GPU Implementation of a Krylov solver Preconditioned by a Shifted Laplace Multigrid Method for Helmholtz Equation

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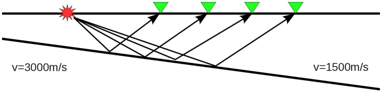
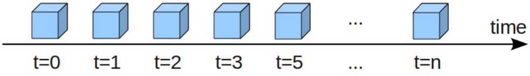
Outline

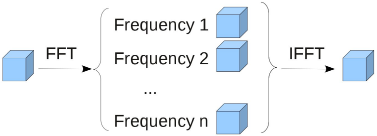
Geophysical Background



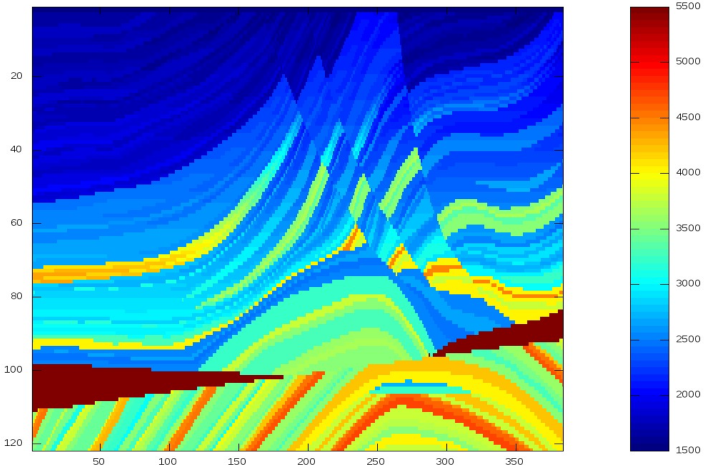
Time Domain



Frequency Domain



Marmousi 2D



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Motivation

Problem Description

The Helmholtz equation in three dimensions is considered

$$-\frac{\partial^2\phi}{\partial x^2} - \frac{\partial^2\phi}{\partial y^2} - \frac{\partial^2\phi}{\partial z^2} - (1-\alpha i)k^2\phi = g, \qquad (1)$$

with non-reflecting first-order radiation boundary conditions

$$\left(-\frac{\partial}{\partial\eta} - ik\right)\phi = 0, \qquad (2)$$

where $\phi = \phi(x, y, z)$ is the wave pressure field, $k = k(x, y, z)$ is the wavenumber, α is the damping coefficient, $g = g(x, y, z)$ is the source term.

Solver

Bi-CGSTAB preconditioned by shifted Laplace multigrid method. Equation solved in preconditioner is

$$-\frac{\partial^2\phi}{\partial x^2}-\frac{\partial^2\phi}{\partial y^2}-\frac{\partial^2\phi}{\partial z^2}-(\beta_1-\beta_2 i)k^2\phi=g, \tag{3}$$

$\beta_1, \beta_2 \in \mathbb{R}$, with the same boundary conditions as the original problem.

Multigrid components:

- Matrix-dependent prolongation (2D: de Zeeuw, 1990, 3D: Zhebel, 2006)
- Standard restriction
- Multi-coloured Gauss-Seidel as a smoother

Preconditioner is computed in single precision.

Next subsection

Little-Green Machine

20 general computing nodes

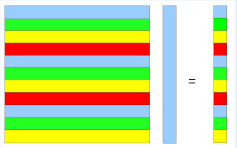
- 2 Intel quadcore E5620
- 24 GB RAM
- 2 TB disk
- 2 NVIDIA GTX480

Founded by

- University of Leiden
- NWO
- TU Delft
- KNMI

Sparse Matrix-Vector Multiplication

$$x_i = \sum_{j=1}^{N} A_{ij} b_j$$

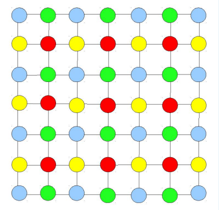


Size	Time 8-cores (ms)	Time GPU (ms)	CPU/GPU
10,000	1.5	0.1	**15**
100,000	8.4	0.1	**84**
1,000,000	64.9	1.1	**59**
5,000,000	328.1	5.4	**61**
20,000,000	1334.6	22	**61**

Gauss-Seidel Smoother

For each color compute

$$x_i = (1-\omega)x_i + \omega\left(b_i - \frac{\sum_j A_{ij}x_j}{A_{ii}}\right),$$

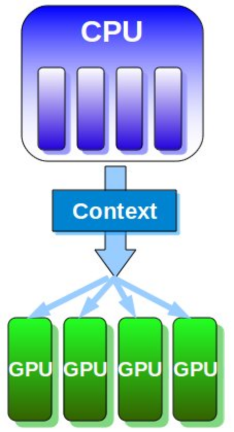


Size	Time 8-cores (ms)	Time GPU (ms)	CPU/GPU
10,000	4	0.6	**7**
100,000	23.4	0.8	**29**
1,000,000	164.5	2.5	**66**
5,000,000	625.9	10.3	**61**
20,000,000	3733.9	39.4	**95**

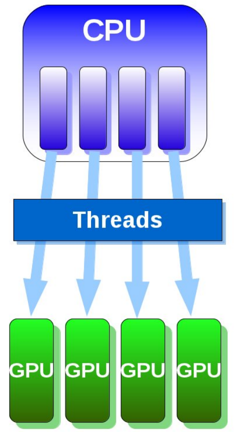
Next subsection

Multi-GPU

Pushing context

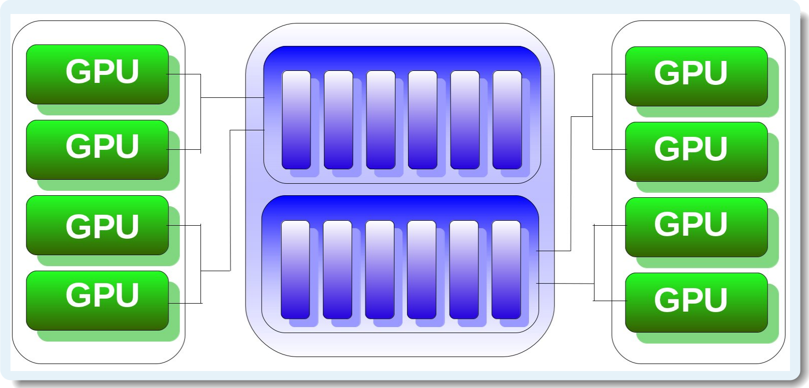


Multi-threading



NVidia Computer

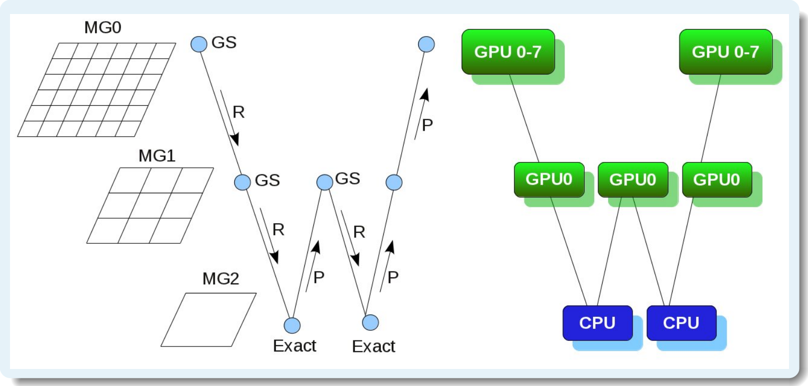
8 GPUs, each GPU has 448 cores, 3 GB RAM
12 cores (2 Westmeres), 48 GB RAM



Multi-GPU Approach

1. Data-parallel approach (e.g. vector operations on multi-GPU)
    1. Relatively easy to implement
    2. CPU$\rightarrow$GPU$\rightarrow$CPU data transfer
2. Split of the algorithm (e.g. solver on one GPU, preconditioner on the another one)
    1. No or little data transfers
    2. Find the best way to split the algorithm
3. Domain-Decomposition approach (e.g. each domain on a different GPU)
    1. Exchange of halos (still data transfer)
    2. Can affect convergence of the preconditioned method

Multigrid on Multi-GPU

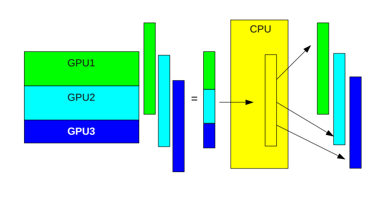


Multi-GPU Issues

- Limited GPU memory size so need multiple GPUs for large problems.
- Efficient memory reusage to avoid allocation/deallocation, e.g. pool of GPU-vectors.
- Limit communications CPU$\rightarrow$GPU and GPU$\rightarrow$CPU.
- Each GPU need separate texture reference.
- Cublas vectors limited to 512 MB.

Speedups for Sparse MVM

Row-wise splitting



Speedups single GPU versus Multi-GPU

Size	12-cores/1 GPU	12-cores/8-GPUs
1,000,000	51.9	10.3
30,000,000	47.4	10.4
100,000,000	-	6.8

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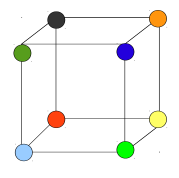
Gauss-Seidel Smoother

For each color

$$x_i = (1-\omega)x_i + \omega(b_i - \frac{\sum_j A_{ij}x_j}{A_{ii}}),$$



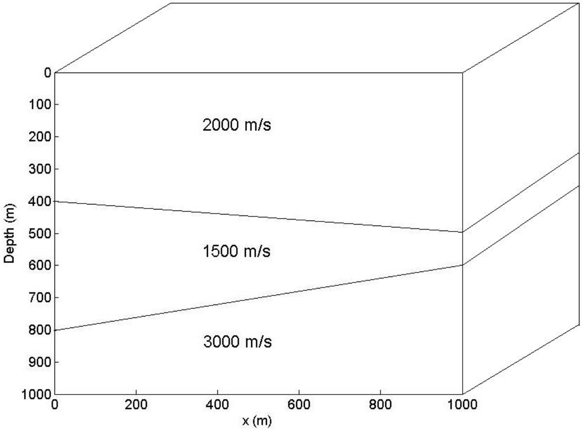
Size	12-cores/1 GPU	12-cores/8-GPUs
5,000,000	16.5	5.2
30,000,000	89.1	6.1
100,000,000	-	4.4

Bi-CGSTAB

Timings for Bi-CGSTAB, single precision

n	12-cores	1 GPU	**Speedup**	8-GPU	**Speedup**
5,000,000	24 s	0.8 s	**29.8**	2.3 s	**10.5**
15,000,000	82 s	2 s	**38.1**	5.8 s	**14.2**
100,000,000	395 s	-	-	28.6 s	**13.8**

Wedge problem



Bi-CGSTAB preconditioned by shifted Laplace multigrid

Problem size $350 \times 350 \times 350 \approx 43,000,000$ unknowns

	Bi-CGSTAB (DP)	Preconditioner (SP)	Total
12-cores	94 s	690 s	784 s
1 GPU	13 s	47 s	60 s
Speedup CPU/GPU	**7.2**	**14.7**	**13.1**
8 GPUs	83 s	86 s	169 s
Speedup CPU/GPUs	**1.1**	**7.9**	**4.6**
2 GPUs+split	12 s	38 s	50 s
Speedup CPU/GPU	**7.8**	**18.2**	**15.5**

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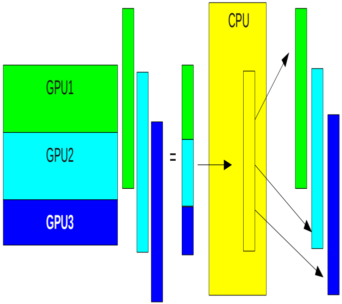
Acknowledgments

- NVIDIA for access to the latest many-core-multi-GPU architecture
- TU Delft for access to the cluster

Thank you for your attention!

Sparse MVM on Multi-GPU

Row-wise splitting



Column-wise splitting