Resolving Divergence: The First Multigrid Scheme for the Highly Indefinite Helmholtz Equation Using Classical Components.

Vandana Dwarka
Numerical Analysis, Delft University of Technology
v.n.s.r.dwarka@tudelft.nl

Cornelis Vuik*
Numerical Analysis, Delft University of Technology
c.vuik@tudelft.nl

In this talk, we present the first stand-alone classical multigrid solver for the highly indefinite 2D Helmholtz equation with constant costs per iteration, addressing a longstanding open problem in numerical analysis [1]. Our work covers both large constant and non-constant wavenumbers up to $k = 500$ in 2D.

We obtain a full $V-$ and $W-$cycle without any level-dependent restrictions. Another powerful feature is that it can be combined with the computationally cheap weighted Jacobi smoother. The key novelty lies in the use of higher-order inter-grid transfer operators [2]. When combined with coarsening on the Complex Shifted Laplacian, rather than the original Helmholtz operator, our solver is $h-$independent and scales linearly with the wavenumber k. If we use GMRES(3) smoothing we obtain $k-$ independent convergence, and can coarsen on the original Helmholtz operator, as long as the higher-order transfer operators are used.

This work opens doors to study robustness of contemporary solvers, such as machine learning solvers inspired by multigrid components, without adding to the black-box complexity.

*Speaker

References

[1] V. Dwarka, C. Vuik, Stand-alone Multigrid for Helmholtz Revisited: Towards Convergence Using Standard Components, *arXiv: 2308.13476*, 2024

[2] V. Dwarka, C. Vuik, Scalable Convergence Using Two-Level Deflation Preconditioning for the Helmholtz Equation, *SIAM Journal on Scientific Computing*, vol. 42, no. 2, pp. A901–A928, 2020.