Efficient Computation of Flow with Cavitation with Compressible Pressure Correction

Objective: Efficient computation of flow with cavitation with simple engineering model

Hydrodynamic Cavitation

Cavitation is the formation of vapor filled bubbles when due to the dynamics of the flow the local pressure in a liquid falls below the vapor pressure. Unsteady cavitation on lifting surfaces gives rise to:

- loss of performance
- noise
- vibrations
- erosion

Model

We use the Homogeneous Equilibrium Model for two-phase liquid/vapor flow. In the HEM the following assumptions are made:

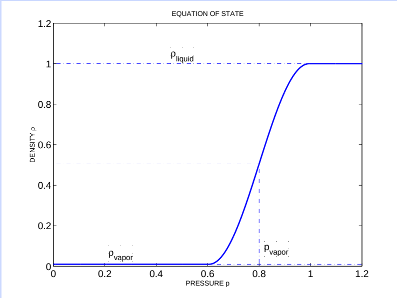
- Both phases are in thermodynamic equilibrium.
- Both phases are at saturation.
- The velocity slip between both phases is negligible.

Governing equations

The HEM for inviscid isothermal flow is described by the following mass and momentum conservation equation and a barotropic equation of state for the water/vapor mixture:

$$\frac{\partial \rho}{\partial t} + (\rho u_\alpha)_{,\alpha} = 0,$$
$$\frac{\partial \rho u_\alpha}{\partial t} + (\rho u_\alpha u_\beta)_{,\beta} = -p_{,\alpha},$$
$$\rho = \rho(p).$$

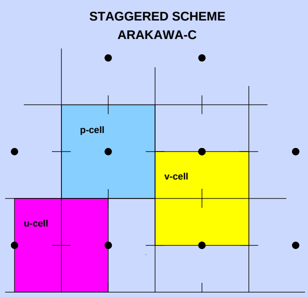
The system is unconditionally hyperbolic and the mixture equation of state is nonlinear and nonconvex:



The pure liquid and pure vapor phase are nearly incompressible, while the mixture phase is highly compressible. As a result the Mach number in the flow varies in the range $0 \leq \mathrm{Ma} \leq 30$. This calls for a Mach uniform method: A method that has accuracy and efficiency nearly uniform in the Mach number[1]. A staggered discretisation in combination with a compressible pressure correction solution method has this desirable property.

Discretisation

We use a finite volume discretisation, according to the staggered ARAKAWA-C arrangement:



Compressible pressure correction

The pressure correction time-stepping method consists of the following stages:
First the momentum equation is advanced in time, using the pressure at t^n.

$$\frac{m^*_\alpha - m^n_\alpha}{\delta t} + \left(u^n_\beta m^*_\alpha\right)_{,\beta} = -p^n_{,\alpha},$$

The following pressure correction is postulated:

$$m^{n+1}_\alpha = m^*_\alpha - \delta t \delta p_{,\alpha}, \quad \delta p = \left(p^{n+1} - p^n\right),$$

and substituted in the mass conservation equation:

$$\frac{\rho(p^n + \delta p) - \rho^n}{\delta t} + \left(m^*_\alpha - \delta t \delta p_{,\alpha}\right)_{,\alpha} = 0.$$

This nonlinear equation for the pressure correction is solved with a nonlinear Gauss-Seidel algorithm, accelerated by GMRES [2].
Finally pressure and momentum are updated:

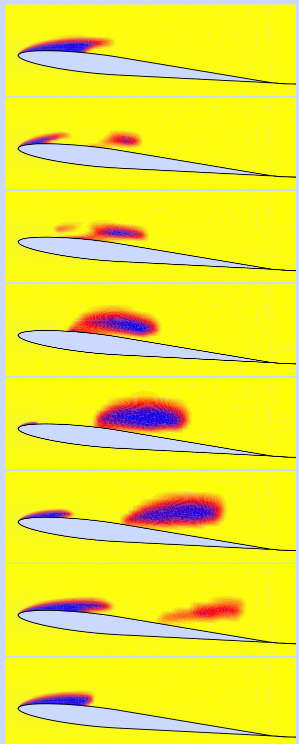
$$p^{n+1} = p^n + \delta p,$$
$$m^{n+1}_\alpha = m^*_\alpha - \delta t \delta p_{,\alpha}.$$

Results

Simulation of cavitation on EN-hydrofoil, cavitation number

$$\sigma = \frac{p_\infty - p_{\mathrm{vapor}}}{\frac{1}{2}\rho_\infty U^2_\infty} = 1.2, \qquad (1)$$

angle of attack $\alpha = 6.2^o$.



References

(1) H. Bijl & P. Wesseling J. Comp .Phys. 141:153-173 1998.
(2) D.R. van der Heul, C. Vuik & P. Wesseling Comput Visual Sci 2:63-68 1999.

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