, $\bar{\eta}$ ‘average’ pce</td></tr>
<tr><td>space occupancy</td><td>Van Lint et. al. [6], Van Wageningen-Kessels et. al. [7]</td><td rowspan="7">weighted summation</td><td>dynamic pce</td></tr>
<tr><td>flow through pores</td><td>Nair et. al. [17]</td><td>novel shape</td><td rowspan="4">constant pce</td></tr>
<tr><td rowspan="3">vehicle length</td><td>2</td><td>Chanut & Buisson [18]</td><td>Smulders</td></tr>
<tr><td rowspan="4">U</td><td rowspan="2">Benzoni-Gavage & Colombo [12]</td><td>Greenshields (quadratic)</td></tr>
<tr><td rowspan="2">Drake (bell-shaped)</td></tr>
<tr><td rowspan="2">unequal velocities</td><td>Wong & Wong [3]</td><td rowspan="2">pce $\eta_u = 1$ (unweighted)</td></tr>
<tr><td>Zhang et. al. [11]</td><td>shape undefined</td></tr>
</table>

Most original model formulations do not include an effective density function and a fundamental relation that only depends on the effective density. In the full paper, all MCKW models known from literature are reformulated such that they do include these functions and fit into the generalized model (1). It appears that the model equations differ with respect to the following three aspects (see also Table 1):

Number of classes Some models only include 2 classes, for others the number of classes is not limited.

Fundamental relation Most models apply a multi class version of fundamental relations that are used more often: the Greenshields, Daganzo, Smulders or Drake fundamental relation.

Effective density function In most models the effective density is a (weighted) summation of all class specific densities: $\rho = \sum_u \eta_u \rho_u$ with η_u the passenger car equivalent (pce) value. The pce value may be one or constant. In [6, 7] the pce value is dynamic, i.e. it depends on the actual traffic state.

4 Model assessment

The MCKW models known from literature are assessed with respect to the model requirements introduced in Section 2. It is relatively straightforward to assess the models with respect to Requirements 1–4. However, the generalized formulation is needed to assess the models with respect to the other requirements. The derivative of the fundamental relation (1b) with respect to the class specific densities $\rho_1, \ldots, \rho_U$ is calculates to asses whether Requirement 5 is satisfied. To asses whether Requirement 6 and 7 are satisfied, the generalized model (1) is reformulated in the Lagrangian (moving) coordinate system and subsequently as a system of equations [13]:

$$\frac{\partial \vec{s}}{\partial t} + \mathbf{J}(\vec{s})\frac{\partial \vec{s}}{\partial n} = 0 \tag{2}$$

with $\vec{s} = (s_1, \ldots, s_U)^T$ the vector of class specific spacings with the class specific spacing $s_u = 1/\rho_u$, $\mathbf{J}(\vec{s})$ the (state dependent) Jacobian matrix that results from the fundamental relation and the effective density function and n the vehicle number. The new formulation (2) simplifies the model assessment because the characteristic velocities in the Lagrangian coordinate system equal the eigenvalues of the Jacobian $\mathbf{J}(\vec{s})$. Therefore, if the eigenvalues are finite, the characteristic velocities are finite as well (Requirement 6). Furthermore, the model is anisotropic (Requirement 7) is the characteristics are not faster than vehicles. Therefore, the characteristic velocities in the Lagrangian formulation must be nonnegative. Therefore, we can assess the model with respect to Requirement 7 by assessing the sign of the eigenvalues of the Jacobian.

Applying the framework introduced above, we analyze all MCKW models known from literature. We conclude that only the models in [6, 7, 18] satisfy all requirements unconditionally. The model in [5] does not allow decreasing velocities in free flow. The model in [4] only satisfies all requirements under rather strict conditions on the pce values. Furthermore, in the models in [3, 17] and the model in [12] with Drake fundamental relation the velocity is not equal to zero for a certain (finite) jam density.

5 Outlook

We introduced a framework for the assessment of MCKW models. It includes a set of requirements that should be satisfied by a MCKW model. The generalized model is reformulated and applied in the assessment of all MCKW models known from literature. Both the framework and the model assessment are discussed in more detail in the full paper. Furthermore, we provide an outline of the steps needed to perform such a model assessment for any new MCKW model.

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