Simplified Dense and Coarse Higher-Order **Surface** Mesh Generation Using ~~Moving~~ Least Squares

Vijai Kumar Suriyababu
Cornelis Vuik
Matthias Möller

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Motivation

- A simplified yet improved higher-order surface reconstruction method
- Works with both regular and irregular meshes without requiring remeshing
- Based on the well-established Least Squares techniques
- (Try to) support both dense and coarse meshes
- Enhances and simplifies existing (and very similar) surface reconstruction techniques [1], [2].

[1] X. Jiao and D. Wang (Oct. 2012). "Reconstructing high-order surfaces for meshing". In: Engineering with Computers 28.4, pp. 361–373. ISSN: 1435-5663. DOI: 10.1007/s00366-011-0244-8

[2] J. Ims and Z. J. Wang (Jan. 2019). "Automated low-order to high-order mesh conversion". In: Engineering with Computers 35.1, pp. 323–335. ISSN: 1435-5663. DOI: 10.1007/s00366-018-0602-x

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Generic Workflow

a. Estimate feature curves (A crucial step)
b. Estimate a local neighbourhood for every vertex (or face)
c. Compute a least squares surface (A polynomial, can also use BSplines / other surfaces)
d. For every node in the surface (lower order to higher order), use weighted least square fits (using some heuristic such as Euclidean distance or local neighbourhood information) and estimate the new points which adapt to the local curvature better
e. Optionally employ some mesh optimization technique to improve the higher order mesh.

We focus on improving the steps **a** and **b**

Existing local neighbourhood (or) stencil selection I

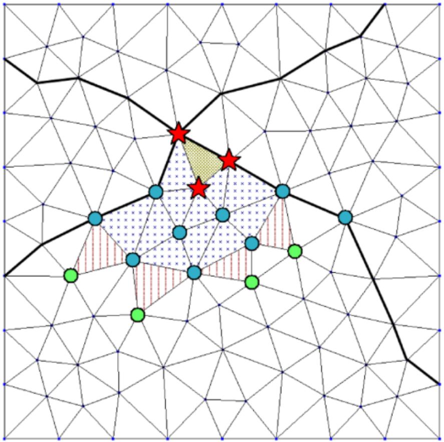
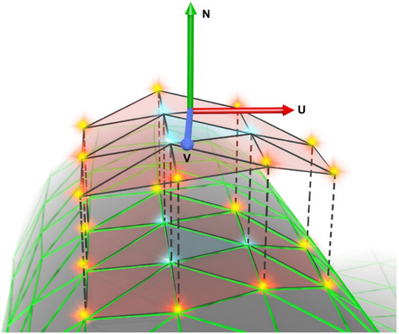


Figure: Local neighbourhood selection [3]

Existing local neighbourhood (or) stencil selection II



[3] J. Ims and Z. J. Wang (Jan. 2019). “Automated low-order to high-order mesh conversion”. In: Engineering with Computers 35.1, pp. 323–335. ISSN: 1435-5663. DOI: 10.1007/s00366-018-0602-x

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Intermediate support structure

Feature distance field

Feature Distance Field I

- Existing literature relies on complex heuristics to estimate discrete feature curves. Except in very trivial cases, this always requires some parameter tuning.
- We start with feature curves defined by a user-chosen angle threshold.
- A feature distance field is then estimated, providing a continuous field instead of discrete curves, allowing direct integration into the reconstruction process.
- We formulate this as surface winding number estimated problem, as proposed by Nicole Feng et al. [4].
- Eliminates heuristic-based stencil selection.
- For feature curves, we can still use direct reconstruction is applied as described in existing literature.

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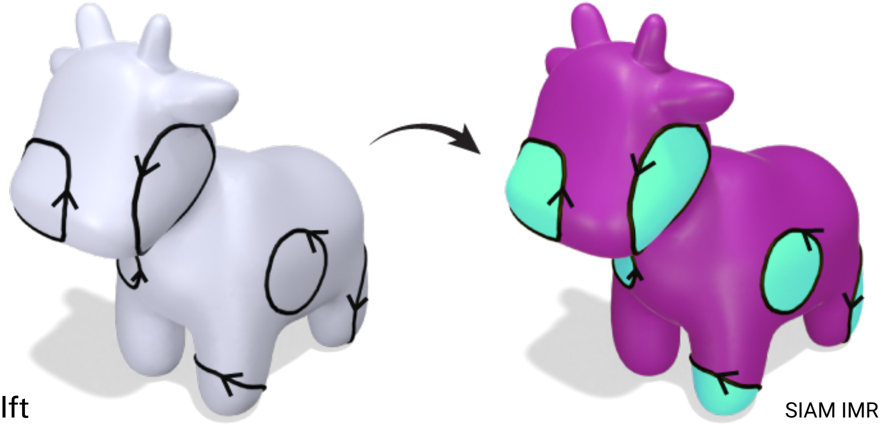
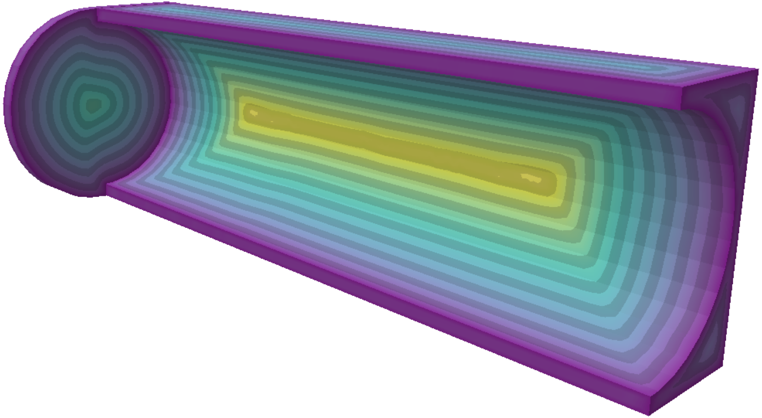
[4] N. Feng, M. Gillespie, and K. Crane (July 2023). "Winding Numbers on Discrete Surfaces". In: ACM Trans. Graph. 42.4. ISSN: 0730-0301. DOI: 10.1145/3592401

Surface winding number[5] of spot (bounded by multiple curves)



[5] N. Feng, M. Gillespie, and K. Crane (July 2023). "Winding Numbers on Discrete Surfaces". In: ACM Trans. Graph. 42.4. ISSN: 0730-0301. DOI: 10.1145/3592401

Feature distance field of a mechanical component generated by bounding feature curves



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Intermediate Support Structure

Series of Local Triangulations

- **Stencil selection** is a useful technique, but relying on it alone is not always ideal.
- This is particularly challenging for **smooth surfaces**, as feature curves alone may not be reliable indicators.
- To address this, we introduce a support structure called **SOLT (Series of Local Triangulations)**.
- SOLT is a **feature-aware resampling algorithm**[6] that estimates a point cloud based on:
  - A given mesh or point cloud
  - Identified feature curves
- Additionally, SOLT enables direct generation of higher-order meshes from point clouds.

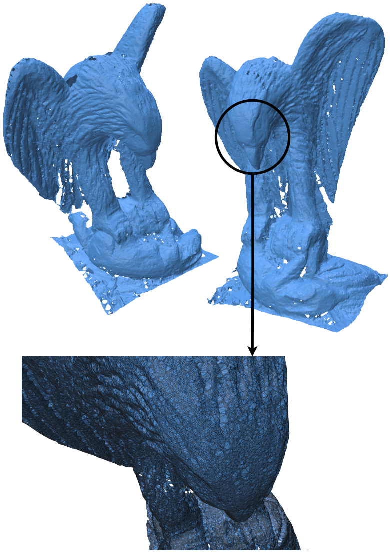
[6] V. K. Suriyababu, C. Vuik, and M. Möller (2025). "Resampling Point Clouds Using Series of Local Triangulations". In: Journal of Imaging 11.2. ISSN: 2313-433X. DOI: 10.3390/jimaging11020049

How Does SOLT Work?

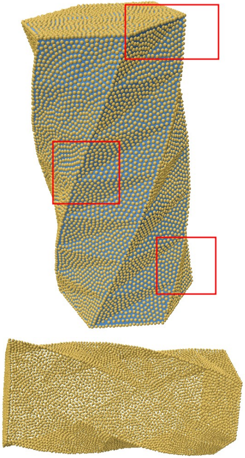
Given a point cloud

- **Step 1: Identify Local Neighborhood** Each point finds its nearby neighbors based on distance or tangent space coordinates.
- **Step 2: Compute Characteristic Length** The characteristic length is determined by the farthest neighbor, adapting to local density.
- **Step 3: Local Triangulation**
  - Arrange neighbors in a counter-clockwise order.
  - Apply **Delaunay triangulation** within this neighborhood.
  - Refine triangulation to meet the Delaunay criterion.
- **Step 4: Merge Local Structures** All local triangulations are combined into a **global mesh**, removing duplicate vertices.
- **Step 5: Optional Smoothing and Denoising** If needed, additional steps refine the structure for better quality.

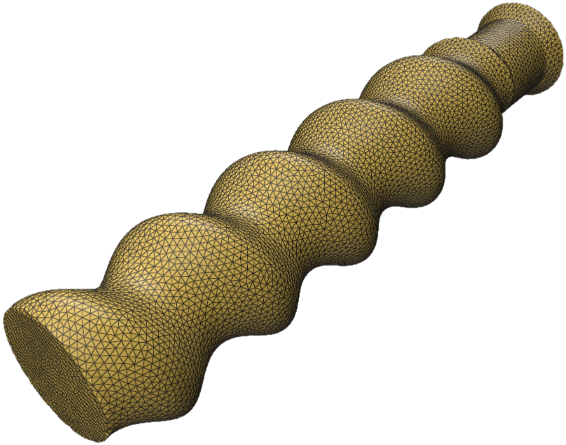
SOLT reconstruction of eagle point cloud



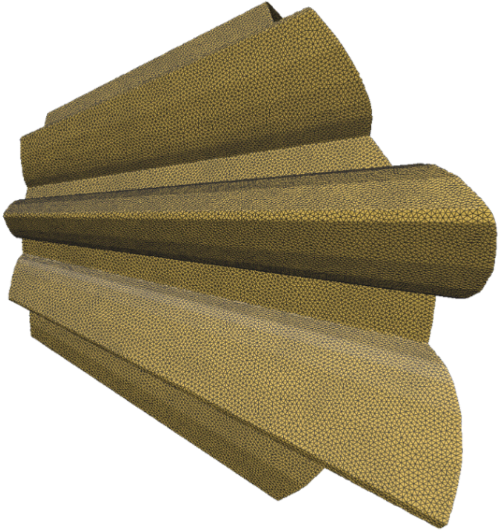
Feature preserving point clouds produced by SOLT



Point cloud Reconstruction (SOLT + BPA) – 10,000 Points



Point cloud Reconstruction (SOLT + BPA) – 100,000 Points



Point cloud Reconstruction (SOLT + BPA) – 25,000 Points

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Workflow

Higher Order Mesh Generation Workflow

- **Input:** Surface mesh
- **Step 1:** Calculate a **feature distance field**.
- **Step 2:** Analyze **surface density** for uniformity.
- **Step 3:** Build an **intermediate support structure** (SOLT based).
- **Step 4:** Adaptive refinement:
  - If **mesh density is uniform**, proceed with mesh stencils alone. If not, include points from **intermediate support structures** for local fittings.
- **Step 5:** Estimate local fittings using **dynamic stencils** based on the feature distance field.
- **Step 6:** Determine **initial positions** for higher-order points.
- **Step 7:** Apply **weighted Least Squares** to estimate higher-order point positions.

Local Approximation with Weighted Polynomial Fitting

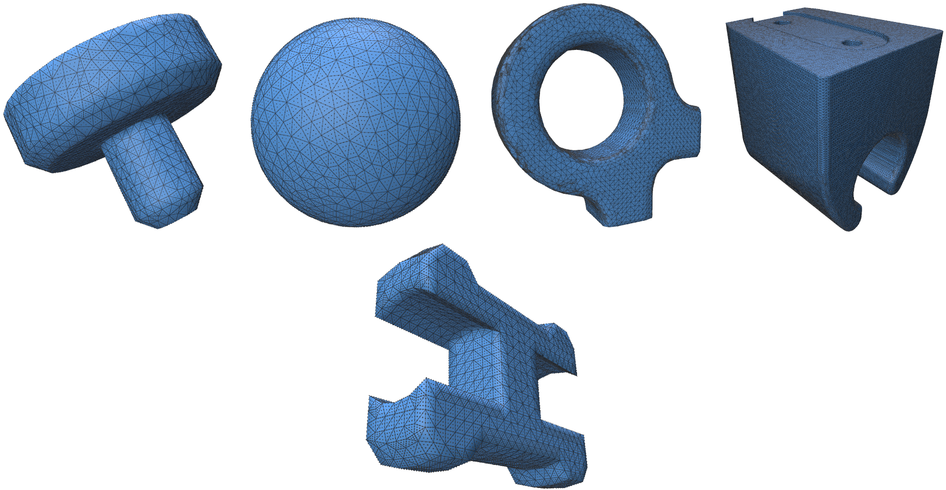
Given a point x_i and its local neighborhood, the local approximation at x_i is achieved by fitting a polynomial $P(x;\beta)$ to the values y_j at neighboring points x_j.

$$\min_{\beta} \sum_{j=1}^{n} w(x_i, x_j)(y_j - P(x_j;\beta))^2 \tag{1a}$$

$$w(x_i, x_j) = \exp\left(-\frac{\|x_i - x_j\|^2}{2\sigma^2}\right) \tag{1b}$$

The weight function $w(x_i, x_j)$ decreases with distance, giving more importance to closer points. The parameter σ controls this influence, ensuring P adapts smoothly to local mesh variations.

Dense Higher-Order Meshes, L_2-norm deviation within 3%



Coarse Mesh Generation Using the Multigrid Framework

- Given a dense linear surface mesh serves as the input.
- The goal is to estimate the positions of higher-order points on a coarse higher order surface.
- This is achieved through:
  - Mesh decimation to reduce complexity.
  - A **coarse-to-fine** mesh mapping using a multigrid framework[7].
  - Reconstruction with least squares (same as before).

[7] H.-T. D. Liu, J. E. Zhang, M. Ben-Chen, and A. Jacobson (2021). "Surface Multigrid via Intrinsic Prolongation". In: ACM Trans. Graph. 40.4

Algorithm for Coarse Mesh Generation

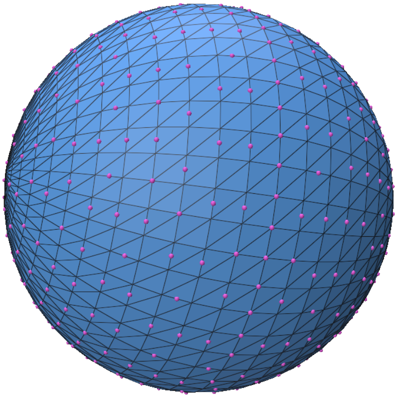
- **Step 1:** Decimate the mesh to reduce resolution.
- **Step 2:** Generate a mapping between the coarse and dense meshes using a multigrid framework.
- **Step 3:** For each point in the coarse mesh:
  - Estimate a least squares surface using mapped points from the dense mesh.
  - Identify nearby points from an intermediate support structure.
  - Use these points to generate a local least square surface.
- **Step 4:** Estimate positions of higher-order points based on weighted average of least square fits on the coarse surface.

Feature Preservation and Considerations

- Feature curves in mechanical parts depend on:
  - The decimation strategy.
  - The number of decimation cycles.
- The coarse-to-fine mapping allows retrieving points from the original dense mesh.
- Controlling the level of decimation is essential to maintain geometric accuracy.
- Best suited for smooth surfaces, but:
  - Careful parameter tuning can extend its use to more complex geometries.
  - Experiments confirm that suitable coarse higher-order meshes can still be effective for analysis.

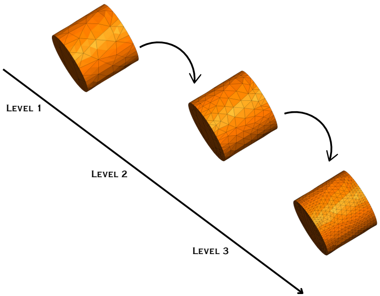
Numerical experiments

- The figure illustrates points from the decimated sphere geometry mapped onto the original dense mesh.
- These points were obtained using the surface multigrid framework.



Coarse Higher-Order Mesh of a Cylinder

- The mesh is displayed with refined elements (Higher order meshes visualized with refined triangles).
- Refinement highlights the smoothness and reconstruction quality.



Conclusion

- **Feature distance fields** provide a viable alternative for surface reconstruction.
- **Intermediate support structures** enhance higher-order surface reconstruction.
- **Surface multigrid mapping** combined with intermediate support structures enables the generation of coarse meshes while preserving detail.
- **Future work:** Improve the robustness of the algorithm and potentially open-source the grid generator.

Thank you for listening!
Any Questions ?

Vijai Kumar Suriyababu Cornelis Vuik Matthias Möller