An Implicit **S**moothed **P**article **H**ydrodynamics Multiscale Method for Porous Media Flow

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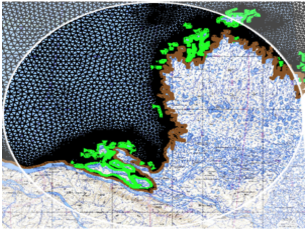


Overview

Reservoir Simulation in a Nutshell

SPH in a Nutshell

Meshless Scheme for Fluid Flow in Porous Media

Multiscale, Multilevel, Multigrid and Deflation Methods

Multiscale Restriction-Smoothed Basis (MsRBS) Method

Multiscale Meshless Based Method (MsMBM)

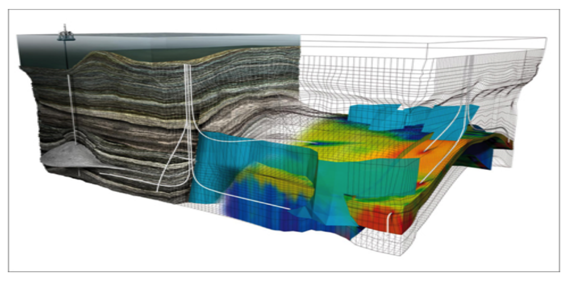
Numerical Results

Summary

Reservoir Simulation in a Nutshell

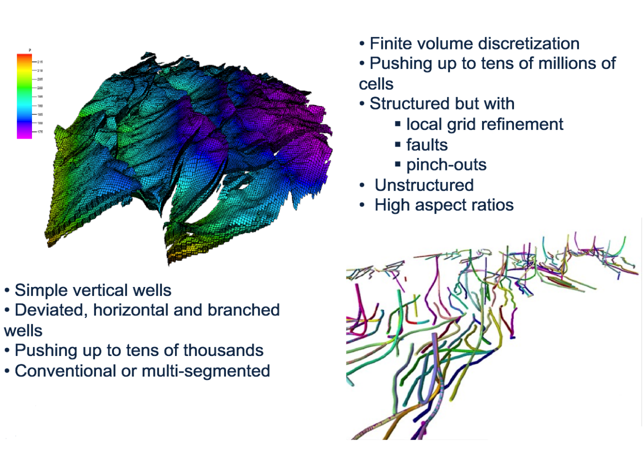
Background

- Need simulation to optimize production
  - Where to drill
  - How many wells
  - How long to produce
  - Where and when to inject fluids or gas
- Need simulation to reduce risks on million dollar investments.



Reservoir Simulation in a Nutshell

Background



- Finite volume discretization
- Pushing up to tens of millions of cells
- Structured but with
  - local grid refinement
  - faults
  - pinch-outs
- Unstructured
- High aspect ratios

- Simple vertical wells
- Deviated, horizontal and branched wells
- Pushing up to tens of thousands
- Conventional or multi-segmented

SPH in a Nutshell

Fundamental Relations $\forall \mathbf{r}', \mathbf{r} \in \mathbb{R}^3$

$$\mathbf{A}(\mathbf{r}) = \langle \mathbf{A}(\mathbf{r}'), \delta(\mathbf{r}-\mathbf{r}') \rangle = \int\limits_{\Omega, \mathbf{r}\in\Omega} \mathbf{A}(\mathbf{r}')\delta(\mathbf{r}-\mathbf{r}')d\mathbf{r}'$$

$$\langle \mathbf{1}, \delta(\mathbf{r}-\mathbf{r}') \rangle = \int\limits_{\Omega, \mathbf{r}\in\Omega} \delta(\mathbf{r}-\mathbf{r}')d\mathbf{r}' = 1,$$

Set of Kernel Functions:$\{W(\mathbf{r}-\mathbf{r}', h)\} \in C^1(\Omega)$

$$\lim_{h\to 0} \{W(\mathbf{r}-\mathbf{r}', h)\} =^{\text{weakly}}= \delta(\mathbf{r}-\mathbf{r}')$$

$$\int\limits_{\Omega, \mathbf{r}\in\Omega} W(\mathbf{r}-\mathbf{r}', h)d\mathbf{r}' = 1,$$

where h is the smoothing length.

SPH in a Nutshell

Basic Equalities:

$$\mathbf{A}(\mathbf{r}) = \lim_{h \to 0} \int\limits_{\Omega, \mathbf{r} \in \Omega} \mathbf{A}(\mathbf{r}') W(\mathbf{r} - \mathbf{r}', h) d\mathbf{r}'$$

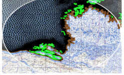
$$\mathbf{A}(\mathbf{r}) = \int\limits_{\Omega, \mathbf{r} \in \Omega} \mathbf{A}(\mathbf{r}') W(\mathbf{r} - \mathbf{r}', h) d\mathbf{r}' + \mathcal{O}(h^2) =$$

$$= \sum_{J \in \Omega_{\mathbf{r},h}} \mathbf{A}(\mathbf{r}_J) W(\mathbf{r} - \mathbf{r}_J, h) V_J + \mathcal{O}(h^2), \forall h \in \Omega_h,$$

$$\nu(\mathbf{r}_I, \mathbf{r}_J) = \sum_{\mathbf{r}_J \in \Omega_{\mathbf{r}_I, \tilde{h}_I}} W(\mathbf{r}_J - \mathbf{r}_I, \tilde{h}_{IJ}) V_{\mathbf{r}_J},$$

where $\mathbf{r}_J$ are the particles in the neighborhood of particle $\mathbf{r}$, V_J is the volume of particle J, $\Omega_{\mathbf{r},h}$ is the entire neighborhood of particle $\mathbf{r}$.

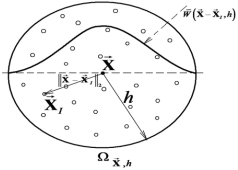
SPH in a Nutshell



Kernel Function:

$$W(z,h) = \frac{\Xi}{h^D} \begin{cases} 1 - \frac{3}{2}z^2 + \frac{3}{4}z^3, & 0 \leq z \leq 1 \\ \frac{1}{4}(2-z)^3, & 1 \leq z \leq 2 \\ 0, & z > 2 \end{cases}$$

where $z = \|\mathbf{r} - \mathbf{r}'\|_2 / h$, $\Xi = \frac{3}{2}, \frac{10}{7\pi}, \frac{1}{\pi}$ in 1D, 2D and 3D respectively.



$\vec{W}(\vec{\mathbf{x}} - \vec{\mathbf{x}}_I, h)$
$\vec{\mathbf{X}}$
$\|\vec{\mathbf{x}} - \vec{\mathbf{x}}_I\|_2$
$\vec{\mathbf{X}}_I$
h
$\Omega_{\vec{\mathbf{x}},h}$

Figure: Neighboring particles of a Kernel support.

Kernel Gradient $\overline{\nabla W}(\mathbf{r}_J - \mathbf{r}_I, \tilde{h}_{IJ})$:

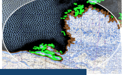
$$(I)\nabla_{\mathbf{r}_J} W(\mathbf{r}_J - \mathbf{r}_I, \tilde{h}_{IJ}) = -\nabla_{\mathbf{r}_I} W(\mathbf{r}_J - \mathbf{r}_I, \tilde{h}_{IJ}),$$

$$(II)\ \nabla \overline{W}(\mathbf{r}_J - \mathbf{r}_I, \tilde{h}_{IJ}) = \frac{\nabla_{\mathbf{r}_I} W(\mathbf{r}_J - \mathbf{r}_I, \tilde{h}_{IJ})}{\nu(\mathbf{r}_I)} - \frac{W(\mathbf{r}_J - \mathbf{r}_I, \tilde{h}_{IJ})\nabla_{\mathbf{r}_I}\nu(\mathbf{r}_I)}{\nu^2(\mathbf{r}_I)},$$

$$(III)\ \overline{\nabla}_{\mathbf{r}_J} \overline{W}(\mathbf{r}_J - \mathbf{r}_I, \tilde{h}_{IJ}) = \frac{\nabla_{\mathbf{r}_J} W(\mathbf{r}_J - \mathbf{r}_I, \tilde{h}_{IJ})}{\nu(\mathbf{r}_I)},$$

$$(IV)\ \widetilde{\nabla}_{\mathbf{r}_J} \overline{W}(\mathbf{r}_J - \mathbf{r}_I, \tilde{h}_{IJ}) = \frac{\nabla_{\mathbf{r}_J} W(\mathbf{r}_J - \mathbf{r}_I, \tilde{h}_{IJ})}{\nu(\mathbf{r}_I, \mathbf{r}_J)} - \frac{W(\mathbf{r}_J - \mathbf{r}_I, \tilde{h}_{IJ})\nabla_{\mathbf{r}_J}\nu(\mathbf{r}_I, \mathbf{r}_J)}{\nu^2(\mathbf{r}_I, \mathbf{r}_J)},$$

Scheme for Fluid Flow in Porous Media



Lukyanov & Vuik (2017) scheme:

$$\frac{n}{\bar{\Gamma}^{-1}_{\beta\beta}} \langle \nabla \left(\mathbf{m}\left(\mathbf{r}_I\right) \nabla \mathbf{F}\left(\mathbf{r}_I\right)\right)\rangle =$$

$$\left\{ \sum_{\Omega_{\mathbf{r}_I,h}} V_{\mathbf{r}_J} \left[\mathbf{F}\left(\mathbf{r}_J\right) - \mathbf{F}\left(\mathbf{r}_I\right)\right] \frac{\left(\mathbf{r}_J - \mathbf{r}_I\right) \cdot \left(m_J + m_I\right) \cdot \overline{\nabla W}\left(\mathbf{r}_J - \mathbf{r}_I, h\right)}{\left\|\mathbf{r}_J - \mathbf{r}_I\right\|^2} \right\} -$$

$$-\left\{ \left(\sum_{\Omega_{\mathbf{r}_I,h}} V_{\mathbf{r}_J} \cdot \left(m_I + m_J\right) \cdot \left[\mathbf{F}\left(\mathbf{r}_J\right) - \mathbf{F}\left(\mathbf{r}_I\right)\right] \overline{\nabla^*_\alpha W}\left(\mathbf{r}_J - \mathbf{r}_I, h\right) \right) \mathbf{N}^\alpha \right\},$$

[Lukyanov & Vuik, Meshfree Methods for Partial Differential Equations VIII, 67-84]

where $n = 1, 2, 3$ is the spatial dimension and tensor $\bar{\Gamma}_{\alpha\beta}$ is defined by

$$\bar{\Gamma}_{\alpha\beta}\left(\mathbf{r}_I\right) = \begin{cases} \Gamma^*_{\alpha\beta}\left(\mathbf{r}_I\right), & \Gamma^*_{\alpha\beta}\left(\mathbf{r}_I\right) \neq 0, \\ \Gamma_{\alpha\beta}\left(\mathbf{r}_I\right), & \Gamma^*_{\alpha\beta}\left(\mathbf{r}_I\right) = 0, \end{cases}$$

Overal Nonlinear Solver Loop

Time integration step:

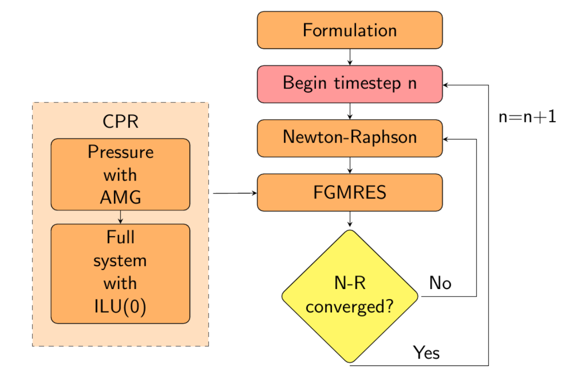


Figure: CPR-based FIM simulation framework: two stage CPR where AMG is used to solve the pressure equation and ILU(0) for the second-stage full residual correction.

Linear Solver Strategy

Linearized system of governing equations:

$$\mathbf{A}x = b \text{ or } \begin{bmatrix} \mathbf{A}_{pp} & \mathbf{A}_{ps} \\ \mathbf{A}_{sp} & \mathbf{A}_{ss} \end{bmatrix} \begin{bmatrix} x_p \\ x_s \end{bmatrix} = \begin{bmatrix} b_p \\ b_s \end{bmatrix}$$

x_p is associated with the pressure (primary) variables
x_s is associated with the other (secondary) variables.

CPR preconditioner:

The pressure equation is constructed by an IMPES-like (i.e., Quasi-IMPES or True-IMPES) reduction using the matrix:

$$\mathbf{A}^*_{pp}\Delta x_p \approx b^*_p, \;\; \mathbf{A}^*_{pp} = \mathbf{C}^{\mathsf{T}}\mathbf{M}_1\mathbf{A}\mathbf{C}, \;\;\; \mathbf{C}^{\mathsf{T}} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix}, \;\; \mathbf{M}_1 = \begin{bmatrix} \mathbf{I} & -\mathbf{Q} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}$$

where

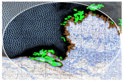
$$\mathbf{Q} = \mathbf{A}_{ps}\mathbf{A}_{ss}^{-1} \approx colsum(\mathbf{A}_{ps}) \cdot colsum(\mathbf{A}_{ss})^{-1}.$$

Linear Solver Strategy

Algorithm 1 Right-Preconditioned GMRES Based Method

```
 1: Setup $\mathbf{A}^*_{pp}$
 2: Compute $r_0 = (b - \mathbf{A}x_0)$, $\beta = \|r_0\|_2$, and $v_1 = r_0/\beta$.
 3: for $j = 1, 2, \ldots, m$ do
 4:     $w_j = \mathbf{A}^*_{pp}\mathbf{M}^{-1}v_j$
 5:     for $i = 1, \ldots, j$ do
 6:         $h_{i,j} = (w_j, v_i)$
 7:         $w_j = w_j - h_{ij}v_i$
 8:     end for
 9:     $h_{j+1,j} = \|w_j\|_2$
10:     if $h_{j+1,j} = 0$ or converged then
11:         set $m = j$ and go to 15
12:     end if
13:     $v_{j+1} = w_j/h_{j+1,j}$
14: end for
15: Fill $\bar{\mathbf{H}}_m = \{h_{ij}\}$ for $1 \leq i \leq m+1$, $1 \leq j \leq m$.
16: Compute the minimizer $u_m$ of $\|\beta e_1 - \bar{\mathbf{H}}_m u\|_2$ and set $x_m = x_0 + \mathbf{M}^{-1}\mathbf{V}_m u_m$.
17: if converged then $x_m$ solution and return else set $x_0 = x_m$ and go to 2
```

Multiscale, Multilevel, Multigrid and Deflation Methods



Common Solution Trends

- Algebraic Multigrid (AMG), Smoothed Aggregation AMG, Non-Galerkin AMG, element-based Algebraic Multigrid (AMGe), Multilevel Solver

  [Ruge and Stuben, 1987], [Vanek et al., 1996], [Falgout and Schroeder, 2014], [Brezina et al., 2001] [Jones et al., 2001], [Lashuk and Vassilevski, 2008], [Griebel and Schweitzer, 2002]

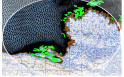
- Multiscale, Algebraic Multiscale (AMS), Multiscale Restriction Smoothed Basis (MsRSB)

  [Hou and Wu, 1997], [Jenny et al., 2003], [Hajibeygi et al., 2008], [Efendiev and Hou, 2009], [Lunati et al.(2011)], [Zhou and Tchelepi, 2012], [Cortinovis and Jenny, 2014], [Wang et al., 2014], [Tene et al., 2014], [Manea et al., 2015], [Cusini et al., 2015], [Møyner and Lie, 2016]

- Deflation Theory

  [Nicolaides, 1987], [Dostál, 1988], [Frank and Vuik, 2001], [Vuik et al., 2002], [Tang and Vuik, 2007], [van der Linden et al., 2016]

Multiscale, Multilevel, Multigrid and Deflation Methods



Galerkin Projection:

A coarse-scale system can be constructed by applying restriction **R** and prolongation **P** operators (a number of times)

$$\mathbf{A}^{k+1} = \mathbf{R}^k \mathbf{A}^k \mathbf{P}^k,$$

where k is the level of the appropriate step of multiscale, multilevel, multigrid, and deflation methods, $\mathbf{A}^{k+1}$ is the matrix on the next level (for the pressure system), $\mathbf{R}^k$, $\mathbf{P}^k$ are the restriction and prolongation operators at the level k.

Post-Galerkin Projection Steps:

- Smoothing (e.g., Gauss-Seidel (GS) or ILU(k) or BILU(k) smoothing method or Krylov-space accelerator)
- Filtering of fills in (Non-Galerkin projection)
- Solving deflated system

$$\prod_{i=1}^{k} \mathbf{D}_1^i \mathbf{A} \hat{x} = \prod_{i=1}^{k} \mathbf{D}_1^i \mathbf{b}.$$

Multiscale, Multilevel, Multigrid and Deflation Methods

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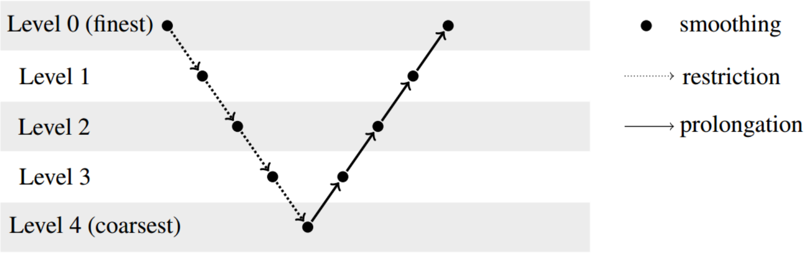
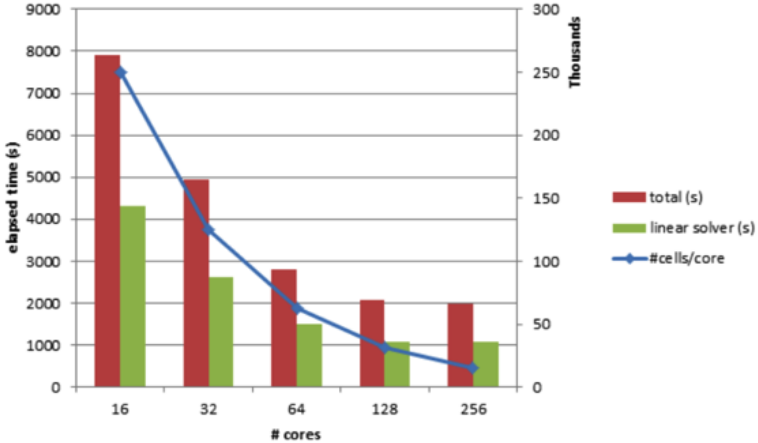
V-cycle

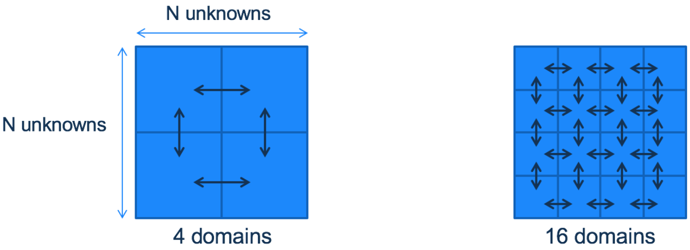
Typical V-cycle scheme of five levels for (multilevel-) multiscale, multigrid, and multilevel deflated method is shown below.



Problem Description: Strong Scalability AMG ?



Problem Description: Strong Scalability AMG ?

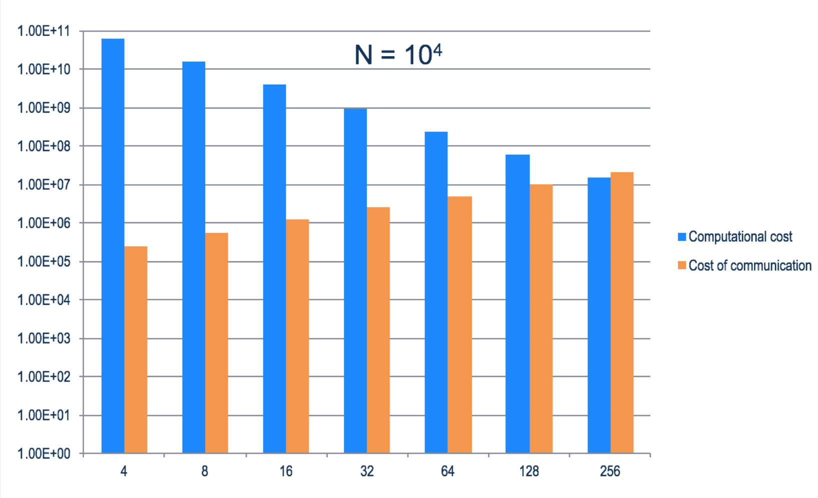


Assume cost of FLOPS is 1 and cost of communication per float is 4. Also assume that we use direct method on each domain. Assume that we communicate 1 float per unknown on boundary of domain.

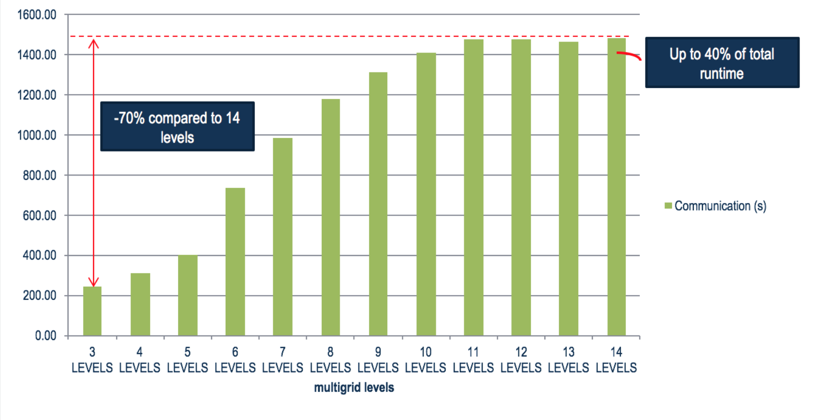
Computational cost: 1 * k * $(N/k)^3$

Cost of communication: 4 * 2 * (k-1)*k*(N/k)

Problem Description: Strong Scalability AMG ?



Problem Description: Strong Scalability AMG ?



-70% compared to 14 levels

Up to 40% of total runtime

Multiscale Finite Volume (MSFV) Method

MSFV: Two-level solver

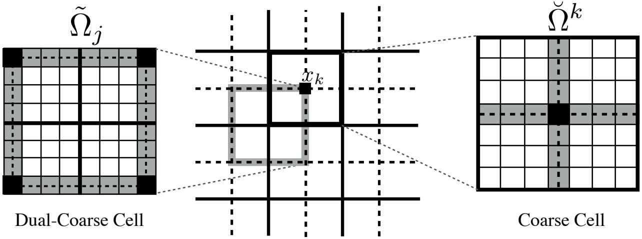


Figure: MSFV grid imposed on the given fine-scale grid (center): N_c coarse (solid lines) and N_d dual-coarse (dashed lines) grids. A coarse and a dual-coarse grid cell are highlighted on the right and left, respectively.

CPR-MS method for Fully Implicit Simulations

Linear Solver Settings:

Table: SPE9 model with capillary pressure: settings. Differences can be seen in the smoothers and in the solver used on the coarse scale pressure system.

Runs	Presmoothing	Post-smoothing	Coarse Solver
Run 1	GS	GS	AMG
Run 2	GS	GS	GMRES-AMG
Run 3	2xGS	2xGS	GMRES-AMG

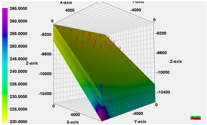


Figure: SPE9 Model.

Simulation Results:

Table: SPE9 with capillary pressure: the table shows the total numbers of nonlinear and of linear iterations and the total and the linear solver CPU time of each run.

		Iteration Count		CPU time (s)	
Runs	Timesteps	Nonlinear	Linear	Linear solver	Total
Run 1	75	284	1888	759	1672
Run 2	75	284	1867	1202	2130
Run 3	75	281	1627	1105	2036
CPR-AMG	75	281	1381	1093	1863

[Lukyanov, 2014], [Cusini et al., 2014]

Multiscale Restriction-Smoothed Basis (MsRSB) Method

MsRSB in a Nutshell

The discretized computational domain Ω is first decomposed into d non-overlapping subdomains $\bar{\Omega}_j$ with $j \in \{1, \ldots, d\}$. The deflation vector $\bar{Z}_j$ forms j-th column of the deflation operator $\mathbf{Z}$ or initial basis functions $\mathcal{P}_j^0$ of MsRSB method, corresponding to $\bar{\Omega}_j$

$$\left(\mathcal{P}_j^0\right)_i = \left(\bar{Z}_j\right)_i = \begin{cases} 1, & x_i \in \bar{\Omega}_j \\ 0, & x_i \in \Omega \setminus \bar{\Omega}_j, \end{cases}$$

where x_j is a fine-scale grid cell center. Based on the above definition, $\bar{Z}_j$ and $\mathcal{P}_j^0$ are piecewise-constant vectors or functions (equal to a constant value of one inside the corresponding coarse domain $\bar{\Omega}_j$), disjoint and orthogonal.

[Frank and Vuik, 2001], [Tang, 2008], [van der Linden, 2013], [Lukyanov et al., 2014], [van der Linden et al., 2016]

[Møyner and Lie, 2015], [Møyner and Lie, 2016], [Shah et al., 2016]

Multiscale Restriction-Smoothed Basis (MsRSB) Method

MsRSB in a Nutshell

The final restriction-smoothed basis functions are computed by employing a modified form of the damped-Jacobi smoothing approach:

$$\delta\mathcal{P}_k^{\eta} = -\omega\mathbf{D}^{-1}\mathbf{A}\mathcal{P}_k^{\eta},$$

[Møyner and Lie, 2016], [Shah et al., 2016]

where $\mathbf{A}$ is the fine-scale matrix, $\mathbf{D} = diag(\mathbf{A})$ is the diagonal part of the matrix $\mathbf{A}$. The final update for prolongation operator is defined as

$$\mathcal{P}_k^{\eta+1} = \mathcal{P}_k^{\eta} + \delta\hat{\mathcal{P}}_k^{\eta},$$

where $\delta\hat{\mathcal{P}}_k^{\eta}$ is the restricted iterative increments. Finally, the basis functions of the MsRSB method can be written in the abstract form as:

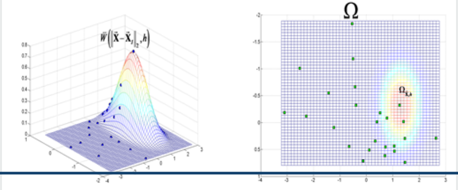
$$\mathcal{P}_k^{\eta+1} = \mathbf{M}_{MsRSB}^{\frac{1}{2}}\mathcal{P}_k^{0},$$

where $\mathbf{M}_{MsRSB}^{\frac{1}{2}}$ is the predefined smoothing matrix of MsRSB method.

Meshless Deflation Theory

Fine and Coarse Spaces:

1. Fine points set S_F (e.g., cell centers of underlying mesh), i.e. $\Omega = \operatorname{span}\left\{\bar{\Omega}_{\mathbf{X}_I,\bar{h}_F} / I = 1, \ldots, N_F\right\}$ consisting of N_F patches which are interior to the support of the kernel $\bar{W}\left(\mathbf{X} - \mathbf{X}_I, \bar{h}_F\right)$, i.e. $\Omega_{\mathbf{X},\bar{h}_F} = \operatorname{supp} \bar{W}(\mathbf{X} - \boldsymbol{\xi}, \bar{h}_F)$, N_F is the number of points, $\bar{h}_F$ is the fine scale diameter (or smoothing length);
2. Coarse points set S_C (e.g., user defined points), i.e. $\Omega = \operatorname{span}\left\{\tilde{\Omega}_{\mathbf{X}_J,\bar{h}_C} / J = 1, \ldots, N_C\right\}$ consisting of N_C patches which are interior to the support of the kernel $\bar{W}\left(\mathbf{X}_I - \mathbf{X}, \bar{h}_C\right)$, i.e. $\Omega_{\mathbf{X},\bar{h}_C} = \operatorname{supp} \bar{W}(\mathbf{X} - \boldsymbol{\xi}, \bar{h}_F)$, $N_C < N_F$ is the number of points, $\bar{h}_C$ is the coarse scale diameter (or smoothing length);



Meshless Deflation Theory

Meshless Deflation Vectors - Zero Order Consistency

$$p_F\left(\mathbf{r}\right) \approx \sum_{J=1}^{N_C} V_{\boldsymbol{\xi}_J} \cdot \bar{W}(\mathbf{r}-\boldsymbol{\xi}_J, \bar{h}_C) \cdot p_C\left(\boldsymbol{\xi}_J\right)$$

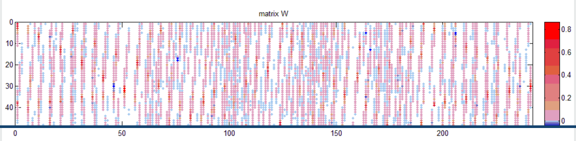
Meshless Deflation Vectors - First Order Consistency

$$p_F\left(\mathbf{r}_I\right) \approx \mathbf{r}_I \cdot \left(\sum_{J=1}^{N_C} V_{\boldsymbol{\xi}_J} \cdot \bar{W}\left(\mathbf{r}_I-\boldsymbol{\xi}_J, \bar{h}_C\right) \mathbf{C}_1\left(\boldsymbol{\xi}_J\right)\right) + \sum_{J=1}^{N_C} \left[V_{\boldsymbol{\xi}_J} \cdot \bar{W}\left(\mathbf{r}_I-\boldsymbol{\xi}_J, \bar{h}_C\right) \cdot \mathbf{C}_2\left(\boldsymbol{\xi}_J\right)\right]$$

where

$$\mathbf{C}_1\left(\boldsymbol{\xi}_J\right) = \left[\sum_{K=1}^{N_C} \left[p_C\left(\boldsymbol{\xi}_K\right) - p_C\left(\boldsymbol{\xi}_J\right)\right] \nabla^* \bar{W}\left(\boldsymbol{\xi}_J - \boldsymbol{\xi}_K, \bar{h}_C\right)\right]$$

$$\mathbf{C}_2\left(\boldsymbol{\xi}_J\right) = \left[p_C\left(\boldsymbol{\xi}_J\right) - \boldsymbol{\xi}_J \cdot \mathbf{C}_1\left(\boldsymbol{\xi}_J\right)\right] ,$$



Multiscale Meshless Based Method (MsMBM)

Richardson Iteration Scheme

$$[\mathbf{p}_F]^{m+1} = [\mathbf{p}_F]^m + \mathbf{V} \cdot (\mathbf{b}_F - \mathbf{A}_F\, [\mathbf{p}_F]^m)$$

where m is the iteration index, $[p_F]^m$ is the pressure vector at the iteration m, $\mathbf{V}$ is the left multiscale meshless based preconditioner defined as an operational object.

Meshless multiscale preconditioner

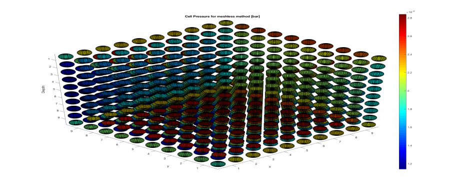
$$[\mathbf{z}_F] = \mathbf{P}\,(\mathbf{A}_C)^{-1}\,\mathbf{R}\,[\mathbf{v}_F]\,, \quad \mathbf{A}_C = \mathbf{R}\mathbf{M}^{-1}\mathbf{A}_F\mathbf{P}$$

$$[\mathbf{w}_F] = [\mathbf{z}_F] + \mathbf{S}_\gamma^{-1} \cdot ([\mathbf{v}_F] - \mathbf{A}_F\,[\mathbf{z}_F])$$

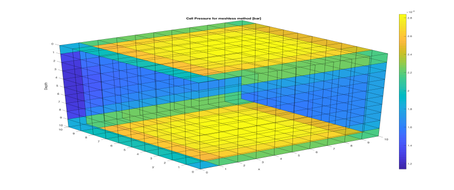
where $\mathbf{S}_\gamma^{-1}$ is the smoothing operator (e.g., Gauss-Seidel (GS) or ILU(k) or BILU(k) smoothing method or Krylov-space accelerator) applied γ times.

Numerical Results: Geometry

Meshless Model:



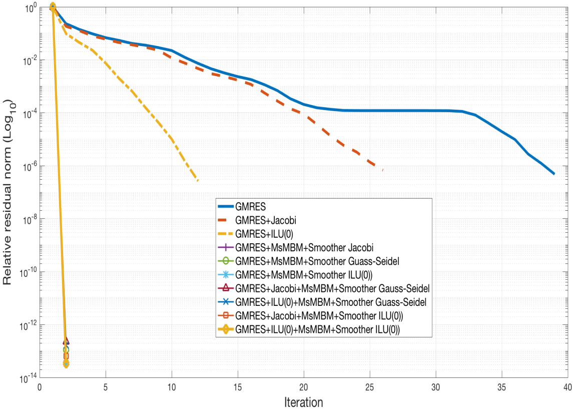
Mesh Model:



Numerical Results: Convergence Analysis - "Perfect Preconditioner"

SIAM GS 2017

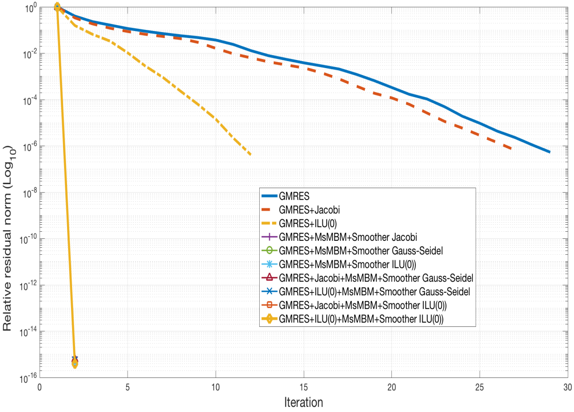
Meshless Model:



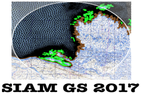
Numerical Results: Convergence Analysis - ”Perfect Preconditioner”

SIAM GS 2017

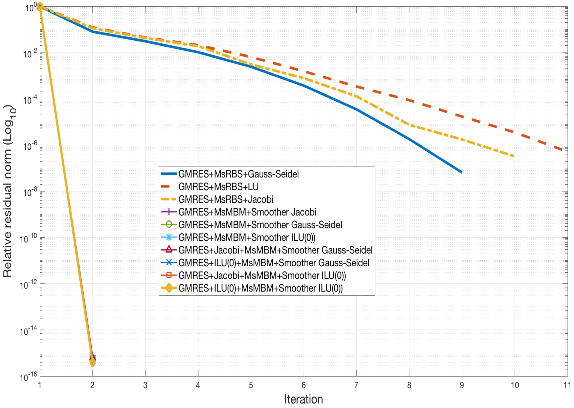
Mesh Model:



Numerical Results: Convergence Analysis - ”Perfect Preconditioner”



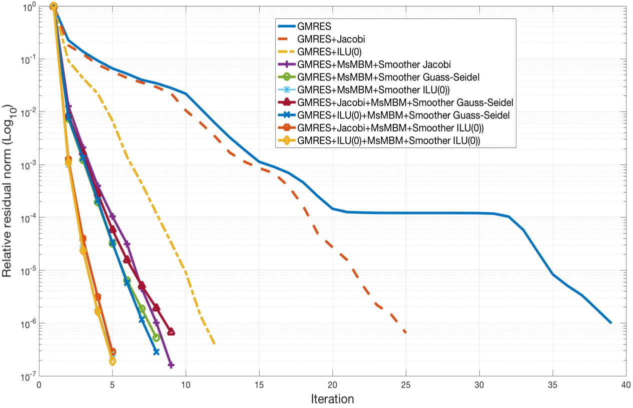
Mesh Model: MsMBM vs. MsRSB



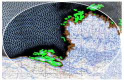
Numerical Results: Convergence Analysis

SIAM GS 2017

Meshless Model:



Numerical Results: Convergence Analysis



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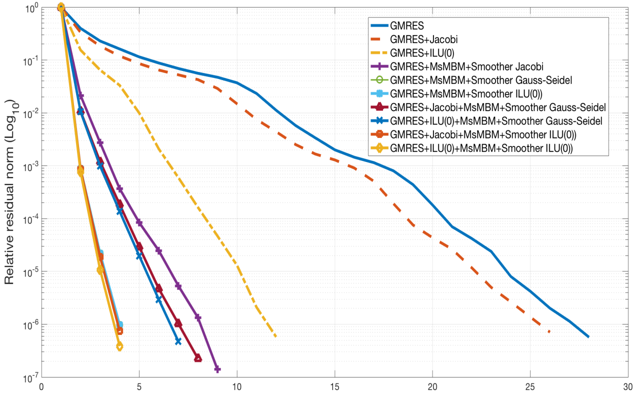
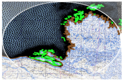
Mesh Model:



Numerical Results: Convergence Analysis



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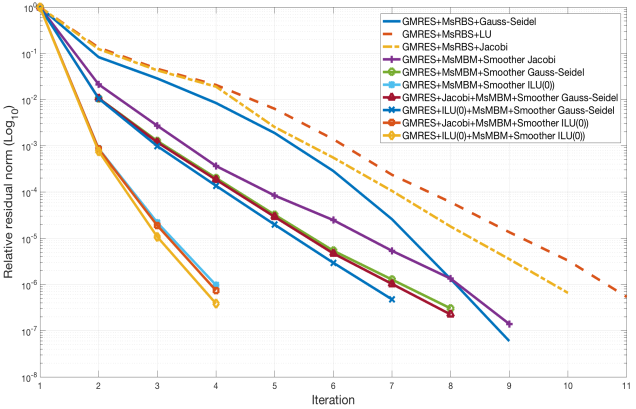
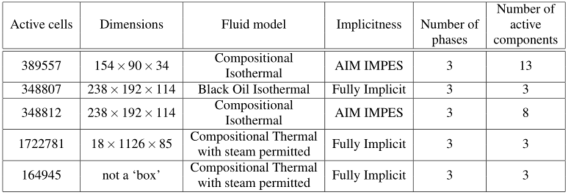
Mesh Model: MSMBM vs. MsRSB



Numerical Results

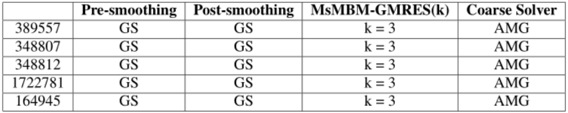
Basic Properties of the Simulated Test Cases

The black oil, iso-thermal and thermal compositional models with varying degree of heterogeneity in the reservoir grid properties are considered in this paper to test the performance of the Multiscale Meshless Based Method (MsMBM)



Active cells	Dimensions	Fluid model	Implicitness	Number of phases	Number of active components
389557	154 × 90 × 34	Compositional Isothermal	AIM IMPES	3	13
348807	238 × 192 × 114	Black Oil Isothermal	Fully Implicit	3	3
348812	238 × 192 × 114	Compositional Isothermal	AIM IMPES	3	8
1722781	18 × 1126 × 85	Compositional Thermal with steam permitted	Fully Implicit	3	3
164945	not a 'box'	Compositional Thermal with steam permitted	Fully Implicit	3	3

Table 1: Basic properties of the simulated test cases.



	Pre-smoothing	**Post-smoothing**	**MsMBM-GMRES(k)**	**Coarse Solver**
389557	GS	GS	k = 3	AMG
348807	GS	GS	k = 3	AMG
348812	GS	GS	k = 3	AMG
1722781	GS	GS	k = 3	AMG
164945	GS	GS	k = 3	AMG

Table 2: Experimental settings.

Numerical Results

Serial Runs

The simulation tests clearly shows that MsMBM leads to a noticeable speedup around 20% in general for serial runs:

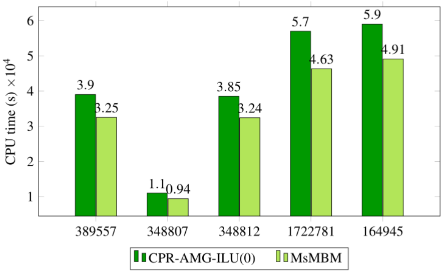


Figure 3: Total time of the linear solver stage for CPR-AMG-ILU(0) and MsMDM solution strategies.

Numerical Results

Parallel Runs

Scalability of the total time of the simulation runs for CPR-AMG-ILU(0) and MsMDM solution strategies in the case 389557:

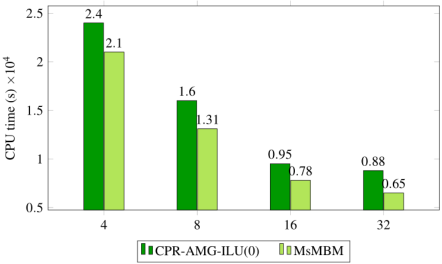


Figure 4: Scalability of the total time of the simulation runs for CPR-AMG-ILU(0) and MsMDM solution strategies in the case 389557.

Summary

Collection of Methods

- Fully Implicit SPH Based Multiscale Method is presented and allows to handle low-frequency modes on the coarse level. High-frequency errors are then resolved by employing a smoother on fine grid.
- Restrictions and prolongation operators reduce to the subdomain-levelset deflation vectors, used in subdomain-levelset deflation method and MsRSB.
- This method does not require a coarse partition and, hence, can be applied to general unstructured topology of the fine scale.
- The SPH based multiscale method provides a reasonably good approximation to the pressure system and speeds up the convergence when used as a preconditioner for an iterative fine-scale solver.
- The method exhibits expected good (not ideal!) scalability during parallel fully implicit simulations.

References

- J.H. van der Linden and T.B. Jonsthovel and A.A. Lukyanov and C. Vuik
  The parallel subdomain-levelset deflation method in reservoir simulation
  Journal of Computational Physics, 304, pp. 340-358, 2016
- A. Lukyanov and C. Vuik
  Parallel Fully Implicit Smoothed Particle Hydrodynamics Based Multiscale Method
  ECMOR XV - 15th European Conference on the Mathematics of Oil Recovery, August 29 - September 1, 2016 Editor: J.D. Jansen EAGE, Houten, 2016
  DOI: 10.3997/2214-4609.201601748

Thank you !
Questions?

Meshless Deflation Theory

Existing Mappings

$$\mathbf{u}_F = \bar{\mathbf{W}}^P \cdot \mathbf{u}_C,\ \bar{\mathbf{W}}^P : S_C \rightarrow S_F \quad \mathbf{u}_C = \bar{\mathbf{W}}^R \cdot \mathbf{u}_F,\ \bar{\mathbf{W}}^R : S_F \rightarrow S_C$$

$$\tilde{\bar{\mathbf{W}}}^R = \mathbf{B}_W^{-1}\bar{\mathbf{W}}^R,\ \mathbf{B}_W = \bar{\mathbf{W}}^R\bar{\mathbf{W}}^P,\ \tilde{\bar{\mathbf{W}}}^R\bar{\mathbf{W}}^P = \mathbf{I},\ \tilde{\bar{\mathbf{W}}}^R : S_F \rightarrow S_C$$

$$\tilde{\bar{\mathbf{W}}}^P = \bar{\mathbf{W}}^P\mathbf{B}_W^{-1},\ \mathbf{B}_W = \bar{\mathbf{W}}^R\bar{\mathbf{W}}^P,\ \bar{\mathbf{W}}^R\tilde{\bar{\mathbf{W}}}^P = \mathbf{I},\ \tilde{\bar{\mathbf{W}}}^P : S_C \rightarrow S_F,$$

where $\bar{\mathbf{W}}^P$ and $\tilde{\bar{\mathbf{W}}}^P$, $\bar{\mathbf{W}}^R$ and $\tilde{\bar{\mathbf{W}}}^R$ are the deflation operators.

Restriction and Prolongation operators

$$(I)\ \ \mathbf{P} = \bar{\mathbf{W}}^P,\ \mathbf{R} = \left(\bar{\mathbf{W}}^P\right)^{\mathsf{T}},\quad (II)\ \ \mathbf{P} = \tilde{\bar{\mathbf{W}}}^P,\ \mathbf{R} = \left(\tilde{\bar{\mathbf{W}}}^P\right)^{\mathsf{T}},$$

$$(III)\ \ \mathbf{P} = \bar{\mathbf{W}}^P,\ \mathbf{R} = \bar{\mathbf{W}}^R,\quad (IV)\ \ \mathbf{P} = \bar{\mathbf{W}}^P,\ \mathbf{R} = \tilde{\bar{\mathbf{W}}}^R,\quad (V)\ \ \mathbf{P} = \tilde{\bar{\mathbf{W}}}^P,\ \mathbf{R} = \bar{\mathbf{W}}^R,$$

$$(VI)\ \ \mathbf{P} = \left(\bar{\mathbf{W}}^R\right)^{\mathsf{T}},\ \mathbf{R} = \bar{\mathbf{W}}^R,\quad (VII)\ \ \mathbf{P} = \left(\tilde{\bar{\mathbf{W}}}^R\right)^{\mathsf{T}},\ \mathbf{R} = \tilde{\bar{\mathbf{W}}}^R,$$

where **P** and **R** are the restriction and prolongation operators.