Solving Large Integrated Electricity Networks

The Power of Numerical Analysis

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Outline

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   - Power Flow Computations
2. Electricity Networks
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3. Results
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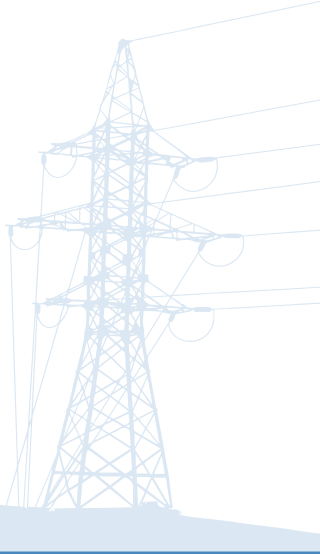
4. Current Developments



Numerical Analysis for Electricity Networks

- Computational simulations of the power system network
- Secure and efficient transmission and distribution of electrical power
- Focus on the steady-state power flow problem
    - Safe operation and planning of the system
    - Contingency analysis to simulate equipment outages
    - Analysis of stochastic behaviour due to solar and wind power generation
- Large and interconnected character of the power systems require fast and robust solution techniques

Our electricity system

Traditionally:

- Transmission: power is transported from large, centralized generators to several **substations**
- Distribution: power is transported from the **substations** to end-consumers

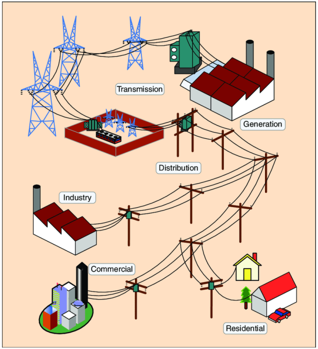


Figure 1: Traditional grid[1]

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[1] Yu, Xinghuo and Cecati, Carlo and Dillon, Tharam and Simoes, Marcelo. (2011). The New Frontier of Smart Grids. Industrial Electronics Magazine, IEEE. 5. 49 - 63. 10.1109/MIE.2011.942176.

Our electricity system

Currently:

- More and more decentralized power generation (solar and wind power)
- Directly connected to distribution networks
- Interconnected power systems

$\Rightarrow$ **Larger power system simulations**

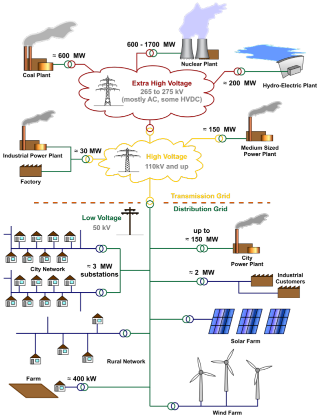

Figure 2: Modern grid[2]

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[2] MBizon, CC BY 3.0 `https://creativecommons.org/licenses/by/3.0`, via Wikimedia Commons

The power flow problem:

Given the generation and consumption, calculate the associated voltages to determine the flow of electrical power.

- Based on physical laws:
  - Kirchoff's Current and Voltage Laws:

$$\sum_k I_k = 0 \quad \text{and} \quad \sum_i V_i = 0$$

  - Ohm's Law: $I = YV$
- Non-linear problem

We need **voltage** V, **power** S and **impedance** Y.
V, S, and Y are complex:

$$V = |V|e^{\iota\delta_V}, \quad S = P + \iota Q, \quad \text{and} \quad Y = \frac{1}{R + \iota X}$$

The power flow equation:

$$S = V\bar{I} = V(\overline{Y\mathbf{V}})$$

Overview

The large scale of power flow problems make traditional solvers obsolete. We need research into new solution techniques.

Timeline of numerical analysis projects on this topic

2008 - 2012	PhD Transmission Networks
2016 - 2020	PhD Distribution Networks
2016 - 2021	PhD Multi-Carrier Energy Networks
2018 - ongoing	PhD Integrated Networks
2020 - 2021	Master project the Dutch Grid
2021 - ongoing	PhD HPC for Multi-Carrier Networks

Overview

PhD Theses

- R. Idema: Newton-Krylov Methods in Power Flow and Contingency Analysis, 2012
- B. Sereeter: Mathematical formulations and algorithms for fast and robust power system simulations, 2020
- S.A. Markensteijn: Mathematical models for simulation and optimization of multi-carrier energy systems, 2021

Power flow computations on transmission networks

R. Idema, 2012

Transmission network properties:

- High-Voltage Alternating Current (AC)
- Meshed network
- Balanced network ⇒ single-phase computations

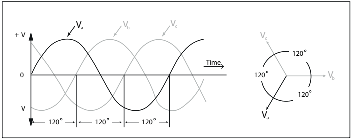


Figure 3: Single-phase voltage representation of AC power flow

Power flow computations on transmission networks

R. Idema, 2012

Traditional solvers for the steady-state power flow problem

- Newton-Raphson (NR)
  - + Quadratic convergence properties
  - - Calculation of Jacobian matrix every iteration
- Fast-Decoupled Load-Flow (FDLF)
  - + Calculation of coefficient matrices only at start
  - + Reduced memory and computational cost
  - - Sometimes fails to converge

In general, NR is preferred over FDLF because of improved robustness.

General agreement to use NR for large, complex power flow problems of the future.

Power flow computations on transmission networks

R. Idema, 2012

- Traditional Newton power flow solvers use a direct solver for the linear systems
- Iterative linear (Newton-Krylov) solvers are generally more efficient than direct solvers
  - They scale much better in problem size
  - They are well suitable for contingency analysis

Power flow computations on transmission networks

R. Idema, 2012

Our contributions:

- Development of robust Newton-Krylov methods
- Comparison of different preconditioners icw Krylov methods
- Large-scale simulations, networks with millions of buses
- Contingency analysis
- Uncertainty analysis

Power flow computations on distribution networks

B. Sereeter, 2021

Distribution network properties:

- Medium/Low-Voltage Alternating Current
- Radial network
- High R/X ratio on cables
- Unbalanced network ⇒ three-phase computations

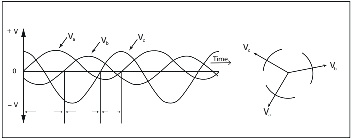


Figure 4: Three-phase voltage representation of unbalanced AC power flow

Power flow computations on distribution networks

B. Sereeter, 2021

Adapted Newton-Raphson solvers for the steady-state power flow problem on distribution networks

- Newton-Raphson using current instead of power mismatches
- More attention to the modeling of three-phase elements such as transformers, loads, and shunts

Power flow computations on distribution networks

B. Sereeter, 2021

Our contributions:

- Comparison of six formulations to solve the power flow problem on distribution networks
- Optimal power flow computations
- Case-study on the large Dutch power grid icw DNO Alliander
  - Analysis of admittance matrix: SPD properties
  - Application of several NA techniques, such as reordering

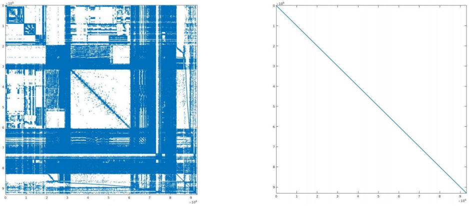

(a) *Original G_{22}* (b) *Reordered G_{22}*

Figure 6.2: *Sparsity of matrix G_{22} and reordered G_{22} using RCM.*

Power flow computations on integrated networks

M.E. Kootte, now

The changing electricity landscape not only require more efficient solvers, but also an integrated approach.

- Simulation of the power flow problem on integrated transmission-distribution networks
- Evaluation of effect of amount of distributed generation and unbalance on transmission network

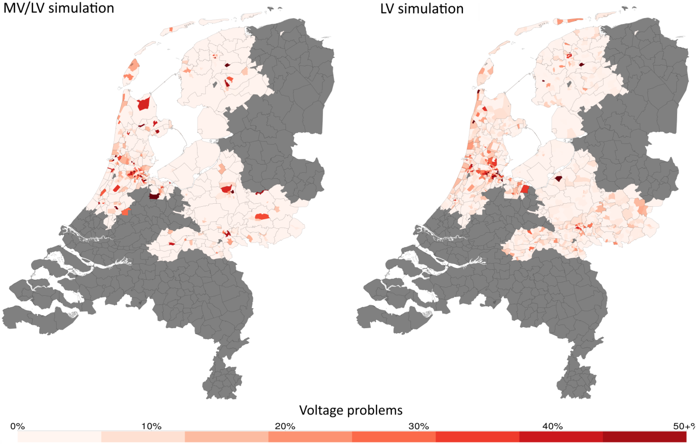
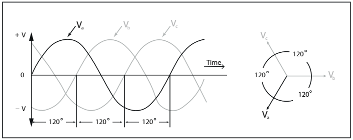


Figure 5: Different locations of Voltage problems on the MV/LV Distribution network. Left: Integrated network. Right: Separate network
Source: B. Sereeter et. al

Transmission Network

- Completely balanced



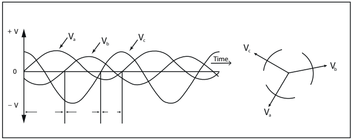
= Single-phase model

$$V_i = [V_a]_i,$$
$$S_i = [S_a]_i,$$

$$Y_{ij} = \begin{bmatrix} \underset{1\times1}{Y_{ii}^{a}} & \underset{1\times1}{Y_{ij}^{a}} \\ \underset{1\times1}{Y_{ji}^{a}} & \underset{1\times1}{Y_{jj}^{a}} \end{bmatrix}$$

Distribution Network

- Completely unbalanced



= Three-phase model

$$V_i = [V_a \ V_b \ V_c]_i^T,$$
$$S_i = [S_a \ S_b \ S_c]_i^T,$$

$$Y_{ij} = \begin{bmatrix} \underset{3\times3}{Y_{ii}^{abc}} & \underset{3\times3}{Y_{ij}^{abc}} \\ \underset{3\times3}{Y_{ji}^{abc}} & \underset{3\times3}{Y_{jj}^{abc}} \end{bmatrix}$$

Power flow computations on integrated networks

M.E. Kootte, now

Our contributions:

Comparison of four different methods to integrate networks

- **Unified methods**
  - **Homogeneous** three-phase network model
  - **Hybrid** single-phase/three-phase network model
- **Splitting methods**
  - **Homogeneous** three-phase network model
  - **Hybrid** single-phase/three-phase network model

Advantages and Disadvantages

Homogeneous networks

+ Intuitive physical approach
- Large system

Hybrid networks

+ Respects the simplifications of the Transmission Network
- Complicated

Unified methods

+ One system
- Should solve the entire system with the same NR-method (NR-P or NR-TCIM).

Splitting methods

+ No need to share network information between System Operators
- Extra iterative scheme

Multi-Carrier Energy Systems

A. Markensteijn, 2021

- Multi-carrier energy systems (MES) consist of several energy carriers
- **Gas, power (electricity), heat,** cooling, biogas, transport, etc.
- Coupled through CHP's, gas-fired generators, boilers, etc.
- Steady-state load flow simulations required for design and operation.

Multi-Carrier Energy Systems

A. Markensteijn, 2021

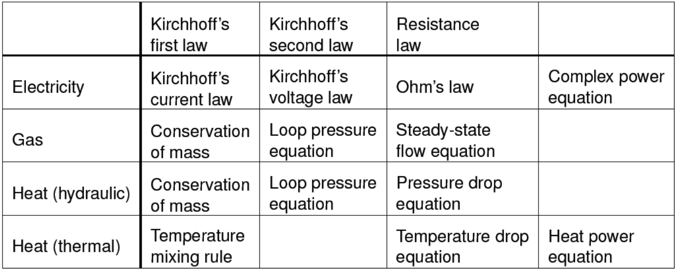
	Node	**Link**	**Terminal link**
Gas	pressure p	flow q	flow q
Electricity	voltage V	current I	current I power S
Heat	pressure p supply temp. T_s return temp. T_r	flow m	flow m outflow temp. T_o heat power ϕ

Table 1: Network parameters

Multi-Carrier Energy Systems

A. Markensteijn, 2021

Network equations



	Kirchhoff's first law	Kirchhoff's second law	Resistance law	
Electricity	Kirchhoff's current law	Kirchhoff's voltage law	Ohm's law	Complex power equation
Gas	Conservation of mass	Loop pressure equation	Steady-state flow equation	
Heat (hydraulic)	Conservation of mass	Loop pressure equation	Pressure drop equation	
Heat (thermal)	Temperature mixing rule		Temperature drop equation	Heat power equation

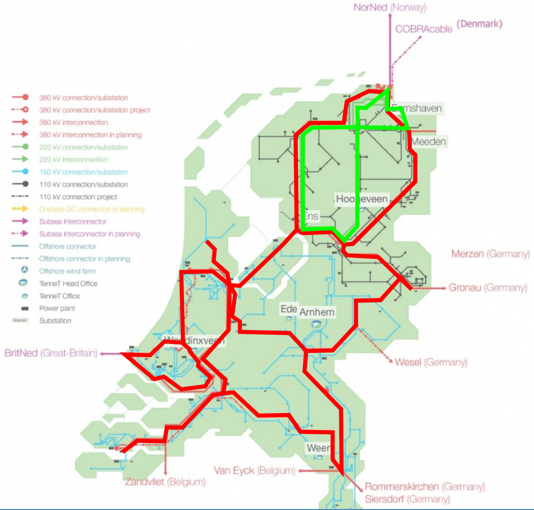
Scalability of Multi-Carrier Networks

B. Nguyen, now

- Modelling and simulating integrated energy networks
- Graph-based model for integrated energy networks based on Anne Markensteijn's PhD thesis (reference to thesis)
- Research on **solvability** and **scalability** of this graph-based model

Automating AC power flow computations on the Dutch grid

S. Chipli, 2021, Tennet



Results

Transmission networks

Results

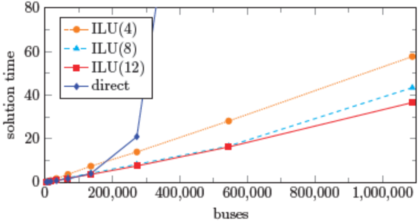


Figure 9.7: Power flow with J_i based preconditioning

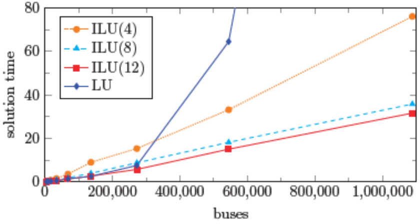
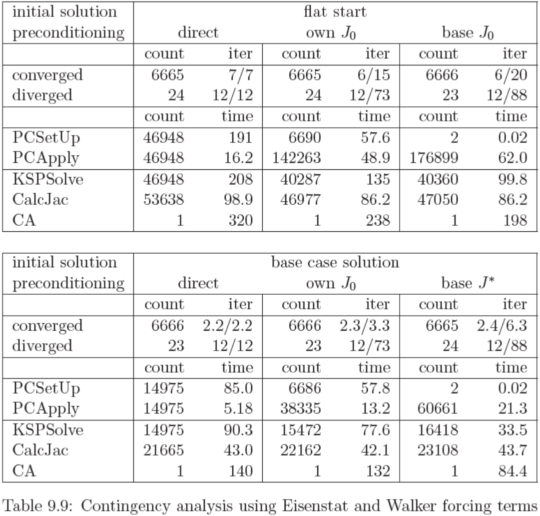


Figure 9.8: Power flow with J_0 based preconditioning

Numerical experiments with several iterative (inexact Newton-Krylov) power flow solver compared to direct method, solving the full nonlinear power flow problem.

sults



initial solution	flat start					
preconditioning	direct		own J_0		base J_0	
	count	iter	count	iter	count	iter
converged	6665	7/7	6665	6/15	6666	6/20
diverged	24	12/12	24	12/73	23	12/88
	count	time	count	time	count	time
PCSetUp	46948	191	6690	57.6	2	0.02
PCApply	46948	16.2	142263	48.9	176899	62.0
KSPSolve	46948	208	40287	135	40360	99.8
CalcJac	53638	98.9	46977	86.2	47050	86.2
CA	1	320	1	238	1	198

initial solution	base case solution					
preconditioning	direct		own J_0		base J^*	
	count	iter	count	iter	count	iter
converged	6666	2.2/2.2	6666	2.3/3.3	6665	2.4/6.3
diverged	23	12/12	23	12/73	24	12/88
	count	time	count	time	count	time
PCSetUp	14975	85.0	6686	57.8	2	0.02
PCApply	14975	5.18	38335	13.2	60661	21.3
KSPSolve	14975	90.3	15472	77.6	16418	33.5
CalcJac	21665	43.0	22162	42.1	23108	43.7
CA	1	140	1	132	1	84.4

Table 9.9: Contingency analysis using Eisenstat and Walker forcing terms

Results of numerical experiments with the Newton-Krylov power flow solver, applied to the contingency analysis problem. The UCTE winter 2008 study model is used as base case. The contingency cases consist of the base case with a single pair of buses (that were connected in the base case) disconnected, simulating branch outages.

Results

Case study of large Dutch power grid, supported by DNO Alliander

- Simulation of the MV/LV grid
- Focus on voltage problems
- Goal of the model: support large-scale investment policy decisions, such as:
    - How many transformers will be overloaded the next 30 years?
    - In which are of the country should more engineers be recruited for cable replacement?
- Test-case: network of Alliander
    - 1/3 of total Dutch power grid
    - 80,000 km cables and three million customers
    - 24 million buses

Results

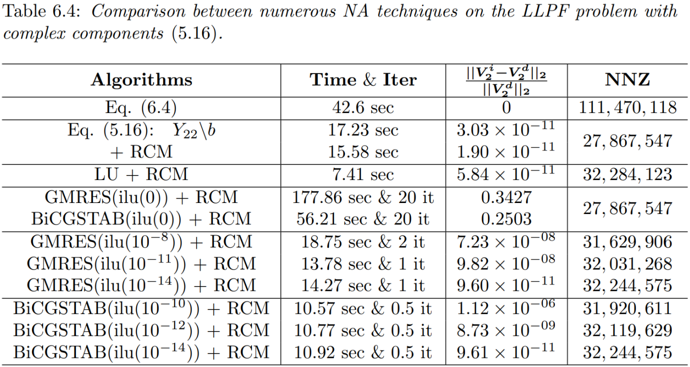


Table 6.4: *Comparison between numerous NA techniques on the LLPF problem with complex components* (5.16).

Algorithms	**Time & Iter**	$\frac{\|\|V_2^i - V_2^d\|\|_2}{\|\|V_2^d\|\|_2}$	**NNZ**
Eq. (6.4)	42.6 sec	0	111, 470, 118
Eq. (5.16): $Y_{22} \backslash b$	17.23 sec	3.03×10^{-11}	27, 867, 547
+ RCM	15.58 sec	1.90×10^{-11}	
LU + RCM	7.41 sec	5.84×10^{-11}	32, 284, 123
GMRES(ilu(0)) + RCM	177.86 sec & 20 it	0.3427	27, 867, 547
BiCGSTAB(ilu(0)) + RCM	56.21 sec & 20 it	0.2503	
GMRES(ilu(10^{-8})) + RCM	18.75 sec & 2 it	7.23×10^{-08}	31, 629, 906
GMRES(ilu(10^{-11})) + RCM	13.78 sec & 1 it	9.82×10^{-08}	32, 031, 268
GMRES(ilu(10^{-14})) + RCM	14.27 sec & 1 it	9.60×10^{-11}	32, 244, 575
BiCGSTAB(ilu(10^{-10})) + RCM	10.57 sec & 0.5 it	1.12×10^{-06}	31, 920, 611
BiCGSTAB(ilu(10^{-12})) + RCM	10.77 sec & 0.5 it	8.73×10^{-09}	32, 119, 629
BiCGSTAB(ilu(10^{-14})) + RCM	10.92 sec & 0.5 it	9.61×10^{-11}	32, 244, 575

Results of several NA techniques applied to the Dutch MV/LV grid.

Results

Integrated networks

Results

	IC		*MFS-hybrid*			
	its	*CPU*	I_{MFS}	I_T	I_D	*CPU*
Test case	#	*sec*	#	#	#	*sec*
T9-3D13 (7-9)	3	0.020	3	4	**5**	1.494
D33-2D37 (30-31)	**10**	0.048	**13**	**5**	**6**	4.974
T118-5D123 (108-112)	4	0.060	3	**7**	**4**	1.691
T3120-10D8500 (2700-2709)	**5**	3.015	3	6	**4**	12.51

	F3P		*MFS-homo*			
	its	*CPU*	I_{MFS}	I_T	I_D	*CPU*
Test case	#	*sec*	#	#	#	*sec*
T9-3D13 (7-9)	3	0.017	3	4	**5**	1.791
D33-2D37 (30-31)	**12**	0.065	**13**	**5**	**6**	6.833
T118-5D123 (108-112)	4	0.073	3	4	**4**	1.973
T3120-10D8500 (2700-2709)	4	3.675	3	6	**4**	14.53

Comparison on number of iterations and CPU-time of the integration methods, applied to four integrated test-cases.

Results

	Original					*Distr. Generation*				
	PV	*IC*	*MFS-hybrid*			PV	*IC*	*MFS-hybrid*		
test case	*buses*	I_U	I_{MFS}	I_T	I_D	*buses*	I_U	I_{MFS}	I_T	I_D
T9-D13	0	3	3	4	4	4	3	**5**	4	**5**
D33-D37	1	5	8	4	5	5	5	**9**	4	5
T118-D123	0	4	3	7	5	5	4	**6**	7	5
T3120-D8500	0	4	3	6	5	5	4	3	6	**4**

	Original					*Distr. Generation*				
	PV	*F3P*	*MFS-homo*			PV	*F3P*	*MFS-homo*		
test case	*buses*	I_U	I_{MFS}	I_T	I_D	*buses*	I_U	I_{MFS}	I_T	I_D
T9-D13	0	3	3	4	4	4	3	**6**	4	**5**
D33-D37	1	5	8	4	5	5	5	**9**	4	5
T118-D123	0	4	3	4	5	5	4	**6**	4	5
T3120-D8500	0	4	3	6	5	5	4	3	6	**4**

Comparison of the influence of the number of PV buses (Distributed Generation) on number of iterations. The left column contains the standard amount of PV buses and the right column contains additional number of PV buses.

Conclusions and Current Developments

- Expertise in running fast and robust simulations of the power flow problem
- Application of NA techniques to very large problems have proven to speed-up the calculations
- The computations on the large Dutch MV/LV grid, showed that these techniques work well on a real network
- We hope to prove the same on the very large HV grid, in corporation with TSO Tennet
- We continue the research into very large integrated networks and multi-carrier energy networks

References

Thank you

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